Modelling Athletic Training and Performance:
A Hybrid Artificial Neural Network Ensemble Approach

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ABSTRACT

How an athlete trains is the most influential factor in the myriad of variables which determine performance in endurance sports. The questions that face all coaches and athletes is how should I train in order to: a) produce peak performance; and b) produce a peak when desired? The aim of this research was to develop a model which enables the accurate prediction of cycling performance using field derived training and racing data. A number of techniques were developed to pre-process raw sensor data; a novel training load quantification technique (PTRIMP) was developed to encapsulate duration and intensity of training load in one metric; the shape of training microcycles was analysed using a data mining approach whereby the time series of PTRIMP was transformed into a symbolic representation; Heart Rate Variability (HRV) indices were calculated from daily orthostatic tests; and a performance quantification technique allowing performance to be calculated from power data obtained from any race / training session where the athlete makes a 100% effort was developed. This data allowed a model to be created that can assist an athlete in manipulating training load and consequently manage fatigue levels, such that they arrive at a competition in a state from which a high performance is likely, and a low performance is unlikely.

Artificial Neural Networks (ANN) were the key modelling technique used to model the relationship between training and performance. A hybrid ANN ensemble model – named HANNEM - was developed. To avoid overfitting of the model, the regularisation technique of bagging and adding noise to each bag was used. A combination model was created by using the output from a linear statistical model as an input for the neural network model, resulting in improved model accuracy. In the final modelling step, an ensemble of neural network models was created, resulting in an improvement of predictive performance over a single model. Model fit was evaluated using leave-one-out testing. Three datasets consisting of longitudinal training and racing power data obtained from three elite cyclists were used to validate the model ($R^2 = 0.51, 0.64$ and $0.78$). These moderate to strong results are in a similar range to those reported in other modelling studies. ANN models on noisy, sparse datasets are prone to
overfitting. A series of experiments were performed on synthetic data to investigate overfitting
issues and validate the use of a bagged ensemble of ANNs on such datasets. Interestingly,
deliberately overtrained individual ANNs resulted in improved ensemble performance for
synthetic datasets with low to moderate data error rates. The novel modelling approach
employed overcame the difficulties associated with using field-derived model inputs, which will
allow the model to be implemented in the real world environment of professional sport. The
model is a practical tool for the planning of training to maximise competition performance.
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CHAPTER 1 - INTRODUCTION

Performance in endurance sports is determined by a myriad of factors - from physiological and psychological parameters to technological and environmental factors. Extensive research effort has gone into attempting to understand the factors that influence performance and the relationship between them. The consensus is that by understanding the factors that influence performance they can be manipulated to produce peak performances when desired.

From the 1970’s onwards, numerous studies have focused on modelling physiological responses to training input using linear mathematical concepts (Banister, Calvert, Savage, & Bach, 1975; Banister, Carter, & Zarkadas, 1999; Busso, 2003; Busso, Carasso, & Lacour, 1991). Unfortunately, however, linear modelling approaches are not likely to be the most appropriate choice for such studies. There is common agreement among researchers that an appropriate model for modelling responses to training needs to consider the complexity and non-linear nature of athletic performance and its response to training (Ganter, Witte, & Edelmann-Nusser, 2011).

There are currently a number of software tools available on the market (CyclingPeaks WKO+, RaceDay Performance Predictor™), which calculate an arbitrary measure of training load to model training and performance. This approach follows on from the TRIMP (training impulse) method proposed by Bannister and Calvert (Banister & Calvert, 1980).

Some more recent researchers have looked at using nonlinear methods - namely neural networks - to model training response relationships (Hohmann, Edelmann-Nusses, & Hennerberg, 2001). The results from this early work is promising, with neural network models outperforming conventional linear models. However, these techniques have not yet been used to model training response relationships in cycling.

Performance modelling for cycling presents a number of unique challenges. These include:
• Difficulty in objectively assessing performance. Performance in road racing involves many different facets, including a strategic component. The rider with the highest average power, for example, is not necessarily the winner of the race (Martin et al., 2001).

• Difficulty in objectively quantifying training load. Cyclists undergo many different types of training sessions, with varying intensities and durations. Interval sessions have varying work to rest ratios, and varying numbers of repeats. These factors affect how fatiguing the training is, but do not indicate how training load, fatigue and fitness should be modelled.

For the purposes of this research, an elite athlete was defined as an athlete who has participated in a World Championship, World Cup, Olympic Games or Commonwealth Games in the last 12 months.

Studying the training and performance of elite athletes involves specific challenges. By definition, elite athletes are rare. Thus investigations inevitably involve dealing with few subjects and low statistical power. Due to the unique requirements of studying elite athletes, researchers in this area have little interest in generalising results to wider, non-elite populations (Sands, 2008).

This review will look at the significant research that has been conducted in modelling training input and performance output in the sports domain, and will examine the potential role of computer science techniques in solving some of the identified issues in performance modelling.

1.1. Objectives

This thesis presents and evaluates a novel hybrid neural network ensemble model called Hybrid Artificial Neural Network Ensemble Model (HANNEM). HANNEM has been developed during
this period of study to provide a system that will enable cycling coaches and athletes to manipulate training loads to prepare cyclists for peak performances in competitions.

The proposed model relies on the development of techniques to quantify training and racing data collected from elite cyclists in the field. Training and performance datasets are inevitably small in this domain. The HANNEM approach has the ability to extract patterns from the small, noisy and incomplete datasets that are typical of the domain. The ultimate aim of this research is to develop a model that predicts real world performance from data obtained in the field with sufficient accuracy to allow the model to be used as a tool to provide decision support when developing a training plan.

In summary, the objectives of this thesis are to:

- Develop techniques to quantify training load from data collected by elite cyclists in the field;
- Develop techniques to quantify performance from field-derived data;
- Develop a model to predict real world cycling performance, using quantified training as an input; and
- Achieve sufficient model accuracy to enable the model to function as a decision support tool for cycling coaches and athletes to plan the manipulation of training loads to prepare cyclists for peak performances in competitions.

1.2. The Problem

In 1996, Banister, Morton and Fitz-Clarke wrote:

“...there is no formal training theory developed for exercise such as [one to identify] the type, quantity or pattern of a training stimulus necessary to produce a prescribed measured effect and the field remains empirical and fallible.”
This topic has received significant research attention; however the situation remains much the same today. The development of a model that will model the relationship between training and performance such that performance output can be predicted from training input and other known and measurable factors still remains as a “holy grail” of exercise physiology research. Figure 1-1 provides a schematic of the components of the problem.

1.3. Quantification of Training Load

When athletes train they apply a series of stimuli which alters their physiological status. The body’s adaptations are moderated by the volume, intensity and duration of the stimuli (Bompa & Carrera, 2005). Exercise physiologists have long sought to measure the training stimulus of athletes so that the relationship between training and adaptation, and training and performance can be examined. Training load can be thought of as the encapsulation of the frequency, intensity and volume of training.

In order to model training input and performance output, training load must be quantified in some way so it can be used as a systems model input. There has been great difficulty in finding
a way to effectively quantify training load using a single term (Foster et al., 2001). Taha and Thomas (2003) suggest that the parameters of intensity, frequency and duration need to be taken into account.

Training-induced adaptations are specific to the type of training stimulus applied. Busso and Thomas (2006) conclude that the specificity of the activity also needs to be considered, possibly by multifactorial models that don’t simplify training loads into a single variable. This approach would, however, increase model complexity significantly.

It is proposed that to accurately reflect the physiological load of training, the quantification approach needs to consider:

- Volume (duration/distance); and
- Intensity.

Considerations which would be of possible benefit include the pattern of load (higher weights for rapid accelerations or at the end of sustained high intensity) and the specificity of training.

As part of this research project, a load quantification method will be developed that will consider volume (duration/distance) and intensity. The possibility of including pattern of load and specificity in the method will also be considered.

There are a number of different measurements that can be used to gain a picture of exercise intensity. The most commonly used for cycling are heart rate and power data.

**Heart rate data**

The body adjusts heart rate (HR) so that cardiac output is sufficient to deliver adequate blood, oxygen and nutrients to working muscles. As such, heart rate is an indicator of physiological load, and has been traditionally used at the basis for load quantification methods (e.g. Banister, 1991).
The occurrence of cardiac drift (the continuous increase in heart rate that usually occurs during prolonged moderate-intensity exercise (Jeukendrup, 2002)) is a common criticism of using HR to measure training load (e.g. (Skiba, 2008). There is a delay between changes in exercise intensity and the corresponding changes in heart rate. Heart rate is also affected by factors unrelated to exercise intensity, including the nervous system and variables such as hydration, temperature and caffeine.

**Power data**

Power is a direct reflection of exercise intensity, while heart rate is a physiological response to the exercise intensity (Jeukendrup, 2002). Some of the newer load quantification techniques developed use power as the input, rather than the traditional heart rate.

Power is the amount of energy generated per unit of time. In cycling it is the amount of energy transferred to the pedals (in watts) per second. SRM power monitors (Schoberer Rad Messtechnik, Jülich, Germany) are commonly used to measure power in cycling. The SRM system provides reliable measurements of power (Gardner et al., 2004), as well as measurements of cadence, speed and distance.

The SRM system is a crank-based device, which measures the deformation of the crankset when torque is applied, by the use of strain gauges. The measured torque and cadence values are digitised, and sent to a microcomputer mounted on the handlebars. The microcomputer calculates the power using the average torque over each complete pedal revolution, and the cadence (Vogt et al., 2006).

Power output is a direct reflection of exercise intensity, and is not influenced by other variables in the way that heart rate is. This makes power data a good candidate to be used as the basis for training load quantification techniques. Training Stress Score (Coggan, n.d.(a)) and BikeScore (Skiba, 2007) are two widely used techniques which use power data.
1.4. Systems Modelling

Researchers who have wished to predict athletic performance have commonly used systems modelling approaches (e.g. Banister, 1991). Load quantification techniques have been a fundamental aspect of systems modelling approaches. These approaches attempt to predict an output (in this case performance) from input(s) (most commonly training load).

Systems theory creates a mathematical model as an abstraction of a dynamic process. The system has at least one input, and one output, which are related by a mathematical representation called a transfer function (Busso & Thomas, 2006).

Banister (1991) proposed a systems model to describe the relationship between training and performance. He argues that conceptually, repeated training bouts contribute to two factors - fitness and fatigue. At any point in time, this relationship can be expressed as the formula: Performance = Fitness – Fatigue. This type of model is commonly referred to as an impulse-response model. Various authors (e.g. Busso, 2003; Coggan, n.d .(c); Bruckner, 2006) have subsequently added different adaptations to the original impulse-response model proposed by Banister.

The impulse-response models generally suffer from a number of limitations. A systems model inevitably requires some level of simplification of the underlying complex system being modelled. Busso and Thomas (2006) argue that the impulse-response models are overly simplified. In addition, the models have not been linked to underlying physiological processes (Taha & Thomas, 2003). A number of researchers have also criticised the inability of the models to accurately predict future performance (Taha & Thomas, 2003; Busso & Thomas, 2006).

Some of the impulse-response models (e.g. Bannister, 1991; Busso, 2003; Coggan (n.d. (c))) assume that a greater amount of training leads to better performance. Previous studies have reported that the impact of training loads on performance has an upper threshold, beyond
which training doesn’t elicit further adaptations (Fry et al., 1992; Morton, 1997 cited in Hellard et al., 2006).

The impulse-response models of Banister (1991), Busso (2003), Bruckner (2006), and to a lesser extent Coggan (n.d. (c)) all rely on frequent performance measure to provide input to the models. Frequent performance tests are often not practical for elite athletes during the competition season. In fact the majority of the studies conducted on these models have been on either untrained subjects, or trained but not elite athletes.

The identified problems with the existing impulse-response models limit their usefulness in predicting performance for elite athletes. Accurately predicting performance from training remains an unsolved challenge; one of the holy grails of sport science.

1.5. Machine Learning for Modelling

The nature of the problem of modelling complex systems (such as biological systems, ecosystems, or in this case an athlete ‘ecosystem’) requires an approach where the essential features of the complex problem are modelled such that system behaviour can be modelled, even under noisy conditions. A paradigm shift has occurred in the modelling to training responses, as it has become clear that biological adaptation is a complex and non-linear dynamical system (Ganter, Witte & Edelmann-Nusser, 2011). Researchers have moved away from the use of linear concepts towards individual non-linear process-oriented concepts (Pfeiffer & Hohmann, 2011).

Machine learning offers a range of non-linear modelling tools. The non-linear modelling techniques of artificial neural networks (ANNs) have been suggested as potentially appropriate for use in modelling the relationship between training and performance (Hellard et al., 2006). Neural networks are flexible, adaptive learning systems that can find patterns in observed data enabling the development of nonlinear systems models that can make reliable predictions (Samarasinghe, 2006).
ANNs have been used to model training and performance in swimming by Edelmann-Nusser, Hohmann, Henneberg (2002). Pfeiffer and Hohmann (2011) used the same dataset to repeat the findings. Despite the encouraging results reported in both studies, due to the very small dataset used to train the neural network, the resultant network is likely to be overtrained (refer to Chapter 4 for further details on ANNs and overtraining).

1.6. Summary

A workable model for predicting athletic performance needs to be based on simplified abstractions of the underlying complex physiological structures. An important question is to decide just how much of the underlying structure should be incorporated into the model (Taha & Thomas, 2003). A balance needs to be struck between complexity of the model and ease of use.

Busso and Thomas (2006) concluded that existing models developed to predict athletic performance are not accurate enough to be used by a particular athlete to monitor their training. We wish to arrive at a model that offers the greatest accuracy for the least cost. The cost of the model includes both computational costs as well as the amount of effort required by the athlete in order to utilise the model. It is proposed that by using machine learning techniques in the modelling process, a more robust model can be developed – one that can overcome some of the identified limitations in the existing models.

1.7. Thesis Outline

The remainder of this thesis is organised as follows:

Chapter 2 introduces further background material by introducing important sport science concepts that underlie the principles of training and performance. These principles form the foundation for the modelling work that is the focus of this thesis. A critical discussion identifies
gaps in the current sport science knowledge which impact the modelling of training and performance.

Chapter 3 lays out the important criteria for effective modelling of training and performance. It introduces the primary modelling approaches that have been used to model the training performance relationship, and evaluates the approaches in light of the criteria developed.

Chapter 4 considers the longstanding problem of using knowledge discovery techniques to improve elite sporting performance. It identifies the issues surrounding the use of knowledge discovery techniques in this domain with a view towards developing a methodology for the generation of an improved prediction model for the training and performance domain.

Chapter 5 discusses the development and initial validation of a novel training load quantification technique called PTRIMP (Power TRaining IMPulse), which allows training load to be quantified from field-derived data. Chapter 5 first appeared as a published paper (refer to Appendix A for a list of publications).

Chapter 6 details the development of a novel performance quantification technique that is practical for use by elite road cyclists. A case study is presented which investigates the relationship between PTRIMP, an indicator of fatigue, and the performance metric developed. Chapter 6 first appeared as a published paper.

Chapter 7 describes the practical application of PTRIMP and the novel performance quantification technique to the problem of identifying the optimal shape of a taper for an athlete prior to a competition. This investigation determines whether the quantification techniques developed provide information accurate enough to be used as the basis of a model which can provide relevant feedback on the optimal design of training programs. Chapter 7 first appeared as a published paper.

Chapter 8 describes the development and validation of HANNEM on datasets collected from three elite road cyclists. The modelling techniques discussed in Chapters 3 and 4 are extended...
and combined, and the novel techniques for quantifying training and performance detailed in Chapters 5 and 6 are utilised. This chapter discusses the results obtained and their implications for the aim of modelling real world training and performance data of elite road cyclists.

Chapter 9 addresses the issues of validating the HANNEM model and describes a set of experiments designed to answer questions raised over the issues associated with using ANNs to model small noisy datasets. It investigates the effects of ensembles on overfitting issues, and also investigates the question of determining the optimal size of an ensemble.

Chapter 10 discusses the contribution of this thesis, identifies the limitations of the work presented and identifies opportunities for future work.

Finally, the appendices list further details. Appendix A provides a list of publications that have been published to date from the thesis. Appendix B contains a glossary of terms used in this work. Appendix C details the algorithm used in HANNEM to generate and validate an ensemble of ANN models. Appendix D provides a detailed visualisation of the results of the experiments conducted in Chapter 9.
CHAPTER 2 – RESEARCH MOTIVATION: SPORTS SCIENCE

2.1. Introduction

This chapter provides an introduction to training principles as they relate to preparing for international road cycling competitions. Coaches and exercise physiologists attempt to design training programs that apply the optimal training stimulus - allowing an athlete to achieve peak performance for targeted competitions. Existing knowledge in some key areas of the science of training is incomplete, resulting in coaches and exercise physiologist relying on anecdotal evidence and trial and error to some extent when attempting to design a training program. These areas will be highlighted in this survey of current research.

The primary aim of this chapter is to draw together existing knowledge about the underlying physiological processes governing athletic response to training and link these processes to commonly accepted training principles. This is done with a view towards developing a novel machine learning model that describes the relationship between training and performance in cyclists. We begin with identifying the characteristics of road race cycling, before introducing the principles of athletic training. The energy systems used to power movement are discussed, before turning to the physiological responses of the body to training. Finally, the components of athletic performance are decomposed and discussed. The gaps in existing knowledge of the science of training are identified in the context of developing a model of training and performance.

2.2. Characteristics of Road Race Cycling

Road cycling competitions offer a complex array of challenges to cyclists. Road cycling can be broadly classified as an endurance sport, with professional cyclists cycling approximately 30,000 to 35,000km each year in training and competition (Lucia, 2001). Male professional cyclists may
race between approximately 80 and 100 days in a year (Cycling Quotient, 2010), while female professional cyclists may race between approximately 30 and 70 days in a year. These races vary from numerous 1-day races, several 1-week tour races, and for the men, one or two 3-week tour races (i.e. Giro d'Italia, Tour de France and Vuelta a Espana) (Lucia, 2001).

Four main stage or race types are possible; flat stages (usually ridden at high speeds with a large group of riders (peloton), often having a sprint finish), individual time trials (often from 40 to 60km on generally flat terrain), hilly stages (involving one or more significant climbs), and criteriums (short, fast races conducted on a short, largely flat circuit). Typical stages may have durations of approximately 1-5 hours (Faria, Parker & Faria, 2005).

Different terrain places differing demands on cyclists. Flat races often involve sprints and surges at a high intensity (to maintain position, and respond to attacks) interspersed with periods at a lower intensity while a rider drafts (sits in the slipstream) of other riders. The climbs in hilly races require the ability to sustain constant submaximal power outputs for extended periods of time. The stochastic nature of cycling requires road cyclists to possess exceptional aerobic and anaerobic capabilities (Ebert et al., 2005).

In addition to terrain variations, road cycling competitions include many uncontrollable variables, such as: weather conditions; altitude; wind direction; and team tactics (Lucia, 2001). These variables will all affect an athlete’s performance and as such will have an impact on the development of a model developed to predict performance from training load. The variables which impact performance are further discussed in Section 2.11 Performance.

2.3. Training Principles

Athletes prepare to meet the demands of competition by performing structured and focused training designed to mimic the demands of competition. The end goal of such training is to optimise athletic performance.
The human body has extensive mechanisms designed to enable it to maintain relatively stable internal physiological conditions, or homeostasis. When physiological conditions are disturbed, the body reacts in an attempt to preserve homeostasis. If the disturbance continues (as in the case of training), the body adapts its functions to a higher level (Whyte, 2006). Performance gains are possible only if the athlete observes this sequence:

Increasing stimulus (load) → adaptation → performance improvement (Bompa & Haff, 2009)

In order for adaptation to occur, appropriate recovery periods need to be scheduled. If the training stimulus is excessive, or recovery is inadequate, the athlete will be unable to adapt and maladaptation will occur. This results in decreased performance (Bompa & Haff, 2009).

Superior adaptation leads to improved performance. The goal therefore is to progressively and systematically increase the training stimulus (by manipulating the intensity, volume and frequency of training), while scheduling in appropriate recovery (Bompa & Haff, 2009).

The effect of training can be classified into three broad categories, each occurring on a different timescale. The immediate training effect occurs during and immediately after a training session. It is characterised by increased fatigue, and elevations of heart rate and blood pressure, as well as depletion of muscle glycogen (Bompa & Haff, 2009).

The positive training benefits of a training session become apparent after the fatigue of the previous stage dissipates; adaptation can then occur, accompanied by improved performance (Bompa & Haff, 2009). The onset of this delayed training effect depends on the size of the training stimulus - the larger the stimulus of a session, the longer it takes for the resultant fatigue to dissipate and performance gains to be realised (Häkkinen et al., 1989).

The cumulative training effect is the long term performance benefit that occurs as the result of training. The effects of long-term endurance training include major adaptations in skeletal muscle, such as increases in mitochondrial content and respiratory capacity of the muscle
fibres. Such adaptations in the muscle have beneficial metabolic consequences, such as a slower utilisation of muscle glycogen and blood glucose, a greater reliance on fat oxidation, and a higher lactate threshold. These adaptations underlie the performance improvements that an athlete realises after undergoing appropriate endurance training (Holloszy & Coyle, 1984).

Training overload needs to be combined with an appropriate period of recovery. Fatigue induced by training results in a perturbation of homeostasis, combined with a reduction in functional capacity. If recovery between training sessions is sufficient, the body dissipates fatigue and replaces energy stores, allowing the body to rebound into a state of supercompensation (Bompa & Haff, 2009).

The rate of recovery follows a curvilinear pattern. 70% of recovery occurs in the first third of the recovery process, while 20% occurs in the second third, and the remaining 10% in the final curve (Bompa & Carrera, 2005). The total duration of the recovery process varies according to the type and intensity of training (Bompa & Haff, 2009).

The recovery period, including the supercompensation phase, from a single training session generally takes about 24 hours (Bompa & Haff, 2009). Disturbances to homeostasis may remain for up to seven days post-exercise, however (Kreider, 1998).

The duration of the recovery curve is affected by such variables as:

- **Age** - in general, the supercompensation phase takes longer in athletes younger than 18 years of age, while athletes older than 25 require more recovery time (Whyte, 2006);
- **Gender** - Males exhibit faster recovery rates than females (Bompa & Carrera, 2005); and
- **Previous training** - Well-trained athletes tend to recover at a faster rate than un-trained athletes (Bompa & Carrera, 2005).
The process of adaption has a number of features which are of importance in understanding the training process. These are: overload, adaptation, accommodation, reversibility, specificity and individualisation.

**Overload**
The overload principle states that training adaptation takes place only if the magnitude of the training load is above the habitual level.

**Accommodation**
The principle of accommodation states that the response of a biological object to a constant stimulus decreases over time (Zatsiorsky & Kraemer, 2006). If a training load is applied at a constant level, adaption occurs initially, before stagnation and a plateau follows (Bompa & Haff, 2009).

**Reversibility**
Adaptations made to training begin to return to pretraining levels when training ceases (Frontera, 2007).

**Specificity**
Specific adaptations are made to the demands placed on the body. The training an athlete undergoes needs to be specific to the demands they will encounter in competition.

**Individualisation**
Individuals will respond differently to the same training session (McArdle, Katch & Katch, 2005).

Coaches and athletes prepare training programs to carefully apply the optimum amount of training overload. Training load is a key concept, fundamental in both prescribing training and in monitoring performed training. Training load can be thought of as the combination of the frequency, intensity and volume of training.

*Frequency* refers to how often training is performed - i.e. the number of training sessions per day or per week.

*Intensity* refers to the level of exertion during a training session. It can be quantified by power output or work performed per unit of time.

*Volume* refers to the duration of a training session. Intensity and duration tend to be inversely related - long training sessions are necessarily conducted at a lower intensity than sessions of short duration where high intensity can be maintained.

In creating a training program, coaches manipulate the variables of frequency, intensity and volume, in an attempt to provide the optimum training stimulus. A training program also needs to be constructed in such a way as to trigger adaptations specific to the demands the athlete
will face in competition. An awareness of the various energy systems that are used to power movement is the foundation for creating stimuli to trigger specific adaptations.

2.4. Energy Systems

The body needs energy in order to survive and perform work. Adenosine triphosphate (ATP) is the building block providing the energy for muscular activity. ATP stores within the body are limited, contributing little to the total supply of energy. As the total store of ATP is sufficient to maintain muscular activity for only a few seconds, this energy source must be regenerated by either aerobic or anaerobic phosphorylation if exercise is to continue.

When the cardio-respiratory system is able to deliver sufficient oxygen to the exercising muscles, aerobic production of ATP occurs within the mitochondria. Through a series of chemical reactions, glycogen and fatty acids are broken down in the presence of oxygen producing molecules of ATP as an end result.

In addition to the aerobic mechanisms for producing ATP, a number of anaerobic pathways exist. With increasing exercise intensity, sufficient oxygen can no longer be delivered by the cardio-respiratory system to the mitochondria of the exercising muscle and thus aerobic energy production fails to meet energy needs.

The primary fuel for anaerobic ATP production is glucose, stored as glycogen in the muscles and the liver. The first anaerobic pathway involves a process referred to as the glycolytic system, where glycogen is broken down into lactate during muscle contraction producing a small number of ATP molecules.

The ATP-CP system provides a second pathway for anaerobic ATP production. Creatine phosphate (CP) is a high-energy phosphagen that acts as an immediate substrate for ATP synthesis. Stores of CP, however, are very limited - enough to sustain maybe 10-15 seconds of
exercise (National Academy of Sports Medicine, 2009). The ATP-CP system is the predominant energy system for maximal activities lasting up to 15 seconds (Brown, Miller & Eason, 2006).

The anaerobic energy pathways provide a rapid means of producing energy - however only limited amounts of energy can be produced. The aerobic pathways, on the other hand, are more complex and are significantly slower to produce energy. They can, however, provide practically limitless supplies of energy.

The use of each energy system occurs on a continuum, with all three systems constantly supplying energy. The duration and intensity of the exercise being performed will determine which of the three systems is predominant at any one time. Refer to Figure 2-1 for a chart showing the estimated proportion of energy supplied by the aerobic and anaerobic energy systems during maximal exercise for time durations between 10 and 240 seconds.
2.5. Physiological Response to Training

Endurance training induces a number of adaptations in previously untrained individuals. These adaptations increase the ability of the athlete to perform prolonged strenuous exercise. Cardiovascular adaptations include: improved peak oxygen uptake (V02max); increased blood volume; and decreased heart rate during exercise of the same intensity. Muscular changes include: greater muscle glycogen storage; increased capillary density of working muscles; and increased mitochondrial density (Kubukeli, Noakes & Dennis, 2002; Bompa & Haff, 2009). The major consequences of these adaptations are a slower utilisation of muscle glycogen and blood glucose, a greater reliance on fat oxidation, and reduced lactate production for a given exercise intensity (Holloszy & Coyle, 1984).

Sprint training also induces specific adaptations. It raises the activity of glycolytic or related enzymes, thus improving anaerobic energy production. In addition, lactate transport capacity is enhanced, expediting lactate clearance rates (Kubukeli, Noakes & Dennis, 2002).

2.6. Detraining

The principle of reversibility means that when training ceases the adaptations the body has made to training begin to quickly reverse. This partial or complete loss of training-induced adaptations when training load provides insufficient stimulus is referred to as detraining (Mujika & Padilla, 2000 (a)).

Short term detraining (less than 4 weeks of an insufficient training stimulus) results in a rapid decline in maximal oxygen uptake (V02max) and blood volume in highly trained athletes. Heart rate during exercise increases, but this is not sufficient to prevent maximal cardiac output being reduced as a result of the lower stroke volume. Ventilatory efficiency is also reduced (Mujika & Padilla, 2000 (a); Mujika & Padilla, 2003).
Glycogen storage levels are rapidly reduced as the result of short term detraining. Muscle capillary density may decrease in as little as 2-3 weeks of insufficient training. Reduced mitochondrial density and a decline in oxidative enzyme activities result in a reduction of aerobic mitochondrial production of ATP. Higher reliance is placed on carbohydrates as a fuel source. Blood lactate concentration increases at submaximal exercise intensities, reflecting the greater contribution of anaerobic pathways to energy supply (Mujika & Padilla, 2000 (a); Mujika & Padilla, 2003).

Following longer term detraining (greater than 4 weeks) muscle fibre changes begin to occur. Oxidative muscle fibre proportion is decreased in endurance athletes. Force production decline relatively slowly, and usually remains above sedentary values for long periods (Mujika & Padilla, 2000 (b)).

The changes brought about by detraining outlined above result in a general loss of cardiorespiratory fitness, metabolic efficiency and muscle respiratory capacity. Endurance performance becomes increasingly impaired the longer the period of inactivity. These negative effects can be limited or avoided by employing reduced training strategies. By maintaining training intensity, a moderate reduction in frequency and marked reduction of training volume can occur while still limiting detraining (Mujika & Padilla, 2000 (b); Mujika & Padilla, 2003).

2.7. Fatigue

Fatigue can be defined as the inability to maintain a power output or force during repeated muscle contractions (Powers & Howley, 2008). Gaining a more precise understanding of the factors underpinning both fatigue and cycling performance may help to determine which training adaptations have the biggest impact on performance, and how best to structure training to maximise these adaptations (Noakes, 2000). In addition, an understanding of the
characteristics of fatigue - particularly its time course - underpins any model of training and performance.

There is conjecture amongst the scientific community about the exact cause of fatigue. Fatigue is a complex phenomenon, influenced by events occurring in both the periphery and the central nervous system (Meeusen, Watson & Dvorak, 2006). Numerous theories and models to explain the cause of fatigue have been proposed, but still no precise explanation exists (Noakes, 2000). Such models include the following:

- Cardiovascular / anaerobic;
- Energy supply / energy depletion;
- Neuromuscular fatigue;
- Muscle trauma;
- Biomechanical;
- Thermoregulatory;
- Psychological / motivational; and
- Central governor (Abbiss & Laursen, 2005).

The central governor model of fatigue, proposed by Lambert, St Claire Gibson & Noakes (2005) is a complex systems model of fatigue that has triggered extensive debate among exercise physiologists (e.g. Weir et al. (2006). In this model, exercise performance is continuously manipulated as the interactions of numerous physiological systems are monitored via feed-forward and feedback loops.

The central governor model hypothesises that a central governor in the brain controls pacing strategies such that physiological systems are not pushed to catastrophic failure. The endpoint of the exercise bout acts as an anchor point, and the athlete's prior experience with similar sessions serves as a template for subconscious setting of exercise intensity (Lambert, St Clair Gibson & Noakes, 2005).
2.8. **Over-reaching and Overtraining**

As we have discussed, in order for performance gains to be realised an athlete needs to apply a training stimulus which overloads the body, disturbing homeostasis and causing fatigue. With adequate recovery, the fatigue dissipates and the body adapts to a higher level of functioning. The balance between the training stimulus and recovery is crucial in this process.

Endurance athletes commonly undergo high training loads with limited recovery for periods of time throughout the season. Chronic maladaptations can occur as a result of this imbalance between training load and recovery, along with a disruption in internal homeostasis. This leads to major declines in performance, with possible symptoms associated with fatigue and mood disturbance (Halson, 2003).

The terms 'overreaching' and 'overtraining' are commonly used in reference to chronic maladaptations. Overreaching can be defined as a short-term decrement in performance capacity which occurs as a result of an imbalance between training load and recovery. The performance decrement may or may not be accompanied by related physiological and psychological symptoms of disturbance. Restoration of performance capacity may take from several days to several weeks (Kreider, 1998).

Overtraining is further along the continuum from overreaching - the imbalance between training load and recovery has been of sufficient magnitude and duration to cause a long-term decrement in performance, from which restoration of performance may take several weeks or months (Kreider, 1998). Other life stressors, such as sleep loss, environmental stressors (heat, cold, altitude) or social, occupational, nutritional or travel stress may combine with training stress to trigger maladaptation (Meeusen *et al.*, 2006).

Studies have shown that an inverted parabolic relationship exists between training load and performance (Busso, 2003). Since the target area for optimal training is small, both undertraining and overtraining are a common problem for athletes (Kreider, 1998). The
quantity of training stimuli which results in either performance enhancement or overreaching/overtraining is currently unknown (Halson, 2003). Approaches (such as a performance modelling) have been developed in an attempt to ascertain for an individual athlete the optimal level of training load to trigger performance improvements, but no such approaches are without limitations. This will be discussed further in Chapter Three.

2.9. Taper

The taper can be defined as a period of reduced training prior to a competition, undertaken with the aim of achieving peak performance at the desired time (Thomas, Mujika & Busso, 2009). The aim of the taper is to reduce accumulated training-induced fatigue, while retaining or further enhancing physical fitness (Bosquet, Montpetit, Arvisais & Mujika, 2007). It is of paramount importance in the preparation of athletes for competitions (Pyne, Mujika & Reilly, 2009; Mujika, 2009).

The effectiveness of a taper as reported in the literature varies, however the improvement in performance is usually in the range of 0.5%-6%. A realistic goal for performance improvement as the result of a taper is about 3% (Mujika & Padilla, 2003). In competitive athletes such modest improvements are important. A worthwhile improvement for top-ranked athletes is estimated to be in the range of 0.5%-3.0% for events such as endurance cycling (Hopkins, Hawley & Burke, 1999).

Uncertainty exists about the optimal design of a taper (Mujika & Padilla, 2003). Coaches often rely on a trial-and-error approach - which inevitably leaves ample possibility for error (Mujika, 2009). The key elements to manipulate in determining an optimal taper include; the magnitude of reduction in training volume; training intensity; duration of the taper, and; the pattern of the taper (Pyne, Mujika & Reilly, 2009).
In a meta-analysis study Bosquet, Montpetit, Arvisais & Mujika (2007) suggested that training volume should be reduced by 41%-60% over a two-week taper, without any modification to training intensity or frequency. They found that reducing training volume elicited a performance improvement approximately twice that gained by modifying either training intensity or frequency alone.

A number of taper patterns have been described and investigated in the literature. Training load can be reduced in the form of a simple step – where the load is suddenly reduced and then maintained at the same low level; or it can be reduced progressively, either with a constant linear slope, or with an exponential decay (Thomas, Mujika & Busso, 2009; Mujika & Padilla, 2003). Figure 2-2 illustrates the most common types of tapers. There is evidence to suggest that a progressive taper is to be preferred (Bosquet, Montpetit, Arvisais & Mujika, 2007; Banister, Carter & Zarkadas, 1999).

Figure 2-2. A demonstration of the pattern of load for the most common types of taper. Adapted from Figure 1 in Mujika & Padilla (2003).
Little research has been done on more complicated taper patterns. One such study looked at the effect of a two-phase taper. This model study found that the last 3 days of the taper were optimised with a 20% to 30% increase in training load. In the modelled response, such a two-phase approach allowed for additional fitness adaptations to be made in the final 3 days, without compromising the removal of fatigue. The magnitude of the performance gain is questionable (1%), however, over the optimal linear taper (Thomas, Mujika & Busso, 2009).

Much of the literature provides generalised guidelines on designing an optimal taper. It must be noted, however, that individual responses to training vary. Not all athletes respond equally to the training undertaken during a taper, and tapering strategies must be individualised (Mujika, 2009). Individual profiles of training adaptation and the time course of de-training need to be considered in determining optimal taper duration (Mujika & Padilla, 2003).

Research into whether the effects of a taper vary between males and females is inconclusive. Mujika (2009) concluded that there is no evidence to suggest that tapering programs should be varied on the basis of gender, or that a gender effect exists concerning the adaptations and taper effects on performance.

A model which accurately relates training load to performance should be able to provide individualised guidance on the optimal type and shape of taper for an athlete. Chapter Seven looks at the effect of the shape of a taper on performance.

2.10. Heart Rate Variability

Heart Rate Variability (HRV) has been proposed as a tool for monitoring training-related fatigue in athletes (Earnest et al., 2004; Atlaoui et al., 2007). By monitoring fatigue levels, it is hoped that fatigue can be managed such that both undertraining and overtraining can be avoided.

Heart rate (HR) is not constant – rather it fluctuates from beat to beat. HRV refers to the variation of the beat-to-beat interval, measured as the time between beats.
Electrocardiographs (ECGs) and some types of heart rate monitors are capable of measuring the time points (in milliseconds) between beats.

HRV is a function of the synergistic action between the two branches of the autonomic nervous system (the parasympathetic nervous system (PNS) and the sympathetic nervous system (SNS)), which act in balance to maintain cardiovascular variables in an optimal range during changing external or internal conditions (Earnest et al., 2004). Activity in the SNS elevates heart rate while the PNS has the opposite effect (Atlaoui et al., 2007). Measuring the beat-to-beat interval during an orthostatic test is a reliable method of assessing HRV in athletes (Wisbury & Montgomery n.d.).

Fatigue alters the balance between the PNS and SNS. Theoretically, the autonomic imbalances observed in fatigued or overtrained athletes should be discernible from HRV indices, and there is some evidence to suggest this is the case (Baumert et al., 2006; Earnest et al., 2004).

It is hypothesised that HRV could be included as a proxy for tracking fatigue in a model relating training to performance. Chapter Five investigates this hypothesis by analysing a dataset from an elite cyclist to identify whether there is a correlation between training load and heart rate variability indices.

2.11. Performance

The concept of performance in road cycling must be examined in order to establish a workable definition to underpin this research. Performance in cycling is affected by numerous factors. Such factors can be classified into six major groups;

- Diet
- Central Nervous System (CNS) function
- Strength / skill
- Environment
Coaches and exercise physiologists require the effects of physiological variables on performance to be isolated, in order to determine whether a particular training regime has triggered an observed variation in performance. The presence of so many confounding variables, however, makes this difficult to achieve.

Performance testing is commonly performed in a lab, where the environment can be controlled to a large extent. There are a number of issues with this approach - firstly the priority for elite
athletes is competition, and lab testing is rarely a practical option for athletes to fit in among their training, travel and race commitments. Secondly, the relationship between performance in tests and performance in events has been questioned (e.g. Hopkins, Hawley & Burke, 1999), and how a change in performance in a test translates to performance change in a competition remains uncertain.

Quantifying cycling performance in competitions presents a number of unique challenges. Cycling competitions are highly variable in terms of terrain, distance and environmental conditions, meaning that time to completion is generally of no use as a performance measure.

Measuring the power a cyclist outputs has become a relatively common practice. In cycling, power is the amount of energy transferred to the pedals (in watts) per second. Power is a direct reflection of exercise intensity, and thus is a possible candidate for use in quantifying performance.

Power is highly stochastic, however, and varies according to the terrain. A study by Ebert et al. (2005) examined the power profiles of top 20 finishers in Women's World Cup races, and compared the profiles of flat versus hilly races. They found that the terrain significantly influenced the power output profile, with hilly races requiring the ability to sustain constant submaximal power outputs for long durations while climbing. Flat races, on the other hand, often involve sprints and surges at a high intensity to maintain position and respond to attacks.

Chapter Six addresses these difficulties, and proposes a novel method for quantifying performance in road cycling competitions.

2.12. Summary

Much is known about the underlying physiological processes governing the human body's response to training, yet much also remains unclear. A set of training principles are commonly accepted by exercise physiologists, and such knowledge underpins the creation of training
programs for athletes. One key principle is that increasing the training stimulus (or load) results in adaptation of the body, leading to an improvement in performance. Training load can be thought of as a combination of frequency, intensity and volume of training, and these variables are manipulated in the creation of training programs with the aim of applying the optimal stimulus at any one time.

Application of a training stimulus leads to fatigue. The precise mechanisms governing fatigue remain unclear. Appropriate recovery, however, must be included in a training program to allow recovery from training induced fatigue, so that the body rebounds into a state of supercompensation. An imbalance between the training stimulus and recovery can lead to states of overreaching and overtraining, both of which result in performance decrements. HRV is a potential tool for tracking fatigue and recovery in athletes.

Fuel is supplied to working muscles through three energy pathways; the aerobic system, the anaerobic ATP-CP system and the anaerobic glycolytic system. An understanding of the energy systems, in combination with an understanding of the physiological responses to training, is used to design workouts to target specific adaptations in an athlete to prepare them for the specific demands that will be met in competition. These adaptations to training begin to reverse when the training stimulus is insufficient. A model of training and performance needs to adequately reflect the time course for both gaining and losing some of the most important physiological adaptations to training.

In the final phase of preparation for a competition, athletes commonly undertake a taper to improve performance by reducing accumulated training-induced fatigue, while retaining or enhancing fitness. Uncertainty still exists about the optimal design of a taper, and in addition, taper advice must consider individual responses to training.

Road cycling performance is a concept which underpins this research. Performance in this context is a complex concept, influenced by numerous factors; it is also particularly difficult to quantify. Exercise physiologists wish to isolate the physiological components of performance so
that the effects a particular training regime has on performance can be determined. In the lab, confounding affects can be controlled to a large extent – however lab testing is frequently not practical. Nor has the relationship between performance changes in exercise tests in the lab and performance changes in competition been reliably established. Field testing is more practical, but also suffers from limited control over confounding factors.

Power output (measured in watts) is a direct measure of the performed by a cyclist, and as such is commonly used in exercise load measures. Both training load quantification and performance quantification techniques need to accurately reflect the physiological load of a particular ride. Terrain significantly influences a power output profile, and so thought needs to be given to a method of accounting for these differences in quantification techniques.

This chapter has identified a number of major unresolved questions which must be answered by this research in order to develop a novel neural network model that describes the relationship between training and performance in cyclists; the ultimate aim of this research. These questions include:

- How can we quantify training load to accurately reflect the physiological load of rides conducted over differing terrain?
- How can we quantify performance in competitions - to close a feedback loop on the efficacy of a training program - without requiring impractical lab testing?
- How can we determine the optimal training stimulus for a given individual for any point in their training regime?
- How can we determine the time course of fatigue for a given individual for a specified training stimulus, and by extension, the optimal amount of recovery required?
CHAPTER 3 – A SURVEY OF SYSTEMS MODELLING RESEARCH

This chapter introduces the primary modelling approaches that have been used to predict performance from training load. In Chapter Two, we identified some criteria as important for developing a useful model to describe the relationship between training and performance in cyclists. Some of the criteria are specific to the domain of road cycling; however many of the models examined have been developed and tested against less complex scenarios – for example swimming, where time and distance are good measures of both training and performance, or in situations where training and performance testing are both completed in the lab. These criteria are still a useful starting point for evaluating the modelling approaches introduced in this chapter. The criteria are as follows:

- A modelling approach needs to incorporate a technique to accurately measure the physiological load of training - taking into account differing terrain, pattern of load and duration of training sessions.

- A performance quantification technique needs to minimise confounding factors (e.g. for cycling, confounding factors include terrain, competition type and team tactics) in order to accurately reflect performance levels.

- The model needs to have the ability to determine the optimal training stimulus for an individual athlete at any one time.

- The model needs to be able to determine the time course of fatigue for an individual athlete given: a current training status; and a training stimulus at time $t$. By extension, the model needs to determine the optimal amount of recovery required.
Models of training and performance in the literature can be classified as belonging to three broad categories: impulse-response models; antagonistic models, and machine learning models. We will begin by considering the key models in each of the three categories.

Systems modelling has been a popular approach in the attempt to model the relationship between training and performance. Impulse-response models are the dominant form of systems model used in this domain, and the key impulse-response models that have been proposed will be discussed first. Antagonistic models are an evolution of the impulse-response models. These models and the promising results that have been reported from some of these approaches will be presented. Machine learning techniques (such as neural networks) have recently been used with some reported success.

The primary aim for this chapter is to identify the most promising of the existing models, and discuss their advantages and limitations. This will be done with a view towards the development of a novel model to describe the relationship between training and performance in cyclists with sufficient accuracy to enable it to be used as a tool in planning training programmes.

### 3.1. Impulse-Response Models

Banister and co-workers (1975) proposed that the relationship between training and performance could be conceptualised by the idea that repeated training bouts contribute to two factors - fitness and fatigue. At any point in time, this relationship can be expressed by the following equation:

\[ \text{Performance} = \text{Fitness} - \text{Fatigue} \]  

This underlying concept gives rise to what are commonly termed the impulse-response models.
Banister (1991) firstly quantified training using a training impulse or TRIMP (further details of this quantification technique are provided in Section 3.4 - Quantification of Training Load). He considered that a proportion of the training impulse at time t (w(t)) defined a fitness impulse (p(t)) and a fatigue impulse (f(t)). The proportion of fitness to fatigue gained from a training impulse is not equal, and will also vary between individuals. The TRIMP is therefore weighted by multipliers k1 and k2 (initially k1 = 1 for fitness and k2 = 2 for fatigue). Between training bouts, fitness and fatigue both decline. This is modelled by an exponential decay equation.

Fitness and fatigue decay at different rates, and rates again vary between individuals. This is modelled by relative decay time constants. Equation (3-2) shows the complete formula in the form presented by Morton, Fitz-Clarke and Banister (1990). Banister (1991) defines the decay constant for fitness (r_1) initially as 45 days and fatigue (r_2) as 15 days. Refer to Figure 3-1 for a graphical example of the fitness and fatigue curves that result from a single training impulse. The performance curve is the difference between the fitness and fatigue elements of Equation (3-2).

\[
p(t) = k_1 [g(t - i)e^{-i/r_1} + w(t)] - k_2 [h(t - i)e^{-i/r_2} + w(t)]
\]

Equation (3-2)

where \(g(t)\) and \(h(t)\) are arbitrary fitness and fatigue response levels, respectively, at the end of day \(t\) and \(i\) is the intervening period between the current day’s training and that previously undertaken.
Figure 3-1. The figure demonstrates the curves for fitness and fatigue resulting from a single training session. The performance curve represents the balance between the fitness and fatigue curves. For this figure the variables were set to: $K_1 = 1$, $K_2 = 2$, $r_1 = 45$ days, $r_2 = 15$ days.

Some good model fits have been reported in studies utilising the model proposed by Banister and co-workers (1975). Morton, Fitz-Clarke and Banister (1990) reported a study where two untrained subjects undertook a 28 day running training program, with timed performance tests occurring at least twice a week. Heart rate monitors were worn for all training sessions, and heart rate was used to calculate the TRIMP for each session (refer to Section 3.4 – Quantification of Training Load for further details on calculating a TRIMP). After the initial 28 day period of intense training, the subjects ceased formal training for a period of 50 days, while continuing the twice weekly criterion performances. The models were fit by iteratively varying model parameters ($K_3$, $K_4$, $r_1$, $r_2$) in order to minimise the sum of squared deviations between...
predicted and actual performances. $R^2$ values of 71% and 96% were reported for the final model fits for the two subjects.

The authors note the difficulty that exists in getting athletes to comply with the strict requirement for collecting reliable and frequent data on training and best effort performances that the model demands. One of the key limitations of this study is the use of untrained subjects, and the short duration of training and performance studied. This raises questions about how transferrable such results are to an elite athlete population. This study does suggest, however, that despite the simplification inherent in such models, they have the potential to provide useful feedback regarding the response to training.

Significant fits for the Banister model have also been reported for athletes involved in hammer throwing (Busso, Candau & Lacour, 1994), weight lifting (Busso et al., 1990), running (Wood, Hayter, Rowbottom & Stewart, 2005) and swimming (Mujika et al., 1996; Hellard et al., 2006).

Busso (2003) proposed a nonlinear model of the effects of training on performance, based on the assumption that the fatigue induced by a training session varies according to the preceding training load. The gain term for the fatigue impulse is mathematically related to training dose using a first-order filter. This refinement of Banister’s model improved the fit of the model for previously untrained subjects trained on a cycle ergometer. Reported adjusted $R^2$ values for the proposed model ranged from 93%-96% for 6 subjects. It is important to note, however, that models estimated using untrained subjects may not be representative of athletes in real situations (Busso, 2003; Busso & Thomas, 2006) – in fact the fit of the same model in a study using highly trained participants in real training conditions was poorer (Thomas, 2008).

The model put forth by Busso uses the following equation to estimate performance:

$$
P_n = p^* + k_1 \sum_{i=1}^{n-1} w_i e^{-\frac{(n-1)}{\tau_1}} - \sum_{i=1}^{n-1} k_2 w_i e^{-\frac{(n-1)}{\tau_2}}
$$  

(3-3)
The value of the gain term $k_2$ at day $i$ is estimated using the following equation:

$$k_2^i = k_3 \sum_{j=1}^{i} w_j e^{-\frac{(i-j)}{\tau_3}}$$

(3-4)

$P_n$ is the estimation of performance on day $n$ from successive training loads $w_i$ with $i$ varying from 1 to $n - 1$. $p^*$ represents the initial performance level of the athlete. $k_1$ and $k_3$ are gain terms which adjust the magnitude of fatigue relative to fitness. The two time constants $\tau_1$ and $\tau_2$ represent the number of days over which fitness and fatigue impulses decay, respectively. $\tau_3$ is also a time constant. Refer to Figure 3-2 for a graphical example of the fitness and fatigue curves that result from a single training impulse.

Figure 3-2. The figure demonstrates the curves for fitness and fatigue resulting from a single training session. The performance curve represents the balance between the fitness and fatigue curves. For this figure the variables were set to: $K_1 = 1$, $K_3 = 0.0148$, $\tau_1 = 40$ days, $\tau_2 = 9$ days, $\tau_3 = 7$ days, $p^* = 10$. 
Model parameters are determined by fitting the model performances to actual performances by the least square method, which minimises the residual sum of squares for \( f(n) \).

\[
f(n) = p_n^{**} - \left( p^* + k_1 \sum_{i=1}^{n-1} w_i e^{-\frac{(n-1)}{r_1}} - \sum_{i=1}^{n-1} k_{2i} w_i e^{-\frac{(n-1)}{r_2}} \right)
\]

Where \( p_n^{**} \) = actual performance on day \( n \).

### 3.1.1. Limitations of Impulse-Response Models

Hellard et al. (2006) demonstrated that the four parameter Banister model suffers from ill-conditioning problems; problems which are likely to affect the accuracy of parameter estimates. Hellard suggests that the following issues contributed to the ill-conditioning:

- Small sample size. With one performance data point being obtained weekly (at most) sample size is an important consideration. The more model parameters that need to be estimated, the larger the sample size required.
- The model parameters are not independent. Inter-dependent parameters mean that several sets of parameters can provide the ‘best’ solution for a given dataset.
- Misspecification of the model. The model assumes that subsequent performances are independent, and that parameter estimates stay stable over time. Both assumptions are unlikely to be correct. (Hellard et al., 2006)

Impulse-response models typically require frequent performance measures to enable estimation of model parameters. Hellard et al. (2006) suggests that greater than 60 observations would be required for estimation of a four parameter Banister model. Frequent testing is impractical for elite athletes (Borrensen & Lambert, 2009; Hellard et al., 2006); even if weekly testing were feasible, it would still take in excess of a year to capture an appropriately sized dataset.

The behaviour of the impulse-response models deviates in a number of areas from known physiological responses to training. Biological adaptation is a complex non-linear problem (Edelmann-Nusser et al., 2002). Busso (2003) suggests that the relationship between daily
amounts of training and performance might be best described as a parabolic relationship. Traditional impulse-response models are based on linear mathematical equations. Although Busso’s (2003) model is non-linear in that the relationship between training and performance is dependent upon previous training loads, all impulse-response models assume that a greater amount of training leads to a better performance. In fact, the positive relationship between training load and performance has an upper limit, beyond which further training doesn’t result in performance gains (Morton, 1997). The authors of a recent modelling study (Mujika, Busso & Geyssant, 2012) report that their findings indicate the possible existence of a training stimulus threshold, beyond which intensity would be the main factor in the production of a training effect. Further to this, the impulse-response model is unable to model the long-term decrement of performance capacity (overtraining) which is the result of an imbalance between training and/or lifestyle stress and recovery (Bruckner & Wilhelm, 2008).

The impulse-response models are abstractions; they don’t assume a relationship to underlying fitness and fatigue processes. Attempts to link the model to underlying processes have had limited success (Taha & Thomas, 2003). It is important to note, however, that knowledge is currently limited concerning the effects of training on each of the potential physiological “markers” that have been identified (Borresen & Lambert, 2009).

Similarly, Bruckner (2006) argues that although impulse-response models model a rise in the fitness impulse immediately after a training session, the underlying physiological processes being modelled can take up to 72 hours to adapt. In the Banister model, the immediate increase in fitness is overshadowed by the concurrent immediate rise in fatigue. The adjustment mechanism of the Banister model appears to differ from what is known about the underlying physiological process. Taha & Thomas (2003) report that the model-based estimated time course of changes in fitness and performance differs from the results observed from experimental observations.

Reported performance of impulse-response models has been extremely varied, both between studies and between subjects in the same study. Mujika et al. (1996) in a study looking at the
effect of training on swimming performance found that the model explained between 45% and 85% of the variations in actual performance.

Hellard et al. (2005) found that training variables explained only 36% of variation in the performance of Olympic swimmers. Thomas, Mujika and Busso (2008) report a fit of between 45% and 63% between modelled and actual performances of eight elite swimmers.

The smallest worthwhile improvement for top-level athletes in the individual sport of road cycling time trials is in the order of approximately 0.6% (Paton & Hopkins, 2006). Paton and Hopkins (2006) concluded that drafting in road races precludes estimation of the smallest worthwhile change in performance for this event. Nevertheless, it is likely that small changes in performance could have a significant effect on the outcome of elite competition. It is thus essential that models are able to predict performance with only small margins of error (Borresen & Lambert, 2009).

Busso and Thomas (2006) provide the strongest criticism of impulse-response models, arguing that the simplifications made in abstracting complex physiological processes into a small number of entities mean that the accuracy of prediction will suffer greatly. They conclude that such modelling processes would barely be compatible with the accuracy required to make such tools useful in designing training programs.

Despite the valid concerns raised by some researchers about the accuracy of impulse-response modelling and by extension their predictive ability, software programs based on these models (such as TrainingPeaks WKO+ and RaceDay) are popular with both recreational and elite cyclists and coaches, who use them in monitoring and developing training plans.
3.1.2. Performance Manager

Andy Coggan developed the Performance Manager concept to attempt to overcome some of the limitations with the impulse-response model. His work is now included in the popular software package TrainingPeaks WKO+.

The Performance Manager is based on the observation that performance is generally greatest when training is first progressively increased to a high level, to build fitness, and then tapered to eliminate residual fatigue. The model eliminates the gain factors $k_1$ and $k_2$ which are used to convert the impulse into separate fitness and fatigue impulses. This makes the model simpler by removing the difficulty of establishing appropriate values for the gain factors. It does, however, mean that the resultant fitness and fatigue components are relative indicators of changes in performance ability, rather than absolute predictors. It also allows the substitution of simpler exponentially-weighted moving averages for the fitness and fatigue components (Coggan, n.d. (c)).

The Performance Manager consists of the following components:

- **Training Stress Score (TSS).** This is a cycling-specific load quantification technique which uses power as the measure of cycling intensity. It provides a way of measuring training load, and is used as the input for the other components of the Performance Manager.

- **Chronic Training Load (CTL).** This is equivalent to the fitness part of the performance = fitness - fatigue equation. It provides a measure of chronic training load by calculating an exponentially-weighted moving average of daily TSS values. The default time constant is set to 42 days. The use of TSS as the load quantification measure means that both volume and intensity of training are taken into consideration (Coggan, 2006).

- **Acute Training Load (ATL).** ATL is equivalent to fatigue in the impulse-response model. It provides a measure of acute training load by calculating an exponentially-weighted
moving average of daily TSS values. The default time constant is set to 7 days (Coggan, 2006).

- Training Stress Balance (TSB) is the difference between CTL and ATL. It serves a similar purpose to the output of the impulse-response model (i.e. a performance predictor), however, as the gain factors have been eliminated from the model it is better viewed as an indicator of the freshness of the athlete - or how well they have adapted to their recent training load (Coggan, n.d. (c)).

The Performance Manager model has not been validated by any scientific studies. Consequently the fit of the model to actual performances has not been reported. As with the impulse – response models, the Performance Manager doesn’t consider the specificity of the training.

The Performance Manager has two time constant parameters which determine the duration of the fitness and fatigue impulses. These parameters are set by trial and error. Banister (1991) suggested that the parameters for the impulse-response model need to be reset every 60-90 days. There is no mechanism within the Performance Manager for recalibrating parameters as the athlete’s fitness changes over time.

Although removing the gain factors and setting the remaining two model parameters by trial and error reduces the requirements for frequent performance tests, the Performance Manager still requires regular performance testing in order to calibrate TSS.

### 3.1.3. Summary of Impulse-Response Models

In order to create a workable model, simplified abstractions need to be made of the underlying complex structures. Determining just how much of the underlying structure should be incorporated into models of the relationship between training and performance is, however, a difficult question (Taha & Thomas, 2003). A balance needs to be struck between complexity and
ease of use - to arrive at a model that derives the greatest accuracy in predicting performance for the least computation cost. Busso and Thomas (2006) argue that the strong simplifications made by the impulse-response models seem to be barely compatible with the accuracy required for designing a training program. They further concluded that in order for systems modelling techniques to be of practical use for coaches developing training programs, new modelling strategies should be considered along with mechanisms for the consideration of the specificity of the training activity.

Existing models have three key limitations:

1. The input for the impulse-response models discussed is training load. None of the training load quantification methods used in the literature accurately describe physiological load for the different types of sessions a cyclist commonly undertakes. Training load quantification algorithms will be discussed in Section 3.4 Quantification of Training Load. Simplification of the model input (training load) into a single variable is likely to negatively affect model accuracy (Taha & Thomas, 2003; Busso & Thomas, 2006).

2. To estimate model parameters regular performance tests are required; this is not practical for professional cyclists. Banister (1991) found that the constants for the model will only serve for a period from 60 to 90 days before the process of iterative modelling and resetting the model constants needs to be repeated. The requirement to regularly reset model constants also reduces the amount of data available for modelling.

3. The inability of the models to accurately predict future performance (Taha & Thomas, 2003; Busso & Thomas, 2006).

3.2. Alternative Modelling Approaches

Some researchers consider that deterministic, linear models based on group statistics are inadequate for understanding and explaining complex systems - such as the training process (Perl, 2004). A shift in approach to modelling training and performance has occurred; away from linear cause-effect oriented models, and towards individual models that allow for complex non-linear interactions between observations (Pfeiffer, 2008; Balague & Torrents, 2005). Such
approaches include a dynamic meta-model named PerPot, an antagonistic model named SimBEA, and the machine learning technique of artificial neural networks.

3.2.1. Dynamic Meta-Model

Perl (2001) describes a Performance-Potential-Metamodel (or PerPot). This model simulates the interaction between training load and performance using a state–event framework with adaptive delays.

Hellard et al. (2006) highlighted the conceptual richness of this model, as it accurately models some principles of training which the impulse-response models fail to account for. It models the collapse in performance following overreaching or overtraining, as well as the reduction in performance that occurs during detraining. It also models the law of diminishing returns as it applies to increasing the training load of well-trained athletes.

The PerPot model is based on the principle of antagonism – the same basic principle underlying the impulse-response model. In the PerPot model a response potential fulfils a similar role to the fitness impulse, while the strain potential is similar to the fatigue impulse of the impulse-response model. Hence each load impulse (training load input) feeds the strain potential and the response potential. The strain and response potentials in turn influence the performance potential. The response potential raises the performance potential (incorporating a delay controlled by the response flow parameter - this is equivalent to the fitness decay constant). The strain potential reduces the performance potential (incorporating a delay controlled by the strain flow parameter which is equivalent to the fatigue decay constant). The strain potential is stored in a reservoir with a finite capacity - if the reservoir is overloaded an overflow is produced, which further reduces the performance potential. Figure 3-3 shows a flow chart describing the structure of the PerPot model.
Pfeiffer and Schrot (2009) obtained an excellent model fit for performance using a PerPot model. The study involved collecting heart rate and power data from two professional road cyclists and one mountain biker in a five month study. TRIMPs were calculated from the heart rate data collected during training and used as the input for the PerPot model. Performance was tested three times per week using cycle ergometer tests. The reported Intraclass Correlation Coefficient (ICC) values for modelled and real performances ranged from 0.61 to 0.86 for the three athletes.

Ganter, Witte and Edelmann-Nusser (2006) reported a wide range of model fits with $R^2$ values of between 13.4% – 92.8% using the PerPot model. The study collected training and performance data from ten athletes during eight weeks of cycling training, using heart rate data.
to record TRIMPs from training and the mean power output during 30s maximal effort exercise tests to measure performance. The range in model fit reported could be due to issues with the reliability of the performance test protocol chosen. Nevertheless, the large variation in modelling results between athletes in this and other studies raises questions about the internal integrity of the modelling.

The PerPot model has a number of limitations. Bruckner (2006) notes that the model does not accurately model short-term supercompensation effects. After exposure to a single training dose, the model shows a delayed decrease in performance, followed by a return to the initial level of performance – with no supercompensation. PerPot models also show a time lag between a training impulse and the resultant fatigue effect (Bruckner & Wilhelm, 2008). This delay is not reflected in the underlying physiological processes. As with the linear mathematical models, PerPot also requires frequent performance testing to enable estimation of parameters.

3.2.2. SimBEA

In 2006, Bruckner proposed an antagonistic model called SimBEA. The mathematical implementation of SimBEA is based on the PerPot model. In addition, it attempts to realistically model the physiological processes that PerPot does not; namely the supercompensation effect - the performance decline associated with overtraining as well as detraining (Bruckner & Wilhelm, 2008).

A maximum adaption rate parameter controls the ‘ceiling’ for training load beyond which maladaptation and a decline in performance consistent with overtraining occurs. A minimum training load parameter determines the minimum training impulse below which detraining occurs. Model parameters are determined by varying them iteratively and optimising the fit of actual performance to modelled performance (Bruckner & Wilhelm, 2008).
The SimBEA model was evaluated in a two phase longitudinal case study undertaken by Bruckner and Wilhelm (2008), using training and performance data from a runner. In phase one of the study, six weeks of daily performance and training data was collected. In the second phase of the study, data from 68 weeks of training and 25 performances were collected. During the 68 weeks the athlete trained as normal and could choose when to schedule performances, thus resulting in a more realistic dataset.

Training load was measured using a TRIMP calculated by multiplying a fifteen-point rate of perceived exertion (RPE) by the duration of the training session. Performance was measured by recording the heart rate during a submaximal effort. The average heart rate for this standard endurance test was taken as the performance.

The first phase of the study showed a good fit between predicted and actual performances. The reported ICC for this phase was 0.86. The second phase of the study saw a moderate fit, with an ICC value of 0.65. It is worth noting that phase two used a data collection protocol which is more realistic for collecting data from elite athletes, and still achieved a moderate fit. These results, while promising, are from a single case study and the SimBEA model requires further research before definitive conclusions can be drawn.

3.2.3. Artificial Neural Networks

Neural networks are flexible, adaptive learning systems that can find patterns in observed data, enabling the development of nonlinear systems models which can make reliable predictions (Samarasinghe, 2006). The non-linear modelling techniques of neural networks have been suggested as potentially appropriate for use in modelling the relationship between training and performance (Hellard et al., 2006).

Silva et al. (2007) used neural network technology to create models of swimming performance. They concluded that this approach was valid for resolving complex problems such as
performance modelling. In order to achieve good results, however, neural networks, in common with other machine learning techniques, require large volumes of data in order to converge on an accurate model. Large datasets are unlikely to be obtainable in this domain. Neural networks are prone to overfitting, particularly with small, noisy datasets such as frequently occur in the training and performance domain. Overfitting occurs when the model identifies the noise in the dataset, rather than the underlying relationship in the data. Overfitting is a known limitation when using neural networks with small datasets.

Neural networks have been used to model the competitive performances of an elite swimmer, on the basis of training data (Hohmann, Edelmann-Nusses & Hennerberg, 2001). The study concluded that neural networks are excellent tools to model and even predict competitive performances on the basis of training data. The precision of the neural network prediction was much higher than that achieved by a conventional regression analysis. The authors identified two key advantages of using neural networks. The approach is robust in handling issues with input data - such as noise; and the neural network can model nonlinear transformation of the relationship between training and performance i.e. as the athlete’s fitness changes over time.

In the Hohmann study (Hohmann, Edelmann-Nusses & Hennerberg, 2001) a very small dataset (only 19 records) was used in the modelling. Recognising the overfitting problems neural networks experience in modelling with very small datasets, the researchers used data from a different swimmer to pre-train the network. This method resulted in a surprisingly small prediction error (0.04%), however doubts remain with the methodology employed in the study. Much evidence exists that training responses are highly individual (Avalos et al., 2003; Busso, 2003; Millet et al., 2002).

Haar (2012) used neural networks to model a small dataset (24 performances) obtained from three highly-trained triathletes. He concluded that ANNs were particularly suitable models for the prediction of athletic performance from training, however there are also doubts about the methodology employed in this study, as it appears not to have addressed issues with
overfitting, nor did it provide details of the architecture of the ANN used, or the validation process used.

Hellard *et al.*, (2006) criticises neural networks for being a “black box”, with a limited ability to explicitly identify causal relationships. This makes it difficult to interpret the results obtained. It is, however, perhaps unreasonable to expect a model to elucidate the extremely complex underlying structures governing the response to training.

### 3.3. Limitations of Current Modelling Approaches

A number of models have been developed in an attempt to quantify the relationship between training and performance. Modelling this relationship as a systems model with training load as the input and performance as the output is an attractive concept – however many of these models have only moderate accuracy (Borresen & Lambert, 2009). The inevitable simplifications of the underlying physiological processes inherent in these models - particularly the reduction of model inputs into a single number representing training load – have been suggested as the likely cause of such model’s poor predictive performance (Busso & Thomas, 2006). The impulse-response models (and their successors, such as SimBEA and PerPot) are limited in their practicality in the real world, due to their requirement for frequent performance tests in order to estimate model parameters.

The impulse-response models fail to model the key physiological processes of supercompensation and overtraining (Bruckner & Wilhelm, 2008). The PerPot model does capture the system collapse effect of overtraining, however it’s modelling of the supercompensation effect has been criticised for its lack of physiological accuracy (Bruckner & Wilhelm, 2008).

Neural network models have the advantage of being able to adapt as the relationship between training and performance changes over time. They are also flexible modelling systems, allowing
more than one input. Datasets in this domain are inevitably small however, and neural networks typically require large datasets to train the model (Jobson et al., 2009). Neural networks remain a promising tool for modelling training and performance, however techniques to avoid overtraining need to be utilised or developed.

3.4. Quantification of Training Load

All the models we have discussed have used training load as a model input. Training load must be quantified in some way, but how should this be done? The frequency, duration and intensity of training all contribute to the nature and magnitude of the training effect (Borresen & Lambert, 2009) and so all need to be considered in a quantification technique (Taha & Thomas, 2003). There has been great difficulty in finding a way to effectively quantify training load using a single term (Foster et al., 2001).

Banister’s TRIMP was introduced in Section 3.1. A TRIMP is derived from multiplying the duration of training by the relative training intensity (measured by heart rate). A multiplying factor is applied in order to weight higher heart rate (HR) ratios proportionally higher.

\[
TRIMP = T(\text{min}) \times \frac{HR_{ex} - HR_{rest}}{HR_{max} - HR_{rest}} \times y
\]

where \(HR_{ex}\) is mean HR during the session, \(HR_{rest}\) is the athlete’s resting HR, and \(HR_{max}\) is the athlete’s maximal HR, and \(y\) is a multiplying factor defined as follows:

\[
y(\text{male}) = 0.64e^{1.92x}
\]

\[
y(\text{female}) = 0.86e^{1.67x}
\]

where \(x\) equals delta HR ratio during exercise.
Taha and Thomas (2003) describe calculating a TRIMP using currently accepted methods of measuring heart rate intensity. An absolute measure of percentage of maximum heart rate is used, while Banister used the Karvonen method of heart rate reserve.

TRIMPs have been successfully used to model the relationship between training and performance in many studies (e.g. Banister et al., 1999; Millet et al., 2002; Ganter, Witte & Edelmann-Nusser, 2006). TRIMPs are a relatively easy measure to calculate, and require only heart rate monitors, which are readily available and moderately priced. They are a useful integrative measure of training load for cyclists (Faria et al., 2005). TRIMPs also have the advantage of being able to be calculated across most sports and training activities, for example running and cycling. They are not an appropriate load quantification method for strength training, however, as heart rate only measures the cardiovascular load of exercise.

Heart rate is a poor method of evaluating very high intensity exercise (Foster et al., 2001). It is likely that long duration, low intensity incorrectly provides a TRIMP score higher than that obtained from high intensity exercise of shorter duration (Hayes & Quinn, 2009; Rusko et al., 2004).

Heart rate is affected by many factors; which impacts on its reliability as a measure of training load. Factors which can affect heart rate include temperature, hydration, sleep, overtraining and caffeine (Jeukendrup, 2002, p. 67). There are claims that the variability of heart rate impacts on the validity of quantification methods based on heart rate (Skiba, 2008).

3.4.1. Rating of Perceived Exertion

Foster et al. (2001) proposed a method of quantitating training using a rating of perceived exertion (RPE). The RPE method involves getting athletes to rate their exertion for a session on Borg’s RPE scale - a scale of 1 to 10 (Borg, 1985, cited in Foster et al., 2001). The 2001 study
suggested that a TRIMP obtained from RPE (duration x RPE) was highly correlated with a modified TRIMP calculated from heart rate zones.

RPE has been used as the input of a systems model to study training and performance in an elite sprinter. The authors concluded that the resultant model was a potentially powerful tool for assessing the effects of training on athletic performance (Suzuki et al., 2006). Bruckner and Wilhelm (2008) also used RPE successfully when they validated their SimBEA model using training data from a runner.

Some advantages of using RPE as a load quantification technique are: it can be used across many different exercise modalities, including strength training; it requires no equipment; and it is simple to calculate. The disadvantages of this approach include: it is a subjective measure; it relies on individual’s memory of the session; differences in rating exist between individuals; and some intra-subject variation exists.

3.4.2. **EPOC**

EPOC (excess post-exercise oxygen consumption) is a measure of the oxygen consumed in excess of resting requirements during recovery from exercise. Measured in litres or ml/kg, it reflects exercise induced disturbance of the body’s homeostasis, and the resultant recovery demand (Wisbey, 2006).

EPOC increases with higher intensity and / or increased duration of exercise (“Indirect EPOC Predication Method Based On Heart Rate Measurement,” 2007). EPOC can thus be used as a measure of training load. It has been suggested that EPOC reflects the cumulative response of the body to the entire training session (Jobson et al., 2009).

It is only possible to measure EPOC in the lab, by analysing respiratory gases. Firstbeat Technologies, however, have developed software to predict EPOC (EPOC_{pred}) based on the inter-beat (RR) interval heart rate data.
In order to make a prediction of EPOC a known maximal heart rate and VO₂ max (the maximal oxygen update) for the user is required. The heart rate and heart rate variability (HRV) measurements during exercise are used to predict respiratory rate and VO₂. Predicted respiratory rate, and VO₂ are then used in conjunction with heart rate to predict EPOC (B. Wisbey, personal communication, June 26, 2006).

Two key advantages of predicted EPOC are: it has a significant correlation with laboratory measured EPOC (Rusko et al., 2003), and thus good physiological accuracy; and it is suited to measuring the physiological load of high intensity exercise. The validation of predicted EPOC against measured EPOC has, however, only been tested for short durations of exercise.

EPOC is more affected by training intensity than volume (Rusko et al., 2004). EPOC is therefore only a useful measure for monitoring training response to short, intense training sessions (Wisbey, 2006). Using EPOC to measure the training load of long endurance exercise will not accurately reflect the physiological load and the recovery time needed from such a session. In addition, the predicted EPOC for a session is the peak EPOCₚₑᵅᵈ reached. This means that repeated efforts with a long recovery will not result in an increased EPOCₚₑᵅᵈ (Wisbey, 2006).

Other disadvantages of using predicted EPOC as a load quantification technique include: the calculation is complicated and requires proprietary software and hardware; and it is based on heart rate, which as previously discussed can be affected by a variety of factors.

3.4.3. Using Power to Quantify Training Load

Power is the amount of energy generated per unit of time. In cycling, it is the amount of energy transferred to the pedals (in watts) per second. SRM power monitors are commonly used to measure power in cycling. They provide reliable measurements of power (Gardner et al., 2004), as well as measurements of cadence, speed and distance.

Power is a direct reflection of exercise intensity as opposed to heart rate, which is a response to the exercise intensity (Jeukendrup, 2002). Heart rate is affected by external factors, and also
experiences a delay in responding to changes in exercise factors. As a direct reflection of work output, power is not affected by these limitations.

Power has good potential to be used as the basis for quantifying training load. It does have some practical limitations however. Power based load metrics can only be used in sports where power can be easily measured (i.e. cycling). Training load from cross training or strength work can’t be measured. Power monitors are also still not in widespread use, due mainly to their relatively high cost – and for athletes with multiple bikes (training, racing and time trial bikes) there is as yet no easy way to switch power monitors between bikes.

In order to use power data as a training load metric, it needs to be pre-processed in some way. Taking the mean power of a session is a common summary statistic. Power output is highly variable however, so the mean doesn’t necessarily accurately reflect the physiological demands of a session.

In an attempt to come up with a better summary of physiological load than mean power, Coggan (2003, 2006) proposed the concept of normalised power. Normalised power is an estimate of the constant power that an athlete could have maintained for the same physiological load.

**Normalised Power**

Normalised power is calculated by:

1. Smoothing the power data by applying a 30 second rolling average filter. This is to account for the fact that physiological responses to changes in exercise are not instantaneous, but follow a characteristic time course. 30 seconds was chosen as the typical half life of many physiological responses (e.g. heart rate, VO2) (Coggan, 2003).

2. Raise the smoothed data to the 4th power. This takes into account the non-linear nature of physiological responses (Coggan & Edwards, 2006), by giving more weighting to higher power values.
3. Average the values obtained in step 2.

4. Take the 4th root of the average to give normalised power (Coggan & Edwards, 2006).

Normalised power is then used in the calculation for a load quantification technique called Training Stress Score (TSS).

**Training Stress Score**

Training Stress Score was proposed by Coggan in 2003 as a load quantification method using power. An intensity factor (IF) is used to normalise TSS to functional threshold power (or the power that can be maintained in a 60 minute time trial). This allows for easier comparison of TSS between athletes. The formulas for calculating TSS are included below:

\[
IF = \frac{\text{normalised power}}{\text{functional threshold power}}
\]  

\[
TSS = \text{duration (h)} \times IF^2 \times 100
\]

TSS has a number of advantages: it is based on power, which is a direct measurement of training stimulus (work rate), rather than a response to the stimulus (such as heart rate); it is relatively simple to calculate (Coggan, n.d (a)); and by using normalised power as the basis for the calculation, TSS applies increased loading for high intensities, which is in keeping with physiological principles.

I argue that TSS places too much emphasis on training duration when quantifying load, and not enough on intensity. For example, when an athlete is coasting downhill, and applying zero force to the pedals, TSS still accumulates.

The model has not been validated by any scientific studies (Coggan, n.d (a)). A similar algorithm (BikeScore), however, has received some initial support from a study which demonstrated the usefulness of the algorithm as an input function for systems-based performance modelling. Using the load quantification technique it was possible to accurately model performance (Skiba, 2007).
3.4.4. Summary of Load Quantification Methods

The load quantification methods we have examined - TRIMPs, RPE, EPOC and TSS - all suffer from drawbacks in physiological accuracy and/or practicality. TSS is widely used by cyclists and has a number of advantages. The key criticism of TSS is that it places too great an emphasis on duration, and consequently too little on intensity.

For the greatest physiological accuracy, a load quantification technique would be multifactorial and include the specificity of the activity, as well as the pattern of load. None of the current load quantification techniques consider these aspects.

Pattern of load is a consideration which impacts on physiological cost. Accelerating to a given speed requires greater power application and hence comes at a greater physiological cost than maintaining a given speed. Similarly, maintaining the same power at the end of an interval comes at greater physiological cost than maintaining that power at the beginning of an interval.

The power output of a cyclist is a direct reflection of exercise intensity. Heart rate responds to the exercise intensity. The load quantification algorithms using power weight a certain power output the same whether it occurs at the start of a session, when an athlete is fresh, or at the end when they are fatigued. The occurrence of cardiac drift (the continuous increase in heart rate that usually occurs during prolonged moderate-intensity exercise (Jeukendrup, 2002, p. 66)) is cited as a criticism of using HR to measure training load (e.g. Skiba, 2008). Maintaining the same power at the end of an effort comes at greater physiological cost than maintaining that power at the beginning of an effort – we suggest that HR more accurately models the physiological cost in this situation than power.

Multifactorial techniques which consider specificity, or include HR data as well as power data while potentially increasing the physiological accuracy of load quantification techniques, do come at the cost of increased complexity in both data collection and model construction. This
compromise between accuracy and usability will need to be considered in the development of a new load quantification technique for this research.

3.5. Quantification of Performance

Training load is the key input for the models we have discussed, and the other critical component is the output: performance. The exact components which constitute the concept of performance are sport-specific. Many of the modelling studies we have discussed have concentrated on sports such as swimming, where performance is logically quantified as a function of time, or have used standardised lab testing, usually on a cycle ergometer, to define performance. We will review the current approaches that have been used for quantifying performance, and discuss their applicability for cycling.

Numerous studies (Thomas, Mujika & Busso, 2008; Mujika et al., 1996; Avalos et al., 2005) have used a method where performances are converted into a percentage of personal best performances. A similar approach in cycling would require the athlete to complete a regular performance test. As discussed, this is generally not practical.

Morton et al. (1990) used a criterion point scale to quantify performance. The equation for the curve of athletic performance records over time is:

$$y = L + ae - \frac{x}{b}$$

(3-10)

where \(y\) is the time or distance recorded for an athletic performance, \(L\) is the perfect performance (e.g. world record), \(a\) is an amplitude factor that is positive for running events and negative for jumping or throwing events, and \(b\) is a time parameter - the difference between \(L\) and the performance of any able-bodied individual (Morton et al., 1990). Road cycling, with its
variety of terrain and the impact of drafting and team tactics, does not have an easily definable concept of perfect performance, however.

A conceptually similar technique called Mercier Scoring is commonly used to compare track and field athletes. The Mercier Scoring Tables are the result of a statistical comparison of all performance in Athletics. A linear fit is applied to the weighted average of the 5th, 10th, 20th, 50th and 100th World-ranked performances in each event over the past 4 years. Performances from more recent years are given a higher weighting. McGregor (2007) used this performance quantification technique in applying a Banister-type model to the training and racing data of a 1500m runner.

Hellard et al. (2006) examined the effect of different performance quantification techniques on the modelling process using a Banister model. Performance was modelled using a logarithm transformation, expressed as a percentage of the world record, as well as the criterion points scale proposed by Morton et al. (1990). These methods resulted in a less reliable model compared to the one obtained when performance was quantified as a percentage of the best performance achieved by the athlete in the course of the study period.

An athlete’s best performances are likely to occur during an actual competition where training induced fatigue was minimised by tapering prior to the event. Ignoring this data eliminates a particularly important component of the training – performance relationship. One study which quantified performance from actual competition data was done by Millet et al. (2002), who used a subjective rating technique to measure performance in triathlon competitions. The validity of this method is questioned by Taha and Thomas (2003) due to its subjective nature. It does however, highlight the difficulty of establishing an accurate method of testing and quantifying performance, which is practical enough to implement in the real world of elite athletes.

It is clear from this review that there is currently no acceptable method for quantifying performance for road cyclists in the field. Such a method would need to: reflect the
physiological component of performance; minimise the impact of external factors such as terrain, drafting and team tactics; and be practical for elite cyclists to implement in terms of data collection.

3.6. Summary

We have discussed the methods available to quantify training load, and their limitations. A training load quantification method needs to:

- Reflect training intensity in accordance with known physiological principles;
- Provide a physiologically accurate balance between the loading of duration and intensity;
- Provide consistent results independent of external factors; and
- Be practical and robust in terms of data collection and processing.

All of the current methods of quantifying training load reflect a compromise in one or more of the above areas.

We have discussed the methods available to quantify performance, and their limitations. A performance quantification method needs to:

- Provide an accurate representation of the physiological component of performance;
- Minimise the impact of external factors such as terrain, drafting and team tactics; and
- Be practical and robust in terms of data collection in the field.

Again, all of the current methods of quantifying performance reflect a compromise in one or more of the above areas.

The methods available to model the relationship between training and performance have also been discussed. Such models need to:

- Accurately reflect known physiological effects, such as overtraining, detraining and supercompensation;
Operate with very small datasets;
Operate with a practically achievable number and type of performance tests; and
Have good predictive performance.

As above, all of the current methods discussed require a compromise in one or more of the requirements.

It is clear that in order to develop a system that will enable cycling coaches and athletes to manipulate training loads to prepare cyclists for peak performances in competitions, a new load quantification technique, a new performance quantification technique, and a new or adapted modelling approach will all need to be developed.

The next chapter, Chapter Four, will investigate computational modelling approaches and assess them for suitability for modelling athletic training and performance.
CHAPTER 4 – KNOWLEDGE DISCOVERY FOR SPORTING PERFORMANCE

In recent years there has been rapid growth in the use of data mining techniques to discover knowledge from datasets from a broad range of industries and domains. As data has become more extensive and pervasive, new techniques have been developed to “mine” information from these sources. Problems in sport science have much in common with the problems encountered in other domains, yet they also have some distinctive characteristics which have not been extensively addressed by existing techniques.

This chapter considers the longstanding problem of knowledge discovery to improve elite sporting performance. The primary aim is to draw together previous modelling work performed on similar problems with a view towards developing a methodology for the generation of a better prediction model for the training and performance domain. We begin with introducing the research problem, and discussing the nature of the data collected. The research design employed in this study is described. The field of data mining is introduced along with a brief introduction to a data mining process commonly employed. Various modelling techniques from both the statistical and machine learning domain are introduced, and their potential applicability discussed. It is not the intention of this study to describe all modelling techniques but rather to concentrate on the concepts and techniques that are relevant to the research problem.

4.1. Research Problem

The aim of this research is to develop a model that will describe the relationship between training and performance such that performance output can be predicted from training input and other known and measurable factors. The model developed needs to be adaptable and
extensible, such that it can adapt to changes or additions to the input data. Figure 4-1 shows a high-level overview of the components of the proposed model.

![Figure 4-1](image)

Figure 4-1. This figure shows the components of the proposed system. Inputs to the model are on the left and model outputs on the right. The model inputs shaded in yellow are designed to be adaptable and extensible, and will be dependent on the data available from an individual athlete. Examples of supporting data that might be useful include RPE values from training sessions, HR data, recovery data such as sleep duration and quality, results from lifestyle stress questionnaires (e.g. Profile of Mood States), and results from any regularly undertaken sport specific standardised testing.

### 4.2. Research Design

This research employs a naturalistic approach, using data from athletes’ normal training and competition performance, rather than a research-driven manipulation of the variables being studied. While this approach has limitations, it allows the research to be conducted on professional cyclists; who are unlikely to consent to participation in experimental trials (Barnett et al., 2010). A time-series design has been used, involving observation of training and racing over time. This design allows both the short term and long term effects of training on performance to be observed over extended periods.
An ideographic approach has been followed. This approach studies each athlete as a separate entity, consequent to the premise that responses to training are individual (as discussed in Chapter 2). The end result is a sequence of successive training data points, generally measured daily, and a sequence of performance data points which are irregularly spaced. The main limitation of study designs which use a case study approach with single or small numbers of participants is difficulty in generalising the findings to a wider population (Kinugasa, 2013).

Barnett et al. (2010) points out the benefits of such naturalistic, idiographic studies, stating that they can potentially provide highly practical and individual information to coaches and athletes on the relationship between training and performance. Further, such a research design is ethically and personally acceptable to professional athletes due to its undisruptive nature.

4.3. Data

Daily training and racing data were captured for several professional cyclists over the course of a season or several seasons. For each training or racing session, the raw sensor data captured was converted into a single unit measurement of training load, using a novel algorithm which will be introduced in Chapter 5. In addition, for races a single unit measurement of the performance was calculated using a novel algorithm which will be introduced in Chapter 6.

One athlete also performed an orthostatic test each morning over the period studied, enabling the athlete’s level of fatigue to be estimated using HRV algorithms. This algorithm will be introduced in Chapter 5.

In Chapter Two, we learnt that the relationship between training load and performance is likely to be an inverted parabola (Busso, 2003). Biological adaptation is a complex non-linear problem, as the adaptation of a biological system leads to changes within the system itself (Edelmann-Nusser, Hohmann & Henneberg, 2002). In other words, inputs into the system lead not only to adaptations, but also to changes in the adaptive behaviour itself.
In a linear system, the magnitude of the system output ($y$) is related to the input ($x$) by a simple equation in the form of $y = mx + b$ where $m$ = the slope of the line and $b$ = the intercept. The behaviour of a linear system can be fully understood and predicted (Goldberger, 1999).

In contrast, small changes of the input variable can have dramatic and unanticipated effects in nonlinear systems. For example, a simple equation that describes a parabola might be: $y = ax(1-x)$. Depending on the value of $a$, this equation can generate steady states, regular oscillations or highly unstable behaviour. A nonlinear system cannot be understood by examining its components individually. Such a reductionist approach fails to consider the interaction between the individual components (Goldberger, 1999).

Data can be considered deterministic if it is possible to fully predict an outcome from a given set of inputs (assuming the underlying relationship is known). In practice, however, most data contains noise (Rabunal & Dorado, 2006). There are a number of potential sources of noise in the training and performance datasets. These include artificial spikes in the SRM sensor data; slight drifts in the sensor readings due to ambient temperature changes during a ride; external factors not considered by the system such as nutrition and sleep; and the simplifications that need to be made of the underlying physiological processes in order to estimate training load. The ability of a modelling technique to deal with noisy data thus needs to be considered.

A limited volume of data is inevitable in this domain. Training data accumulates with one data point per day, per athlete. Performances occur at most every week. The underlying relationship between training and performance changes over time so larger datasets consisting of years of data may have reduced accuracy for time periods where the underlying relationship is changing, or is different to that of the majority of the dataset. Missing data is also an issue – human error, equipment malfunction, changes of bikes and difficulties associated with frequent travelling can all result in missing data.
4.4. The Data Mining Process

Data mining can be defined as the “process of discovering meaningful new correlations, patterns and trends by sifting through large amounts of data... Data mining employs pattern recognition technologies as well as statistical and mathematical techniques” (Gartner, 2012).

A widely used methodology for undertaking data mining projects is the CRoss-Industry Standard Process for Data Mining (CRISP-DM). CRISP-DM involves six interrelated phases:

- Business Understanding – identifying the problem and developing an understanding of the business objectives for the project;
- Data Understanding – Developing an understanding of the data including how to source it, and identifying data quality issues. This phase involves performing an initial exploration of the data;
- Data Preparation – Clean and pre-process the data to prepare it for modelling;
- Modelling – Select, apply and tune various modelling techniques;
- Evaluation – Assess the performance of the models, and how well they achieve project objectives; and
- Deployment – Deliver the results of the data mining project to the client / end user.

(Muehlen & Su, 2012)

The choice of a particular modelling technique or algorithm to apply for a particular situation depends on the nature of the problem and the properties of the dataset. It is often difficult to choose the right technique for a particular situation. Typically a number of models are trialled, and the most accurate result is selected. A number of potential models will be considered in the following sections.
4.5. Statistical Models

The use of statistical and mathematical models is well established within the field of sport science. Statistical models tend not to require large datasets to be effective, however they do tend to rely on prior knowledge of the form of the relationship between variables, and also tend to require the dataset to meet certain conditions, such as normality and serial independence. Regression and ARIMA (autoregressive integrated moving average) models are two techniques that are routinely employed.

4.5.1. Regression

A popular, well-understood statistical technique used to model a relationship between a set of input variables and an output variable is multiple regression. Regression attempts to find the line of best fit for the given data. Few authors have used multiple regression in modelling the relationship between training and performance (Hellard et al., 2006), however Mujika et al. (1996) used stepwise regression to model training load and performance, and reported a close match with the Banister model. Hellard et al. (2006) suggests multiple linear regression as a viable alternative to the Banister model, and found improved statistical accuracy in estimated parameters and modelled performances using this technique.

Hellard et al. (2006) proposes transforming each training variable by a quadratic (or higher-order) function, in order to model the potentially parabolic relationship between training load and performance. The relationship between training and performance, however, is unlikely to be as simplistic as a parabola, and such modelling is not easily able to account for the change in the relationship between training and performance over time.

Jobson et al. (2009) suggest applying mixed linear models for repeated measures data, such as that collected by Busso (2003). Mixed models allow a model of popular behaviour to be
constructed, rather than creating an individual model for each subject. Parameters of the model are varied to take into account intra-individual variability (Avalos et al., 2003). Avalos et al. (2003) reported moderate model performances using a mixed model with swimmers with an average $R^2$ of 37.7% and a range of 15.1% to 65.5%. Yeh (2007) states, however, that mixed linear models are usually unreliable. The exact value of model parameters typically need to be estimated from the data set, which is difficult as the assumption of normality may be violated in small data sets (Yeh, 2007).

The relationship between training and performance is likely to be too complex to be modelled most effectively by regression approaches alone. They may, however, offer some benefits when combined with more complex non-linear modelling approaches.

4.5.2. **ARIMA**

Training and performance time series have a temporal aspect. Sequential performances are not likely to be independent – indeed it is likely that previous performances contain important information that can be used to predict subsequent performances.

Autoregressive integrated moving average (ARIMA) models are a popular linear model for time series forecasting. In an ARIMA model, the future value of a variable is assumed to be a linear function of several past observations, plus random errors.

Stationarity is a necessary condition for building useful ARIMA models. A time series displaying a trend or heteroscedasticity requires preprocessing techniques such as differencing and power transformation to remove the trend and stabilise the variance (Zhang, 2003).

One key advantage for the domain of training adaptation is that the ARIMA technique does not require large amounts of data. It also takes into account the time ordered nature of the data. The major limitation of ARIMA models is the pre-assumed linear form of the model. We know that biological adaptation is a complex non-linear problem.
As we have discussed, the relationship between training and performance cannot be described by a linear equation. Therefore multiple regression, mixed linear models and ARIMA models on their own, are unlikely to be the most effective approach in describing this relationship. They do, however, have a potential place as components in a hybrid model that includes other non-linear techniques.

4.5.3. Gaussian Process

A central challenge in modelling time-series data is determining a model that can capture the nonlinear nature of the data, while avoiding overfitting. Gaussian process models are nonlinear regression models that can obtain relatively good results with only small training data sets (Ruano, 2005). Gaussian process models account for the fact that input data contains noise. The default assumption is that the noise has a Gaussian distribution (Schwaighofer et al., 2007).

Gaussian process modelling works by considering a family of functions that could potentially model the system in question. This collection of functions reflects a prior belief about the nature of the system. This prior belief is then updated in the light of new information (i.e. the observed data). Principles of statistical inference are used to identify the most likely posterior function – the function that most likely describes the true relationship of the system, in the light of both prior assumptions and the observed data (Schwaighofer et al., 2007).

Gaussian process models have a number of advantages. One key advantage is that unlike ANNs, they perform well with comparatively small data sets. They are robust models, requiring relatively few tuning parameters. They also provide an estimate of the uncertainty in the model predictions. This uncertainty estimate is provided for each input point on which the prediction is made (Ruano, 2005).

These advantages do come with some costs, however. Gaussian process models are computationally expensive. This is likely to be less of an issue when modelling with small data sets. Lack of transparency is another limitation. They are ‘black box’ models, and don’t provide any information about the nature of the relationships of the underlying system (Ruano, 2005).
Gaussian process models also tend not to be as adept at modelling complex relationships as machine learning models.

To the author’s knowledge, Gaussian process models have not been applied to the domain of sports training and performance modelling. Some authors have used Gaussian process models successfully for modelling biological processes – e.g Zhou et al. (2010) who reported that a Gaussian process model was superior to both an ANN and a Support Vector Machine (SVM) model in the modelling of peptides.

4.6. Machine Learning

In recent years there has been rapid growth in the successful use of machine learning techniques for knowledge discovery applications in a diverse range of problem domains such as economics, medicine and government. These flexible and powerful techniques are ideally suited to dealing with large volumes of data where little is known about the underlying relationships between data elements. The following sections will introduce some algorithms which are potentially applicable to the problem of modelling the relationship between training and performance.

4.6.1. Pattern Mining

Sequential pattern discovery has emerged as an important research topic in knowledge discovery (Ruan, Xu & Pan, 2005). It is a technique that aims to discover correlations between events through their order of appearance (Cohen, Adams & Berthold, 2010). These subsequences of frequently occurring ordered events can be thought of as patterns.

Training data is essentially a time series as it consists of a sequence of ordered values that change with time. Time series can be transformed (or symbolised) into curves or shapes. For example, the shapes made by a series of training might be described as up, sharp up, sharp
down, down. Training series that follow a similar pattern can be identified and grouped together.

Searching for patterns in real-valued data is difficult (McGovern et al., 2011). Discretising or symbolising the time series is a technique that is used to reduce the dimensionality of real-valued time series data to enable pattern discovery. Many high level representations of time series have been proposed (Lin et al., 2007).

A number of time series representations have been introduced, including the Discrete Fourier Transform (DFT) (Faloutsos et al., 1994) and Discrete Wavelet Transform (DWT) (Chan & Fu, 2002). These real-numbered transformations are limited in the algorithms that can be applied to them. A method of discretising time series data into symbolic strings has been proposed by Lin et al. (2003), called Symbolic Aggregate approXimation, or SAX. Data that has been discretised can utilise a number of existing data mining algorithms, such as Markov models and decision trees (Lin et al., 2007).

SAX allows a time-series of arbitrary length $n$ to be reduced to a string of arbitrary length $w$. The alphabet size used is also an arbitrary integer (Lin et al., 2003). The process involves firstly normalising the time series to have a mean of zero and a standard deviation of one. A series of breakpoints are identified that divide a Gaussian distribution up into $n$ number of equiprobable regions. These breakpoints are used to map each data point into symbols, such that a data point lower than the smallest breakpoint will be mapped to the symbol ‘a’; a point greater than or equal to the smallest breakpoint but smaller than the next breakpoint will be mapped to ‘b’ and so on (refer to Figure 4-2).
Figure 4-2. Predetermined breakpoints are used to break the feature space into equiprobable regions. Each data point is mapped to the symbol of the region it falls in.

Pham, Le and Dang (2010) propose an improvement to SAX, called adaptive Symbolic Aggregate approximtion (aSAX), which uses an adaptive vector of breakpoints, determined by a preprocessing phase using a $k$-means clustering algorithm. This technique enables appropriate breakpoints to be set for non-Gaussian distributions. Lin et al. (2007) claim, however, that the correctness of the SAX algorithm remains when the Gaussian assumption is violated and only a slight drop in efficiency is observed.

Constructing a training program is really about choosing the optimal combination and sequencing of training patterns, made up of individual training sessions. These patterns exist on different timescales. In training periodisation literature these cycles on different timescales are sometimes referred to as microcycles, mesocycles and macrocycles. Through techniques such as SAX, it is possible that frequent patterns in training and performance data can be identified and isolated, leading to an improved understanding of which patterns or combination of patterns have been successful with an individual athlete.
4.6.2. Artificial Neural Networks

In Chapter Three, we introduced artificial neural networks (ANN) as a modelling tool that has been used by some researchers to model the relationship between training and performance. In this section further details will be provided on the implementation details and characteristics of ANNs.

Artificial neural networks are non-linear statistical data modelling tools. They can be used to model complex relationships between inputs and outputs or to find patterns in data. They involve a network of simple processing elements (neurons or nodes) which communicate by sending signals to each other over a large number of weighted connections.

A neural network consists of a set of interconnected nodes (or neurons). Each input variable corresponds to a node at the first or input layer. Each variable to be predicted corresponds to a node at the final or output layer. One or more hidden layers, consisting of nodes connected between the input and output layers, are frequently included in a neural network architecture. Refer to Figure 4-3 for an example of a neural network with two nodes in the input layer, a hidden layer with 3 modes and an output layer with a single node.

Weights express the strength, or relative importance of each node input. A neural network “learns” by a process of repeated adjustments to these weights (Negnevitsky, 2002).
Nodes include an activation or transfer function that controls the type of relationship that the neural network can model. Sigmoid functions are the most popular activation functions – they behave linearly around 0 (when weights are small) and non-linearly around the limits. This means that both linear and non-linear phenomena can be modelled (Tuffery, 2011).

There are various types of neural network models. A multilayer perceptron is the archetypal neural network. This network is made up of several layers, as described earlier – an input layer, one or more hidden layers, and an output layer. Each node in a level is connected to the set of nodes in the previous layer. This type of network is particularly suitable for the discovery of complex non-linear relationships.

Neural networks can be used to create adaptive systems - a system that can change its behaviour according to changes in the information that flows through the network. The main advantage of neural networks is their excellent predictive performance. They are more useful
where the structure of relationships in the data is not clear (Tuffery, 2011). The ANN algorithm enables complex relationships between inputs and outputs to be captured (Shmueli, Patel & Bruce, 2008). Neural networks can also take into account complex interactions between variables, as each input variable is automatically assigned a weight as part of the learning process (Tuffery, 2011). Techniques such as linear regression do not have the ability to deal with such complexities in the data.

Neural networks are effective in the presence of multicollinearity (De Veaux & Ungar, 1994) while the hidden nodes of the network exploit the contribution of each predictor (Ingrassia & Morlini, 2005). This is an important benefit in a domain where sources of information are limited, and system inputs are likely to be highly correlated. Neural networks are also non-parametric; they don’t assume that variables adhere to a particular probability distribution (Tuffery, 2011).

The error of a neural network model can be expressed in terms of the bias squared plus the variance. The bias is the measure of a model's ability to generalise to an unseen test set after the model has been trained on the training sample. The variance can be characterised at a measure of the extent to which the same predictions would have been obtained from a model trained on a different training sample (Sharkey, 1996).

To achieve a model that performs well a balance needs to be struck between the conflicting requirements of small variance and small bias. Attempts to reduce the bias (by fitting the model more closely to the training sample) are likely to result in higher variance, while decreasing the variance (reducing the closeness of fit to the training sample) usually results in increased bias (Sharkey, 1996).

There are three main drawbacks of the neural network approach that are relevant in the current domain. Firstly, neural networks can be described as “black boxes”; the outcome is predicted while the cause and effect relationship between inputs and outputs is not identified. This is a valid approach where a system is too complex to fully understand, although prediction
of behaviour is possible (Rabunal & Dorado, 2006). It means, however, that the system output cannot be validated by relating the model to what is known about the underlying physiological processes. Nor can examination of the model, in this domain, add to our understanding of the mechanics of the relationship between training and performance.

To alleviate this “black box” limitation to some extent, it is possible, as some researchers have done (such as Perl, Dauscher & Hawlitzky, 2003) to generate synthetic data to check the system’s performance against expected physiological behaviour.

The second limitation of ANNs is that of extrapolation. Although ANNs are capable of making predictions based on generalisations made from a given set of data, extrapolations are likely to be unreliable. If the network has been trained upon data within a certain range, its predictions outside of this range may be completely invalid (Shmueli, Patel & Bruce, 2008). It is also important to realise that convergence towards a global solution is not always achieved. Neural networks can get trapped in local minima, and fail to find the global best solution.

The final limitation presents the most serious problem in the application of neural networks to this domain. The flexibility in modelling offered by an ANN comes at a cost: it relies heavily on having a large amount of data for training of the model. ANNs perform poorly when data is insufficient (Shmueli, Patel & Bruce, 2008).

Learning from small data sets is fundamentally difficult. Estimating the underlying probability distribution of the observations is problematic, and models constructed are prone to overfitting (Yeh, 2007). An overfitted model fits the data used for model building well, but has poor generalisation ability for new data (Zhang, 2003). Small data sets also cause problems in estimating the exact relationship between the inputs and outputs (Li & Liu, 2009).

How much data is enough? A common rule of thumb for the minimum size of data sets is to double the number of connections between neurons in the ANN (Edelmann-Nusser, Hohmann & Hennebarg, 2002). Definitions of what constitutes a ‘small’ data set vary between authors. Tsai and Li (2008) defines a training set as large enough if it provides enough information to
enable stable learning accuracy. Modelling a complex, noisy or unstable system generally requires more samples than a simple, stable system. In this domain minimum datasets of 25 or so and upward are likely.

One approach proposed in recent years to alleviate some of the problems associated with modelling small data sets is to generate some virtual data to increase the completeness of the data set (Li and Liu, 2009; Tsai & Li, 2008).

A statistical resampling technique called bootstrapping is particularly useful for small datasets (Sharkey, 1996). Bootstrapping involves sampling, with replacement, from the training set. The limitation of traditional bootstrapping is that no new information is added in the process, so some of the issues of a small dataset remain (Zhang, 2007).

One strategy that can be used in conjunction with bootstrapping to prevent overfitting is the regularisation technique of jittering. Jittering consists of adding artificial noise to bootstrapped data, thus supplementing the training sample with additional artificially created data which is similar to, but different from, the original data. This typically smoothes the dataset, assisting the neural network to learn the true underlying pattern, rather than overfitting to the idiosyncrasies of the sample data (Zhang, 2007). Zur, Jiang and Metz (2004), found that adding additional data samples with jitter to a small dataset prior to training can increase the performance of ANNs.

One of the major developments occurring in neural networks is ensemble modelling. Ensembles combine the output of several individual models. Combining different models can result in an improvement of predictive performance over that of an individual model (Peronne & Cooper, 1992; Sharkey, 1996; Zhang, 2007). The smoothing property of the ensemble method can remove overfitting (Perrone, 1992).

Ensembles formed of models which are quite different (e.g. trained with different data) tend to be most effective. Creating different training datasets can be achieved by a number of different techniques, for example: bootstrapping; using different data sources; or different pre-
processing techniques (Sharkey, 1996). Bagging is a term frequently used to refer to the
technique of training an ensemble of models on a number of different - although overlapping -
datasets created using bootstrapping (Tuffery, 2011). Such techniques are particularly useful
where there is a shortage of data (Sharkey, 1996).

As discussed previously, the error of a model can be expressed in terms of the bias squared plus
the variance. In the context of evaluating an ensemble of nets, the bias term refers to the
measure of the extent to which the ensemble output differs from the target function, while the
variance measures the extent to which the individual members of the ensemble disagree
(Sharkey, 1996). Ensemble techniques can reduce the variance, but not the bias, of a model;
improving the model generalisation (Tuffery, 2011). An effective approach, therefore, is to
create a set of nets with high variance, but low bias - as the variance component will be
removed when combining the models (Sharkey, 1996).

To create models with high variance (i.e. models which disagree in their predictions) the
model’s learning needs to be disrupted in some way. This can be achieved by varying the
learning sample, or by using the same sample but varying the learning parameters (Tuffery,
2011). The consensus emerging from the field is that it is by varying the training data in some
way, models with high variance are more likely to be achieved (Sharkey, 1996).

There is no established methodology for choosing the most appropriate size of an ensemble, as
it is dependent on the number of distinct models that can be created from the dataset
found that an ensemble size of 5 or 10 appeared to be a reasonable choice for a time series
forecasting problem, achieving significant performance gains over a single model, while still
keeping computational effort low.

Once an ensemble of models has been created, an effective way of combining the outputs must
be found. Simple averaging and weighted averaging are two possible techniques (Sharkey,
1996). A type of weighted averaging called boosting – a technique which is commonly applied
to ensembles of decision trees - involves modifying the training set of each model iteratively, to counter errors made in the previous iteration’s model. These modifications generally include a technique to overweight or increase the importance of those observations which were modelled poorly (Tuffery, 2011). Boosting tends to improve model accuracy over a simple ensemble or bagging, but is prone to overfitting, particularly with noisy data (Han & Kamber, 2006). Simple averaging has been shown to be effective and efficient approach (e.g. Zhang, 2003).

Zhang (2007) proposes a neural ensemble model based on the idea of adding jitter to the input data, and forming several different training sets with the noisy data. The jittered ensemble method is based on the concept that for each data point many possible discrete observations could have been made. Creating multiple jittered instantiations for each data point can be viewed as multiple alternate realisations of the underlying data generation process (Zhang, 2007).

A single neural network model - such as that used in the training and performance modelling literature - is likely to result in overfitting problems when used on the smaller datasets gathered in this research. Techniques such as bagging, and applying jitter to the bagged samples should reduce the impact of overfitting, thus allowing access to the benefits of ANNs; particularly their ability to model complex relationships and handle complex interactions between variables. It is clear from this review that the development of new algorithms and techniques should be considered, including the use of other models in conjunction with ANN models (creating a hybrid model) as well as refinements of the ANN algorithm.

4.6.3. Genetic Algorithms

Genetic algorithms aim to reproduce the mechanisms of natural selection. The most appropriate rules for the solution to a prediction or classification problem are selected, and
after crossing and mutating, are transmitted to a child generation. The process is repeated for a number of generations until a sufficiently predictive model is obtained (Tuffery, 2011).

Genetic algorithms are particularly useful for solving optimisation problems. The creation of athlete’s training programs can be thought of as an optimisation problem. Genetic algorithms have not been used in this domain to the author’s knowledge, although they are widely used for solving optimisation problems such as timetabling. Used in combination with pattern mining techniques, genetic algorithms could assist in developing optimal training sub-sequences.

4.6.4. Other Approaches

Other machine learning approaches that are potentially applicable to modelling the relationship between training and performance include Decision Trees and Support Vector Machines. Although there are subtle variations between the different types of machine learning algorithms, the basic process of learning from training data is the same – which means that the results from any individual dataset are usually similar.

4.6.5. Hybrid Models

Hybrid models apply a number of different modelling methods to the same data. They can make a synthesis of the results, or use the output from one model as the input for a second. These approaches can potentially combine the best parts of each model, using each model’s unique features to capture different patterns in the data. It is nearly always possible to build a more robust and more precise model in this manner (Tuffery, 2011).

There is a body of research into the use of combination models for time series forecasting. Some researchers have found that predictive performance improves with the use of combined
models over that of single time series models (e.g. Zhang 2003, Zhang & Qi, 2005), while other researchers have shown a hybrid model can underperform its constituent model performances (Taskaya-Temizel & Casey, 2005).

Hybrid models alleviate to some extent the difficulty of finding the single best performing model for an individual dataset. In theory the combination of a statistical model and a machine learning model could combine the effectiveness of statistical models on relatively small datasets, with the ability of machine learning models to capture complex data patterns.

4.6.6. Evaluation of Machine Learning Models

Machine learning models all involve training the model on representative sample of the dataset; the models learn from this training sample. In order to evaluate the performance of a model, the model is tested against new, unseen data. A dataset is commonly partitioned into a number of subsets, usually a training and a testing set. The testing set of previously unseen data is used to predict the unbiased performance of the model.

There are different techniques for partitioning a dataset. The most common technique is called holdout sampling - the dataset is simply divided randomly into two unequal subsets (commonly 70% training and 30% testing). This technique is not suitable for very small datasets, however, where the size of the training dataset will simply become too small for effective training.

K-folds cross-validation is more suitable for small datasets as it uses all the available data for both training and testing. In this technique data is randomly partitioned into k subsamples. In an iterative process, a model is created where one subsample is used for testing, and the rest are used for training, until all subsets have been used as the testing set. The performance of each of the models is averaged, to give an estimate on how the model might perform on new observations.
Leave-one-out cross-validation (also called jack-knifing) is a form of k-folds cross validation where $k$ is equal to the number of observations in the dataset. In an iterative process a model is created using one randomly chosen data point as the testing set and the rest of the dataset as the training set, until all data points have been used as the testing point once. Leave-one-out cross-validation is effective for very small datasets.

4.7. Summary and Proposed Model

Data mining and machine learning techniques have become increasingly important fields for the discovery of knowledge from datasets in a wide range of domains. The problems and datasets involved in the sport science domain have some distinctive characteristics. Training and performance datasets for individual athletes are inevitably small in size and noisy. The relationship between training and performance is highly individual, so aggregating data between individuals is not likely to be useful.

This chapter presented selected statistical and machine learning techniques that have been, or have the potential to be, used in sport science knowledge discovery projects. Table 4.1 shows summaries of the techniques that have been described. Historically, statistical and mathematical techniques have been used by sport science researchers. Statistical techniques are able to handle relatively small datasets. They tend, however, to be sensitive to noise and they require assumptions to be made beforehand about the form of the relationship in the data. Statistical techniques also tend to be limited in their ability to model highly complex relationships in data.

More recently, machine learning techniques have been successfully applied to problems in a wide variety of domains. Such techniques have not been widely used in the sport science domain as yet. Machine learning techniques tend to be flexible and are generally able to model highly complex data relationships. No assumptions need to be made about the form of the data.
prior to modelling. They do, however, require large amounts of data to achieve such flexibility, and are prone to overfitting in the absence of adequate data. Techniques such as bagging can be used to reduce the impact of overfitting issues.

The literature shows that the majority of models used for the prediction of performance have been mathematical and statistical techniques. A smaller number of researchers have looked at using machine learning techniques, namely ANNs, but there is little evidence of the development of new techniques and algorithms to overcome the current data issues. It is well known that datasets in the training and performance domain are highly complex however little attention has been paid to the issues of size and noise in the data. In fact there is limited research from any domain into the use of machine learning techniques on small datasets. Work such as that of Li, Chen and Lin (2003) and Li et al. (2005, 2009), has focused on generating virtual data in order to create a larger dataset, however this is problematic where little is known about the properties of the underlying population from which the sample is taken. It is therefore desirable to have a method that can alleviate issues with data quality and extract data points that contain higher information content from noisy and incomplete datasets.

Combinations of techniques and models have great potential to be used in predicting performance from training data. Multiple techniques can be used to combine the best features from statistical and machine learning approaches, and improve the robustness of models created on small noisy datasets.

The work of this thesis extends current sport science and data mining research through the development of a model called HANNEM (Hybrid Artificial Neural Network Ensemble Model). HANNEM is a hybrid model, combining a statistical model with neural network ensemble technology. It combines a linear statistical modelling technique to capture linear patterns in the data; with the SimBEA model developed by Bruckner (2006) to capture important domain knowledge regarding the physiological processes underlying training and its impact on performance; and an ANN model to capture non-linear patterns in the data as well as complex interactions between variables. To reduce overfitting and increase the generalisability of the
model the technique of bagging is used, with noise or jitter added to the bags, and an ensemble of ANNs is trained on these slightly different bags of data. Overall the goal of HANNEM is to model the relationship between training and performance with a view towards obtaining sufficient accuracy in predicting performance to permit the model to be used for optimisation of training programs.

<table>
<thead>
<tr>
<th>Technique</th>
<th>Comments</th>
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<tbody>
<tr>
<td>Regression</td>
<td>- Can’t model complex non-linear phenomena</td>
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<tr>
<td></td>
<td>- Is sensitive to collinearity issues</td>
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<td></td>
<td>- Can handle relatively small datasets</td>
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<tr>
<td>ARIMA</td>
<td>- Popular model for time series forecasting</td>
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<td></td>
<td>- Stationarity is a necessary condition</td>
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<td></td>
<td>- Can handle relatively small datasets</td>
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<td></td>
<td>- Describes linear relationships only</td>
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<tr>
<td>Gaussian Process</td>
<td>- Nonlinear regression model</td>
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<tr>
<td></td>
<td>- Considers family of functions that potentially model the system in question</td>
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<td></td>
<td>- Can handle relatively small datasets</td>
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<td></td>
<td>- Provide estimate of uncertainty in predictions</td>
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<td></td>
<td>- Black box</td>
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<tr>
<td>SAX</td>
<td>- Identifies patterns in time series data</td>
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<tr>
<td>ANN</td>
<td>- Models non-linear phenomena and complex interactions between variables</td>
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<td></td>
<td>- Can handle noisy data</td>
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<td>- Can be trapped in local minima</td>
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<td>- Black box</td>
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<td>- Requires large amounts of data to avoid overfitting</td>
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<tr>
<td>Genetic Algorithms</td>
<td>- Solves optimisation problems</td>
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<td></td>
<td>- Uses mutation to avoid premature convergence towards local optima</td>
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### Decision Trees
- Creates rules which are easy for users to understand
- Variables can be collinear
- The model selects the most appropriate variables
- Relatively unaffected by extreme values
- Models non-linear phenomena
- Greedy algorithm detecting local not global optima at each step
- Requires large amounts of data to avoid overfitting

### Support Vector Machines
- Models non-linear phenomena
- Black box
- Requires large amounts of data to avoid overfitting

### Hybrid Models
- Can potentially combine the best parts of a number of models
- Is nearly always possible to create a more robust and precise hybrid model than the best single model
- Alleviates difficulty in choosing best single model

**Table 4.1. Summary of modelling techniques**
CHAPTER 5 – CORRELATION OF TRAINING LOAD AND HEART RATE VARIABILITY INDICES

Over the years researchers have grappled with finding a way to effectively quantify training load using a single term. Training load is primarily a function of the intensity of a session and its volume (duration or distance), yet difficulties remain in determining the appropriate relative weightings between these two variables. A training load quantification technique must be practical and easy to obtain as part of an athlete's normal training routine.

The current chapter discusses the development of a novel training load quantification technique called PTRIMP (Power TRaining IMPulse). PTRIMP has been developed to provide a physiologically consistent weighting of intensity and fatigue. The raw data for the measurement is obtained from power data captured during training rides.

The relationship between PTRIMP and heart rate variability indices (indicative of fatigue) is also investigated. The study aims to determine whether PTRIMP as observed in the field correlates with fatigue measures in a manner consistent with known physiological responses to training. In this way the internal coherence of the PTRIMP algorithm can be evaluated.

5.1. Heart Rate Variability

Heart rate variability has been proposed as a tool for monitoring training-related fatigue in athletes (Earnest et al., 2004; Atlouai et al., 2007). HRV is a function of the synergistic action between the two branches of the autonomic nervous system (the parasympathetic nervous system (PNS) and the sympathetic nervous system (SNS)). They act in balance through neural, mechanical, humoral and other physiological mechanisms to maintain cardiovascular variables in an optimal range during changing external or internal conditions (Earnest et al., 2004).
Activity in the SNS increases heart rate, constricts blood vessels, decreases gastrointestinal motility and constricts sphincters, while activity in the PNS has the opposite effect (Atlaoui et al., 2007). Heart rate (HR) is not constant – rather it fluctuates from beat to beat. HRV refers to the variation of the beat-to-beat interval, measured as the time between beats.

Electrocardiographs (ECGs) and some types of heart rate monitors are capable of measuring the time points in milliseconds between beats. Measuring the beat-to-beat interval during an orthostatic test is a reliable method of assessing HRV in athletes. An orthostatic test involves being supine, then standing up. It is effective as it combines information on parasympathetic tone during the supine portion of the test, where the PNS is dominant, with the interaction of the SNS upon standing (Wisbey & Montgomery, n.d.). While standing, heart rate is increased and cardiac contractility and vascular tone are decreased by a decrease in PNS activity and an increase in SNS activity (Atlaoui et al., 2007).

Fatigue alters the balance between the PNS and SNS. There are suggestions of two types of overtraining – one characterised by SNS dominance, and the other by PNS dominance (Lehmann et al., 1996; Atlaoui et al., 2007). Theoretically, the autonomic imbalances observed in fatigued or overtrained athletes should be discernible in HRV indices, and there is some evidence to suggest this is the case (Baumert et al., 2006; Earnest et al., 2004).

Earnest et al. (2004) found little correlation between HRV and exercise volume of professional cyclists during Stages 1-9 of the Tour of Spain. A strong inverse correlation, however, was observed between HRV and exercise volume and intensity in stages 10-15. If in fact training-induced fatigue is detectable using HRV indices, a correlation should exist between training load and a Recovery Index (RI) derived from HRV. In this study, we examine the HRV response, and compare it to the training load of a female professional road cyclist during normal training and racing over a period of 250 days.
5.2. Training Load Quantification

There has been great difficulty in finding a way to effectively quantify training load using a single term (Foster et al., 2001). A number of different measurements have been used by researchers to estimate exercise intensity. Two of the most appropriate for cycling are heart rate and power data. Banister proposed a quantifying training load using a unit of training termed a TRIMP (TRaining IMPulse) (Banister, 1991). A TRIMP is derived from multiplying the duration of training by the relative training intensity (measured by heart rate). A multiplying factor is applied in order to weight higher heart rate (HR) ratios proportionally higher.

Heart rate is a poor method of evaluating very high intensity exercise (Foster et al., 2001). Although TRIMPs can be calculated for a range of sports, they are not an appropriate load quantification method for strength training, as heart rate only measures the cardiovascular load of exercise.

Heart rate is affected by many factors, which impacts on its reliability as a measure of load. Factors which can affect heart rate include temperature, hydration, sleep, overtraining and caffeine (Jeukendrup, 2002). There are claims that the variability of heart rate impacts on the validity of quantification methods based on heart rate (Skiba, 2008).

5.3. Using Power to Quantify Training Load

Power is the amount of energy generated per unit of time. In cycling, it is the amount of energy transferred to the pedals (in watts) per second. SRM power monitors are commonly used to measure power in cycling. The SRM system provides reliable measurements of power (Gardner et al., 2004), as well as measurements of cadence, speed and distance.

Power is a direct reflection of exercise intensity- whereas heart rate is a response to the exercise intensity (Jeukendrup, 2002). Heart rate is affected by external factors, and also
experiences a delay in responding to changes in exercise factors. As a direct reflection of work output, power is not affected by these limitations.

Power has potential as the basis for quantifying training load, but in order to be useful, the power data needs to be analysed correctly. Taking the mean power of a session is a common summary statistic. Power output is highly variable, however, so the mean doesn’t necessarily accurately reflect the physiological demands of a session.

Training Stress Score (TSS) was proposed by Coggan in 2003 as a load quantification method using power. An intensity factor (IF) is used to normalise the TSS to functional threshold power (or the power that can be maintained in a 60 minute time trial). This allows for easier comparison of TSS between athletes.

The relationship between intensity and physiological load is an exponential curve. We propose that, along with TRIMPS, TSS places too much emphasis on the duration of a session, while not weighting high intensity training enough.

The load quantification technique we have named PTRIMP (Power TRaining IMPulse) was developed in an attempt to overcome this limitation, and better reflect the physiological cost of high intensity efforts.

5.4. Modelling

5.4.1. Data

A female professional cyclist provided sensor data from training and racing, as well as sensor data from a daily orthostatic test over a 250 day period. This period includes the European racing season, as well as a break from training over the Christmas period, and subsequent early season training. The following subsections provide details of the methodology used to collect the data.
5.4.2. **Orthostatic Test**

The subject completed an orthostatic test each morning upon first arising. Wearing a Polar CS600 heart rate monitor (Polar Electro Oy), set to record R-R intervals, the subject lay down in a quiet place for 3 minutes. Recording started after the HR settled. At the three minute mark, the subject raised themselves into a standing position, and recording continued for a further 2 minutes. The HR data was then downloaded, and transmitted for analysis.

5.4.3. **HRV Analysis**

Custom-built software (HRV Athlete, FitSense Australia) was used to analyse the HR file of R-R intervals (recorded in milliseconds). The software uses a Fast Fourier Transform (FFT) and a power spectral density analysis to decompose the signal into its sinusoidal components, allowing plotting of the power of each component as a function of its frequency. Power was broken down into two frequency bands – low frequency (LF, 0.04 to 0.15 Hz) and high frequency (HF, 0.15 to 0.4 Hz) (Wisbey & Montgomery, n.d). Parasympathetic activity is considered responsible for HF, and both parasympathetic and sympathetic outflows are considered to determine LF (Aubert et al., 2003). The LF/HF ratio thus gives an indication of the status of the autonomic nervous system. The LFHF data was smoothed, and then standardised using a z-score to give a Recovery Index (RI).

5.4.4. **Training Load**

A professional model SRM power monitor (SRM, Julich, Welldorf, Germany) was fitted to the subject’s road and time trial bikes. Data from all rides (training and racing) was captured. Data was imported into MATLAB 2007b (Mathworks Inc., Boston, MA) for the calculation of PTRIMP.
A record of the Maximal Mean Power (MMPs), or the highest mean power achieved by the athlete for a defined duration, was kept for 5s, 30s, and 4mins. An MMP was updated at the completion of a ride when a new personal best was achieved. The following steps were taken to calculate PTRIMP. The power data was first smoothed by taking 3 rolling averages, of 5s, 30s and 4mins durations. Each of the smoothed points was then given a weight, which was calculated by dividing the point by the athlete’s Maximal Mean Power (MMP) for that duration and expressing the result as a percentage. The function form of PTRIMP(s), where $s = 5, 30$ or 240 seconds, is a 4<sup>th</sup> degree polynomial of this percentage ($X_i$) (refer to Equation (5-1)). Each PTRIMP(s) is then added together to determine PTRIMP (refer to Equation (5-2)).

$$\text{PTRIMP}(s) = \sum_{i=1}^{n-1} (-9.615e-008)X_i^4 + (2.263e-005)X_i^3 - (0.0004612)X_i^2 + (0.01615)X_i - 0.01119$$  \hspace{1cm} (5-1)

$$\text{PTRIMP} = \frac{\text{PTRIMP}(5s) + \text{PTRIMP}(30s) + \text{PTRIMP}(240s)}{1,000}$$  \hspace{1cm} (5-2)

### 5.4.5. Data Analysis

Coggan (n.d. (c)) suggests using a time constant of 7 days to model the time course of training induced fatigue. PTRIMP data was thus smoothed over 7 days using an exponential moving average.

RI data had some missing days, for instance when the athlete was travelling, or when equipment failed. A linear interpolation was applied, where at most two consecutive days of data was missing. Data was missing for an extended period (21 and 12 days) at two points in the dataset. Values from this period were removed from analysis.

Wavelet-based semblance analysis allows two datasets to be compared on the basis of their phase, as a function of frequency. It is suitable for use with non-stationary data, unlike many other statistical correlation measures (Cooper & Cowan, 2008). This type of analysis is
appropriate for describing the relationship between physiological time series, where describing the entire dataset with a single number is an oversimplification. Datasets might be uncorrelated on short timescales due to noise, while being strongly correlated in larger timescales. The cyclical nature of training programs means it is also possible that a correlation exists during some phases of training, and not others.

The wavelet-based semblance analysis was calculated using Matlab source code developed by Cooper (referred to in Cooper & Cowan, 2008). A time scale of 30 days was found to give the best resolution.

5.5. Results

RI and PTRIMP datasets were strongly negatively correlated on larger time scales of 30 days and above. In smaller scales, there appeared to be small cycles of positive correlation interspersed with negative correlation. The first 40 days were moderately positively correlated (Figure 5-1 (c)).

5.6. Discussion of Results

The hours of training [mean ± standard deviation (SD)] completed over the 250 days studied was 1.5hrs ± 1.3. Average power output was 134 watts ± 34. PTRIMP values were 11.8a.u (arbitrary values) ± 13.

The semblance analysis showed a high negative correlation between PTRIMP and RI over much of the dataset. This is in accordance with the physiological principles underlying the values – when training levels over the last 7 days have been high, it is expected that fatigue is high, and thus the RI is low. Although additional work with more subjects is required, this data indicates
that higher training loads as measured by PTRIMP are associated with higher fatigue levels as measured by the RI.

At smaller time scales, (around 7 days) it becomes apparent that regular periods of positive correlation are interspersed with periods of negative correlation. One possible explanation for this phenomenon is that there is a delay between increased training and the onset of long-term fatigue. Halson (2002) found that it took between 3-7 days of intensified training before overreaching developed.

It is suggested that future investigations looking at whether the periods of positive correlation occur during a particular stage of a training microcycle could provide more insight into this phenomenon. From these results, it is as yet unclear whether HRV data provides useful indications of fatigue at the micro level – or during a normal 7 day microcycle of training.

The first 40 days of the dataset show a moderate positive correlation. This is likely to be a result of both the PTRIMP and the RI calculations being more unstable in this period; both calculations rely on a body of previous data to individualise the scores. Thus PTRIMP may appear higher in this period (Figure 5-1(a)), as multiple new MMP records were set during this time.

![Figure 5-1](image)

Figure 5-1  (a) Values over time of PTRIMP after wavelet transform. (b) Values over time of RI after wavelet transform. (c) Semblance. White corresponds to a semblance of +1 (high positive correlation), 50% grey to a semblance of 0, and black to a semblance of -1 (high negative correlation).
5.7. Summary

This section documented the development of the PTRIMP algorithm for the quantification of training load. The PTRIMP algorithm was developed to accurately reflect the physiological cost of high intensity efforts. The algorithm reflects the exponential relationship between intensity and work performed as well as considering the interaction between intensity and the duration of efforts performed within a training session.

In a novel application wave based semblance analysis was used to compare training load (PTRIMP) with a fatigue indicator (RI) on the basis of their phase as a function of frequency. PTRIMP and RI were strongly negatively correlated on time scales of 30 days or above, showing that high training levels (over the previous 7 days) where linked to high fatigue levels. These results were expected, and provide evidence to support the claims for the physiological accuracy of the PTRIMP algorithm. This work also provides support for suggestions that heart rate variability (HRV) indices may be useful in predicting fatigue in athletes, allowing training to be adjusted to reduce the risk of injury / sickness / overtraining.
CHAPTER 6 – CORRELATION OF NOVEL TRAINING LOAD AND PERFORMANCE METRICS IN ELITE CYCLISTS

Researchers have used many different techniques to quantify performance in the training and performance modelling literature. Many approaches require the athlete to undertake regular performance tests: something that is not practical for elite athletes. Other performance quantification techniques rely on time or other performance measures which are inappropriate for road cycling. A key point of difference for this research is that no change in routine or additional work was required in order for athletes to achieve compliance with data collection requirements.

This chapter considers the development of a novel performance metric that is practical for use by elite road cyclists. The performance quantification technique developed is based on the understanding that performance in a road race is determined by one or more critical periods, of differing durations, during a race.

A case study is presented in this chapter which investigates the relationships between PTRIMP - the novel training load quantification technique developed in Chapter Five; an indicator of fatigue (RI); and the novel performance metric developed. This is done with a view to ascertaining whether the dataset collected (and the techniques developed for quantifying training load and performance) have the potential to be used in the modelling and prediction of performance with sufficient accuracy for the model to be used as a tool in optimising training programs.

6.1. Training Load and Performance

Athletes and coaches seek to optimise athletic performance for key competitions by maximising fitness and minimising fatigue. An understanding of the relationship between training and performance is required to achieve this. From the 1970’s onwards, numerous studies have
focused on modelling physiological responses to training input using linear mathematical concepts in an attempt to better understand the training - performance relationship (Banister et al., 1975; Banister, Carter, & Zarkadas, 1999; Busso, 2003; Busso, Carasso, & Lacour, 1991). These impulse-response models require frequent performance testing to enable estimation of model parameters. Hellard et al. (2006) suggests that greater than 60 observations would be required for the estimation of a four parameter Banister model. Such frequent testing, however, is generally not feasible due to the pressures of competition, travel and training placed on elite cyclists.

A technique to simulate the interaction between training load and performance using a state – event model with adaptive delays has been developed (referred to as the PerPot model) by Perl (2001). Further details on the implementation of the PerPot model are included in Chapter Three. Pfeiffer and Schrot (2009) obtained an excellent model fit for performance using a PerPot model. As with the linear mathematical models, however, this type of model requires frequent performance testing to enable estimation of parameters.

Other nonlinear methods to model training-response relationships have encountered practical limitations. Neural networks can effectively model training and performance (Hohmann, Edelmann-Nusses, and Hennerberg 2001) but similar to other artificial intelligence techniques require large volumes of data in order to converge on an accurate model. A limited volume of data is inevitable in this domain. Training data generally accumulates with one data point per day, per athlete. Performances occur, at most, every week.

We propose using multiple linear regression to create a model for the relationship between training and performance. It is suggested that a realistic number of observations (approximately 20) would be required for such a model (Soper, 2009). This type of model can also take into account possible interactions between input variables (Sen & Shrivastava, 1990).

In order to model training load and performance, techniques to quantify these variables must be utilised. Chapter 5 introduced the novel training load quantification technique called
Training load alone does not provide a complete picture of likely fatigue levels, however, as other factors such as sleep will affect the fatigue levels of an athlete. HRV has been proposed as a tool for monitoring training-related fatigue in athletes (Earnest et al., 2004; Atlaoui et al., 2007).

Heart rate (HR) is not constant – rather it fluctuates from beat to beat. HRV refers to the variation of the beat-to-beat interval, measured as the time between beats. Electrocardiographs (ECGs) and some types of heart rate monitors are capable of measuring the time points in milliseconds between beats. HRV is a function of the synergistic action between the two branches of the autonomic nervous system (the parasympathetic nervous system (PNS) and the sympathetic nervous system (SNS)), which act in balance to maintain cardiovascular variables in an optimal range during changing external or internal conditions (Earnest et al., 2004). Activity in the SNS elevates heart rate while the PNS has the opposite effect (Atlaoui et al., 2007). Measuring the beat-to-beat interval during an orthostatic test is a reliable method of assessing HRV in athletes (Wisbury & Montgomery n.d.) (refer to Chapter 5 for further details on conducting an orthostatic test).

Fatigue alters the balance between the PNS and SNS. Theoretically, the autonomic imbalances observed in fatigued or overtrained athletes should be discernible from HRV indices, and there is some evidence to suggest this is indeed the case (Baumert et al., 2006; Earnest et al., 2004). We propose that adding HRV indices of fatigue, which reflect other stressors such as illness and lack of sleep, will increase the fit of the model.

The final input needed to build up the model we propose is performance – and a technique for quantifying performance is required. Numerous studies (Thomas, Mujika & Busso, 2008; Mujika et al., 1996; Avalos et al., 2005) have used a method where performances are converted into a percentage of personal best performances. A similar approach in cycling would require the athlete to complete a regular performance test. As discussed, this is generally not practical. Other researchers (such as Morton et al., 1990) use methods where an individual’s performance is compared to a ‘perfect performance’ such as a world record performance.
Hellard *et al.* (2006) found these techniques to be unreliable, and in addition they are problematic when it comes to road cycling as an easily definable perfect performance does not exist in this context.

Quod, Martin, Martin and Laursen (2010) examined the maximal power produced by experienced cyclists for durations of 60-600s during a series of mass start cycle races. They found that these maximal powers were similar to the power produced over the same durations in laboratory testing, and conclude that field Maximal Mean Power (MMP) analysis may provide coaches with a useful tool for the quantification of changes in road cycling performance capacity. They note, however, that it is unlikely that a rider will give a maximal effort across each of the durations examined in a single race.

The results of a road race are typically determined by one or more critical periods during the race. The cyclist who can produce the greatest maximal effort for these race-specific critical time periods is potentially the winner (Quod, Martin, Martin & Laursen, 2010). MMP analysis intends to capture the output of a cyclist during these critical periods.

A technique which enables performance data to be collected from training and racing data greatly increases the volume of usable data available to a researcher. We propose such a technique, which builds on the concept of MMP analysis. The proposed technique identifies the shape of an athlete’s Maximal Mean Power (MMP) curve of personal bests over a range of durations. The area under the personal best MMP curve is then compared to the area under the curve for the MMP curve from an individual performance.

The aims of the present study were to use a novel training load quantification method (PTRIMP) and HRV indices (RI) as predictors in a multiple regression model to predict cycling performance calculated using a novel algorithm. PTRIMP, RI and performance are all calculated from field derived training and racing data.
6.2. Modelling

A professional female cyclist provided data for the study over a 250 day period. This period included the European racing season, as well as a break from training over the Christmas period, and subsequent early season training.

6.2.1. Training Load

An SRM power monitor (professional model, SRM, Julich, Welldorf, Germany) was fitted to the subject’s road and time trial bikes over the data collection period. Power data from all rides (training and racing) was captured at 1Hz. In accordance with operator instructions, the SRM was zeroed prior to the start of each session. Data was imported into MATLAB 2007b (Mathworks Inc., Boston, MA) for the calculation of PTRIMP. PTRIMP was calculated as per the method outlined in Chapter 5. A record of MMPs for 5s, 30s, and 4mins achieved by the athlete was kept, and updated when a new personal best was achieved. TSS was calculated using MATLAB, according to the method presented by Coggan (n.d. (a)).

6.2.2. Quantifying Performance

The athlete kept a training diary where a subjective rating of the percentage of effort expended for each session was recorded. This was used to identify races and training sessions where a 100% effort (‘I gave everything I had’) was made.

The durations for which MMP were recorded were at regular intervals from 5s to 20min. These durations are chosen to represent a spectrum of energy system contributions. For durations of up to approximately 10 seconds, energy is predominantly supplied by the anaerobic alactic system. From around 10 seconds to approximately 60 seconds, energy is predominantly supplied by the anaerobic lactic system. Beyond this, the contribution of the aerobic energy
system increasingly becomes the major contributor (Gore et al., 2000; 45). The range in durations thus theoretically balanced out the effects of different types of races.

A curve was created from the MMPs for time durations from 5s to 20min for the power file from which performance was to be measured. The area under the curve was then compared to that of the athlete’s maximum MMP profile at the time of the performance (refer to Figure 6-1). The definite integral was calculated using the trapezium rule.

![Figure 6-1. Comparison of the area under the MMP curve between an athlete's personal best, and a Performance A.](image)

6.3. Orthostatic Test

The subject completed an orthostatic test each morning upon first arising (Wisbey & Montgomery, n.d). Wearing a Polar CS600 heart rate monitor (Polar Electro Oy), set to record R-R intervals, the subject lay down in a quiet place for 3 minutes. Recording started after the HR settled. At the three minute mark, the subject raised themselves into a standing position, and recording continued for a further 2 minutes. The HR data was then downloaded, and transmitted for analysis.
6.3.1. HRV Analysis

Custom-built software (HRV Athlete, FitSense Australia) was used to analyse the HR file and calculate the RI for each day as per the method outlined in Chapter 5.

6.3.2. Data Pre-processing

PTRIMP data was smoothed over 3, 7, 14 and 30 days using a rolling average. Missing values for PTRIMP and RI (caused by equipment failure etc.) were interpolated. A linear interpolation was used for RI. Missing PTRIMP values were interpolated by using HR data captured on the same day. A HR TRIMP was calculated using the equation:

\[
TRIMP = T \text{ (min)} \times \frac{HR_{ex}}{HR_{max}}
\]

where \( T \) = time in minutes, \( HR_{ex} \) = average HR for the session and \( HR_{max} \) = the athlete’s maximum HR. HR TRIMP values were then standardised by calculating z-scores. The following equation was then solved for PTRIMP:

\[
t = \frac{PTRIMP - \mu}{\sigma}
\]

where \( t \) = HR TRIMP z-score, \( \mu \) = mean of PTRIMP sample and \( \sigma \) = SD of PTRIMP sample.

Scatterplots indicated that the relationship between PTRIMP smoothed over 3 days (PTRIMP(3)) and performance is non-linear (inverted parabola). The data was transformed using the natural log of Performance to improve the linear relationship. A constant was added to Performance so that all values were positive prior to transformation. As performance is measured in arbitrary units (a.u), it was judged no information was lost in this process.

The dataset contained outliers. Trials showed that one outlier with very poor performance was artificially inflating the fit of the model. This worst performance was capped at -155,000(a.u).
Successive data points were treated as independent, as in most cases the performance measurements are separated in time.

6.3.3. Multiple Linear Regression Modelling

Mean and standard deviation (SD) were calculated for all variables. For log transformed data the mean was back-transformed ($e^{\text{mean}}$) and SD is presented as a coefficient of variation (100($e^{\text{sd}} - 1$)). The Shapiro-Wilk normality test was performed to verify the normality of the distribution as both z-scores and linear regression techniques assume normality. Pearson’s product-moment correlation coefficient was used to determine the association between variables. The Breusch-Pagan test was used to test for heteroskedasticity of residuals. A Durbin-Watson test was used to test for autocorrelation in the residuals.

Correlation analysis was performed to give an initial indication of whether PTRIMP data smoothed over 3, 7, 14 or 30 days was likely to have the strongest influence on performance. From this analysis, PTRIMP(3) and RI were selected as the variables showing the highest correlation with performance. PTRIMP(3) and RI were not found to be significantly correlated.

A multiple linear regression model was created using linear least squares to minimise the residual sum of squares. The adjusted coefficient of determination ($R^2$) was calculated. A process of data exploration was followed to find the best model. The outcome of this process was the inclusion of interaction terms between RI and PTRIMP(3) and a quadratic term for PTRIMP(3) increased the fit of the model.

A $k$-means cluster analysis was performed on the performance data to derive three clusters (interpreted as low, moderate and high performance). Models were fitted using the R Statistical Package Version 2.6.1 (The R Development Core) and the package Rcmdr (version 1.3-15).
6.4. Results

Table 6.1 outlines the general characteristics of the data. The test for normality indicated that PTRIMP(3) and RI were not normally distributed ($W = 0.8840, P = 0.0084$ and $W = 0.8946, P = 0.014$ respectively). As these values are near-normal however, for the purposes of this study a transformation was not applied.

<table>
<thead>
<tr>
<th>Performance(log)</th>
<th>PTRIMP(3)</th>
<th>RI</th>
</tr>
</thead>
<tbody>
<tr>
<td>99,681.96 ± 46.26</td>
<td>13.75 ± 7.58</td>
<td>75.76 ± 25.53</td>
</tr>
</tbody>
</table>

Table 6.1. Characteristics of PTRIMP(3) and RI for records paired with a performance. $N = 25$. All parameters are recorded in arbitrary units (a.u)

The model providing the best fit for performance included a term for interaction between PTRIMP3 and RI and was of the form $PTRIMP(3) + PTRIMP(3)^2 \cdot RI$ (Adjusted $R^2 = 40\%$, $P = 0.006$).

PTRIMP and TSS were highly correlated over the entire dataset of rides collected ($N = 163$). The correlation was 0.98, and a 95% confidence interval of [0.98, 0.99].

6.5. Discussion and Analysis

The results indicate that PTRIMP(3) and RI can be used as variables in a multiple regression model to predict performance ranges. The model obtained explained 40% of the variation in the performance measured. Statistics relating to the goodness of the fit obtained in modelling training and performance in the literature are quite variable, but help to establish a comparison for the fit obtained by this model. Ranges between 45% - 85% and 67% - 68% for model explained variances in performance have been reported for Banister type models (Mujika et al., 1996; Busso et al., 1997). One study reported a fit for the PerPot model as between 13% and 92% (Ganter, Witte and Edelmann-Nusser, 2006). These models require large numbers of performances in order to estimate model parameters (Hellard, et al., 2006; Ganter, Witte and Edelmann-Nusser, 2006), however, limiting their practicality.
Comparatively few researchers have investigated the relationship between training and performance outside of the laboratory setting. Performance testing in the laboratory attempts to control for as many lurking variables as possible. Factors such as nutrition, hydration, temperature, and pacing strategies have not been controlled in this research using real-world performances. As such, the fit of this model is not expected to be as good as those reported for studies using performances measured by frequent laboratory testing. The aim of this study, however, is to use field-derived data to develop a model which can be used as a practical tool by athletes and coaches.

The significant explanatory power of the model suggests that the methodology used to calculate performance has potential to be used in real world modelling. The non-linear relationship between PTRIMP and RI (refer to Figure 6-2) indicates that for peak performance, an athlete should be fresh, but not too fresh. This is in accordance with common tapering strategies. Figure 6-3 demonstrates the effectiveness of the model in correctly distinguishing between low and high performances. There is some cross-over between low and medium and medium and high performances, however.

The fit of the model was significantly improved by adding RI as an explanatory variable. HRV indices capture more information about an athlete’s state of fatigue than training load alone. HRV is affected by other factors such as sleep, life stress and illness (Fletcher, 2007; Wisbey & Montgomery, n.d.). The addition of the quadratic term for PTRIMP(3) improved the fit of the model due to the non-linear relationship between training and performance.

Two findings of this study were not in accordance with the literature. We found that PTRIMP smoothed with rolling average over 3 days showed the highest correlation with performance. The values for the time course of fatigue parameter (an input into the Banister model) have been reported as 15 days (Morton et al., 1990) and 12.4 days (Mujika et al., 1996). Coggan (n.d. (c)) suggests using a time constant of 7 days to model the time course of training induced fatigue. The findings in the present study suggest a shorter time frame for the time course of fatigue than has been generally reported in the literature.
Banister (1991) proposed that the relationship between training and performance could be conceptualised by Equation Error! Reference source not found.. A variant of this concept was tested for significance in the model, by introducing a variable obtained by subtracting PTRIMP(7) (representing fatigue) from PTRIMP(30) (representing fitness). The time courses chosen to represent fitness and fatigue were as suggested by Coggan (n.d. (c)). The resultant fit was poor.

These findings appear to be interrelated. The correlation between performance and training load decreased as the number of days PTRIMP was averaged over increased. This is most likely related to the algorithm used for quantifying performance. Performance was calculated relative to the athlete’s best performance at that time. As the athlete became fitter, the level required to achieve a ‘good’ performance also rose, thus decreasing the impact of ‘fitness’ (dependent on long term training) and increasing the impact of ‘fatigue’ (induced by short term training).

It is suggested that further work be conducted in a number of areas. The hypothesis that the algorithm used to quantify performance reduced the impact of long term training on the model can be tested by constructing another model. By adjusting the performance algorithm to be relative to the best performance achieved by the athlete over the entire course of the study, the correlation between long term training and performance can be examined.

The relationship between training and performance is complex, non-linear, and changes over time. The variation over time was not captured in the present model. Use of this data with a dynamic systems model (PerPot for example) may provide a better fit. Machine learning techniques are typically suitable for modelling complex non-linear behaviour, and should be considered for use in this domain.

PTRIMP is highly correlated with TSS. PTRIMP does have an advantage over TSS in achieving a more physiologically accurate relationship between the relative weighting of duration and intensity. TSS and PTRIMP both calculate load additively, however TSS has ride duration (T) as a
multiplicative constant (refer to Equations (6-3) and (6-4)). TSS thus places more emphasis on training duration at the expense of intensity compared to PTRIMP.

\[
TSS \approx T \times \sum_{\tau=1}^{n} f(P_T^\tau)
\]  

(6-3)

\[
PTRIMP \approx \sum_{\tau=1}^{n} g(P_T^\tau)
\]  

(6-4)

where \( T = \) total duration time
\( t = \) time measure interval
\( P_T^\tau = \) raw power at time \( t \)
\( f, g = \) functions
\( n = \) number of observations
\( \tau = \) number of rolling average intervals

For example consider the following two training rides:

<table>
<thead>
<tr>
<th>Ride</th>
<th>( t_1 )</th>
<th>( t_2 )</th>
<th>...</th>
<th>( t_n )</th>
<th>Duration (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200w</td>
<td>0</td>
<td>0</td>
<td>1hr</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>200w</td>
<td>0</td>
<td>0</td>
<td>2hr</td>
<td></td>
</tr>
</tbody>
</table>

TSS for ride 2 will be greater than TSS for ride 1. PTRIMP, however, calculates the two rides as equal, which is an accurate reflection of the actual physiological work load.
Figure 6-2 3d graph showing relationship of performance to PTRIMP(3) and RI. The yellow balls represent the location of each observation in 3d space. The surface is fitted using additive regression, allowing the nonlinearity of the relationships to be demonstrated. The lines indicate the residuals (red for negative, green for positive).

Figure 6-3. Fitted performances compared to actual performances, sorted by fitted performance. Actual performances have been separated into 3 groups – low, high and moderate performances.
6.6. Summary

This chapter describes the development of a novel performance metric. The performance metric is founded on the understanding that performance in a road race is determined by one or more critical periods. These critical periods require maximal power outputs over varying durations. The performance metric developed allows performance to be estimated from data collected in real world races. The chapter further describes a model which enables the prediction of performance (quantified using the novel algorithm developed) from an athlete’s training and HRV indices. The model provides athletes with target ranges for training load and fatigue levels to increase the likelihood of a high performance. The use of field-derived inputs makes this model a practical tool for the planning of a taper to maximise competition performance.

This research is an initial case study, and additional work with a larger sample size is required. The results indicate, however, that the novel techniques developed to quantify training load and performance have the potential provide the basis for models of sufficient accuracy to enable their use as a tool in optimising training programs. It is suggested that other, non-linear modelling techniques, such as dynamic systems models or machine learning techniques, be considered to optimise modelling performance.
CHAPTER 7 - IDENTIFYING TAPER SHAPE AND ITS EFFECT ON PERFORMANCE

Athletes and coaches manipulate training load with the aim of achieving the physical state most conducive to optimal performance at the desired time for key competitions. Part of the manipulation of training load involves the reduction of training for a short period prior to a competition - this is known as a taper.

This chapter considers the practical application of the novel performance metric PTRIMP and the novel performance quantification technique (detailed in Chapters 4, 5 and 6) to the problem of identifying the optimal taper for an athlete. Two datasets obtained from two female elite cyclists are modelled to determine each athlete's optimal taper shape. The aim of this investigation was to firstly identify the effect the shape of a taper has on an elite cyclists’ performance, and secondly, to determine if individual differences in optimal taper shape exist between athletes. Furthermore a model which demonstrates that taper shape has a significant effect on performance provides validation of the PTRIMP and performance metrics. It also provides confirmation that such information can be the basis of models accurate enough to be able to provide relevant feedback on the optimal design of training programs.

7.1. The Taper

The taper can be defined as a period of reduced training prior to a competition, undertaken with the aim of achieving peak performance at the desired time (Thomas, Mujika & Busso, 2009). It is of paramount importance in the preparation of athletes for competitions (Pyne, Mujika & Reilly, 2009).

The effectiveness of a taper as reported in the literature varies, however the improvement in performance is usually in the range of 0.5% - 6%. A realistic goal for performance improvement
as the result of a taper is about 3% (Mujika & Padilla, 2003). In competitive athletes such modest improvements are important. A worthwhile improvement for top-ranked athletes is estimated to be in the range of 0.5% - 3.0% for events such as endurance cycling (Hopkins, Hawley & Burke, 1999).

The aim of the taper is to reduce accumulated training-induced fatigue, while retaining or further enhancing physical fitness (Bosquet, Montpetit, Arvisais & Mujika, 2007). The key elements to manipulate in determining an optimal taper include; the magnitude of reduction in training volume; training intensity; duration of the taper; and the pattern of the taper (Pyne, Mujika & Reilly, 2009).

Uncertainty exists about the optimal design of a taper (Mujika & Padilla, 2003). In a meta-analysis study Bosquet, Montpetit, Arvisais & Mujika (2007) suggested that training volume should be reduced by 41% - 60% over a two-week taper, without any modification to training intensity or frequency. They found that reducing training volume elicited a performance improvement approximately twice that gained by modifying either training intensity or frequency.

A number of taper patterns have been described and investigated in the literature. Training load can be reduced in the form of a simple step – where the load is suddenly reduced and then maintained at the same low level; or it can be reduced progressively, either with a constant linear slope, or with an exponential decay (Thomas, Mujika & Busso, 2009; Mujika & Padilla, 2003). There is evidence to suggest that a progressive taper is to be preferred (Bosquet, Montpetit, Arvisais & Mujika, 2007; Banister, Carter & Zarkadas, 1999).

Little research has been done on more complicated taper patterns. One such study looked at the effect of a two-phase taper. This model study found that the last 3 days of the taper were optimised with a 20% to 30% increase in training load. In the modelled response such a two-phase approach allowed for additional fitness adaptations to be made in the final 3 days, without compromising the removal of fatigue. The magnitude of the performance gain is
marginal (0.01%), however, compared to the optimal linear taper (Thomas, Mujika & Busso, 2009).

Much of the literature provides generalised guidelines on designing an optimal taper. It must be noted, however, that individual responses to training vary. Not all athletes respond equally to the training undertaken during a taper, and tapering strategies must be individualised (Mujika, 2009). Individual profiles of training adaptation and the time course of de-training need to be considered in determining optimal taper duration (Mujika & Padilla, 2003). The positive performance results of the two-phase taper in modelling work done by Thomas, Mujika and Busso (2009) suggests that further investigation into optimal taper shapes is warranted.

The purpose of this study was to provide validation of the PTRIMP and performance metrics by investigating the effect of the shape of a taper on elite cyclists’ performance, and determining if individual differences in optimal taper shape exist between athletes. We aim to investigate these aspects from a novel angle – through symbolisation of taper time series information and subsequent relation of the taper shape to performance.

7.2. Modelling the Taper

Two female elite cyclists provided data for the study over a period of 250 and 870 days respectively. An SRM power monitor (professional model, Schoeberer Rad Messtechnik, Germany) was fitted to each subject’s bike(s) over the data collection period. Power data from all rides (training and racing) was captured at 1Hz. In accordance with operator instructions, the SRM was zeroed prior to the start of each session. SRM data files were imported into a custom-built program for the calculation of training load (PTRIMP) and performance.

7.2.1. Quantifying Training Load

PTRIMP was calculated as per the method described in Chapter Five (refer to Equations (5-1) and (5-2)). A record of MMPs for 5s, 30s, and 4mins achieved by the athlete was kept, and updated when a new personal best was achieved.
7.2.2. Quantifying Performance

Each athlete kept a training diary, which was used to identify races. Performances were included in the dataset when training data existed for the three days prior to the performance. Twenty-six performances were identified for Subject A, and 22 for Subject B.

A curve was created from the MMPs for time durations from 5s to 20min for the power file from which performance was calculated. The area under the curve was then compared to that of the athlete’s maximum MMP profile at the time of the performance (refer to Figure 7-1). The definite integral was calculated using the trapezium rule.

![Figure 7-1. Comparison of the area under the MMP curve between an athlete’s personal best, and a Performance A.](image)

The durations for which MMP were recorded were at regular intervals from 5s to 20min. These durations were chosen to represent a spectrum of energy system contributions. For durations of up to approximately 10 seconds, energy is predominantly supplied by the anaerobic alactic system. From around 10 seconds to approximately 60 seconds, energy is predominantly supplied by the anaerobic lactic system. Beyond this, the contribution of the aerobic energy
system increasingly becomes the major contributor (Gore, 2000). The range in durations thus theoretically balanced out the effects of different types of races.

### 7.2.3. Symbolic Aggregate Approximation

Symbolic Aggregate approXimation (SAX) allows a time-series of arbitrary length $n$ to be reduced to a string of arbitrary length $w$. The alphabet size used is also an arbitrary integer (Lin, Keogh, Lonardi & Chiu, 2003). The process involves firstly normalising the time-series to have a mean of zero and a standard deviation of one. A series of breakpoints are identified that divide a Gaussian distribution up into $n$ number of equiprobable regions. These breakpoints are used to map each data point into symbols, such that a data point lower than the smallest breakpoint will be mapped to the symbol ‘a’, a point greater than or equal to the smallest breakpoint but smaller than the next breakpoint will be mapped to ‘b’ and so on (refer to Figure 7-2).

A time series consisting of the three days training load (PTRIMP) in the lead up to a performance was created. The Matlab code of Lin, Keogh, Lonardi and Chiu (2002 & 2003) was used to discretise the time series of PTRIMP data. The code was customised in two areas. A change was made such that normalisation of the time series was performed based on the entire series, rather than just on the current ‘window’ of data. The code which performs numerosity reduction was also removed. A window size of three days and an alphabet size of three were used. An alphabet size of three was selected as a good compromise enabling the creation of a usable number of patterns whilst maintaining reasonable statistical power.
Figure 7-2. Predetermined breakpoints are used to break the feature space into equiprobable regions. Each data point is mapped to the symbol of the region it falls in.

The result was a sequence of three strings for each 3 day taper period, representing the shape of the taper. A result of ‘321’, for example, represented consecutive days of high, medium and low training respectively.

7.2.4. Taper Shapes

The symbolised taper time series allowed different taper “shapes” to be identified. A number of categorisation schemes were tested. Analysis of the data suggested that the training load of the day before a performance had the greatest effect on the subsequent performance. It also suggested that a training load of 3 (high) on the final taper day was highly represented in tapers resulting in poor performances. Using this knowledge a grouping scheme with four categories was developed (low, high-tail, low-tail and high). A low taper contained a combination of low to medium training loads (refer to Figure 7-3). A low tail taper consisted of a high training load on days one and/or two, and low to medium loads on day three (refer to Figure 7-4). A high taper
included tapers with a high training load on days one and/or two, and a high load on day three (refer to Figure 7-5). A high tail taper contained low to medium training loads on days one and two, and a high load on day three (refer to Figure 7-6).

Figure 7-3. Low taper shapes contain the above combinations of low to medium training loads.
Figure 7-4. Low tail taper shapes consist of a high training load on days one and/or two, and low to medium loads on day three.
Figure 7-5. High taper shapes included tapers with a high training load on days one and/or two, and a high load on day three.
Figure 7-6. High tail taper shapes contained low to medium training loads on days one and two, and a high load on day three.

7.2.5. Data Pre-Processing

SRM files occasionally include short durations of spurious power readings. Such spurious data points were identified and removed. The Performance dataset for one subject contained an outlier. This (worst) performance was capped at -155,000(au).

The Performance data was transformed by taking the natural log of Performance. A constant was added to Performance so that all values were positive prior to transformation. As performance is measured in arbitrary values (au), it was judged no information was lost in this process.
7.2.6. \textit{Statistical Analysis}

The Shapiro-Wilk normality test was performed to verify the normality of the distribution. A two-way analysis of variance (ANOVA) confirmed that a significant interaction effect between subject and taper shape was present. Subsequently, the difference between the taper shapes was compared using a one-way analysis of variance (ANOVA). The scale proposed by Cohen (1988) was used for interpretation: the magnitude of the difference was considered either small (0.2), moderate (0.5), or large (0.8). The statistical power for the effect size was determined to indicate the probability of correctly rejecting a false null hypothesis. Statistics were calculated using the R Statistical Package Version 2.6.1 (The R Development Core) and the package Rcmdr (version 1.3-15).

7.3. \textit{Results}

Statistically significant differences were observed in the mean of performances grouped by taper shape between subjects (for raw data refer to Table 7.1). The main effect of taper shape on performance was significant for both Subject A ($F(3, 22) = 4.2, p < 0.05, MSE = 0.5$) and Subject B ($F(3,18) = 3.9, p < 0.05, MSE = 0.3$). The effect of each taper group on performance showed considerable variation between subjects (refer to Figure 7-7).
Table 7.1 Mean and standard deviation (SD) of performance grouped by taper shape for each subject.

<table>
<thead>
<tr>
<th>Group</th>
<th>Subject A</th>
<th>Subject B</th>
<th>Subject A</th>
<th>Subject B</th>
<th>Subject A</th>
<th>Subject B</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>11.28</td>
<td>11.36</td>
<td>0.36</td>
<td>0.27</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>High tail</td>
<td>11.15</td>
<td>11.48</td>
<td>0.23</td>
<td>0.33</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Low</td>
<td>11.83</td>
<td>11.29</td>
<td>0.38</td>
<td>0.25</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Low tail</td>
<td>11.56</td>
<td>11.78</td>
<td>0.33</td>
<td>0.28</td>
<td>10</td>
<td>6</td>
</tr>
</tbody>
</table>

Figure 7-7. Plot of mean performance for each subject, grouped by category. Error bars show 95% confidence intervals. This plot shows the variation between subjects in their reaction to different taper shapes.
The most effective taper for Subject A was a “low” taper, where combinations of low to medium training days were performed. The least effective taper was a “high tail” taper, where low to medium training was performed in days one and two, and a high training load was undertaken on day three. Using Cohen’s scale (Cohen, 1988) the effect size between the “low” and the “high tail” taper was classified as large. The statistical power was 97.5% (5% error level). When expressed as a percentage, this was a 6.1% improvement in performance.

The most effective taper for Subject B was a “low tail” where a high training load was undertaken on days one and/or two, and a low training load undertaken on day three. The least effective taper for Subject B was the “low” taper. Using Cohen’s scale (Cohen, 1988) the effect size between the “low tail” and the “low” taper was classified as large. The statistical power was 92.4% (5% error level). When expressed as a percentage, this was a 4.4% improvement in performance.

7.4. Discussion and Analysis

The results show considerable variance between individuals in their response to a taper. A two-way ANOVA showed that the interaction effects between taper shape and subject was significant. Combining the results of both subjects was considered inappropriate, even though the subjects for this study were reasonably homogenous – both female elite cyclists. Tapering advice in the literature is frequently generalised across genders, between different sports, and between trained athletes and elite athletes. These results suggest that specific tapering advice cannot be provided using a generalist model.

The “low” taper shape was associated with the highest mean performance for Subject A, and the lowest mean performance for Subject B. The optimal taper may be influenced by the intensity and volume of the training preceding the taper. Those who train harder and longer may require a longer taper to enable them to recover, while those with less training require a shorter taper to minimise loss of fitness (Kubukeli, Noakes, & Dennis, 2002). One
possible explanation for the variance in taper effect observed is that Subject A tended to carry more fatigue into the final 3 days of training, due to a higher training load in the preceding training cycle. Thus, a low training load in the taper allowed for more effective dissipation of fatigue, resulting in higher performance.

The most effective taper shape for Subject B was the “low tail” shape. If Subject B went into the final 3 days with relatively low levels of fatigue, higher training loads would not have the negative effect on overall fatigue that they would for Subject A. Thomas, Mujika and Busso (2009) suggest that a moderate increase in training load in the final 3 days of a taper can allow adaptations to training to occur without compromising fatigue minimisation.

The most effective taper shape for Subject A represented a 6.1% improvement, while that of Subject B represented a 4.4% improvement. Paton & Hopkins (2006) estimate that the smallest worthwhile improvement for top-level athletes in the individual sport of road cycling time trials is in the order of approximately 0.6%. They concluded, however, that drafting in road races precludes estimation of the smallest worthwhile change in performance for this event. Nevertheless, it is likely that small improvements in performance, as observed in this study, could have a significant effect on the outcome of elite competition.

The experimental design did not consider the training load prior to the final 3 days of taper before a performance. The study reported in Chapter 6 suggested that training load over the 3 days before a performance had the highest correlation with performance. The current research could be extended by modelling the training load in the 4-11 days prior to the 3-day period studied. Such an extension could potentially determine whether the level of training-induced fatigue brought into the taper period affects the optimal shape of the taper, as hypothesised.
The training load quantification method (PTRIMP) aggregates the volume and intensity of training. This means it is not possible to distinguish the specific influence of training intensity during the taper. The model we have developed can determine the optimal shape of training load within a taper, but cannot provide guidance on the optimal balance between training duration and intensity during the taper. There is general agreement in the literature that intensity should be maintained in the taper (e.g. Mujika & Padilla, 2003; Pyne, Mujika & Reilly, 2009; Bosquet, Montpetit, Arvisais & Mujika, 2007), and this is the general practice followed by the subjects.

In this study, crisp boundaries were used in determining the breakpoints in the symbolisation process. Crisp boundaries result in some loss of data, as two similar training load values positioned either side of a boundary will be treated as different categories although the actual difference in values is small. Removing crisp boundaries in favour of fuzzy boundaries would remove this potential issue.

This research studied the effect of different taper shapes on performance in actual competitions. Few studies have used data from real world training and performance data. The relationship between performance in tests and performance in events was questioned by Hopkins, Hawley & Burke (1999), and how a change in performance in a test translates to performance change in a competition remains uncertain.

Performance tests in competition are affected by external factors, such as climatic conditions, tactics, drafting, terrain and varying competition types (time trials, hilly road races, flat road races and criteriums are all race types included in this study’s dataset). Such factors are either not applicable, or can be controlled for, in a lab situation. The fact that the effect size of the different taper shape treatments is significant for both subjects, however, indicates that the model developed is robust.

The use of field-derived model inputs makes this model a practical tool for the planning of a taper to maximise competition performance. The relatively minimal data collection
requirements make it feasible for elite athletes to provide data without interfering with their normal training and racing.

7.5. Summary

This chapter has built upon the previous investigations by performing an experiment to determine whether the strategies - detailed in Chapters 5 and 6 - for estimating training load and performance from data obtained in the field could be used to provide relevant feedback on optimal training strategies. This investigation describes a model which uses power data from a cyclist’s training and races to determine an optimal individualised taper strategy for the final 3 days before a competition. The model uses a novel application of a symbolisation technique to enable examination of the shape of a taper.

The investigation aimed to identify whether the shape of a taper had a significant effect on the subsequent performance of two elite cyclists. Consequent to this, it aimed to determine whether individual differences in optimal taper shape exist between athletes. The small but significant performance improvements observed for each athlete as the result of undertaking the optimal taper shape are of a magnitude likely to have a significant effect on the outcome of elite competition. The results of this study also suggest that taper responses are highly individual, and cannot be generalised.

The use of field-derived training and performance metrics mean that athletes can provide data without interrupting their normal training and racing schedules. This makes the model a practical tool for the planning of a taper to maximise competition performance. The internal coherence of the model developed, as well as the statistical significance of the effect between taper shape and performance provides validation of the novel training and performance metrics developed as part of this work. Future work could extend the technique described in this chapter to identify the patterns of training load in training microcycles, and relate these patterns to fatigue and performance measures.
CHAPTER 8 – HANNEM: A HYBRID ANN ENSEMBLE FOR SYSTEMS MODELLING

8.1. Introduction

This chapter brings together the load quantification algorithm (PTRIMP) research described in Chapter 5 and the performance quantification algorithm research described in Chapter 6, by creating a model to describe the relationship between training and performance for elite road cyclists. The physiological processes underlying an athlete’s response to training (as described in Chapter 2) form the theoretical underpinning for this chapter. The modelling techniques discussed in Chapters 3 and 4 are combined and extended.

A novel hybrid artificial neural network ensemble model (HANNEM) is developed in this chapter. The model overcomes issues in the domain concerning: limited amounts of noisy and often incomplete data; ambiguity as to the form of the patterns present in the data; and practical concerns limiting the number of independent parameters available for modelling. HANNEM combines the SimBEA model (Bruckner, 2006) - an evolution of the impulse-response models described in Chapter 3, which brings valuable information regarding physiological responses such as supercompensation, accommodation and reversibility - with linear and nonlinear modelling techniques. In the proposed HANNEM model, a linear model identifies the linear patterns in the data, while a neural network model captures the nonlinear patterns in the data.

This investigation tests HANNEM using three datasets consisting of longitudinal training and racing power data, collected from three elite road cyclists. The results of these tests are presented along with a discussion of the effectiveness of the model.
8.1.1. The SimBEA Model

SimBEA is conceptualised as an evolution of the traditional impulse-response model. The model incorporates three important training principles; adaptation and supercompensation; accommodation; and reversibility. A given training impulse causes an immediate performance decrement due to the effect of fatigue, followed by a delayed increase in performance as the result of an increase in fitness. SimBEA thus models the supercompensation curve (refer to Chapter 2 - Training Principles for a discussion on the theory of supercompensation). Figure 8-1 shows how the model behaves in response to a single training impulse - captured as the blue bar. The resulting performance estimate of the model (the red bar) initially decreases in response to the training impulse that occurs on day 2, then rebounds by day 4. This behaviour is consistent with what is known about the supercompensation curve.

The SimBEA model also behaves in accordance with the training principle of reversibility. Figure 8-1 demonstrates that the adaptations made to training gradually return to pre-training levels once training ceases. As no further training stimulus is applied after day 2, once the initial supercompensation effect triggered by this stimulus dissipates performance gradually declines.

![Figure 8-1. SimBEA models the decline in performance once training ceases. The blue bar represents a training impulse, and the red bars the performance estimate of the model. After the initial supercompensation effect triggered by the training impulse, performance declines to pre-training levels.](image-url)
The principle of accommodation states that if a training stimulus is applied at a constant level, following an initial period of adaptation, a plateau will occur with no further improvements in performance. Figure 8-2 shows the response of the SimBEA model to a training load (blue bars) applied at a constant level. After an initial increase, performance (red bars) plateaus. Upon cessation of the training impulse on day 11, there is an initial supercompensation effect as fatigue from the previous training load dissipates.

Chronic training maladaptations, or overreaching / overtraining can occur where there is an imbalance between the training load and recovery (refer to Chapter 2 - Overreaching and Overtraining for further details). The SimBEA model includes a parameter for the maximum rate of adaptation. This parameter in effect places a cap or upper limit on the fitness benefit that results from a training impulse - meaning once the cap is reached the fatigue response dominates and performance declines. Figure 8-3 shows the response of the model to increasing the training load by a factor of 3 on day 4. Performance (red bars) declines. On cessation of training on day 11, performance rebounds. The behaviour of the model is most appropriate for modelling overreaching - a short-term decrement in performance that occurs as the result of brief period of training load and recovery imbalance.
Overtraining is likely to be more appropriately modelled by a system collapse as the restoration of performance following overtraining may take several weeks or months (Kreider, 1998). Such a system collapse is modelled in the PerPot model (Perl, 2004). The relationship between training and performance is dramatically altered during periods of overtraining - a fact not incorporated into the SimBEA model. Identifying when an athlete is overtrained is a complex issue, however, and it is worth questioning how much a more physiologically realistic modelling of overtraining would add to the accuracy of the modelling using the present dataset.

Bruckner (2006, 2008) describes the calculations involved in the SimBEA model (these papers are written in German, so some adaptations have been made to the terms to enhance readability in English). Refer to Table 8.1 for a definition of all terms.

SimBEA stores training impulses in the term AP (adaptive potential). The rate of the release of AP (in the form of fitness) is controlled by the term APR (representing the potential rate of fitness growth). APR at time $t$ is calculated by:

$$APR(t) = \frac{AP(t)}{VA}$$  \hspace{1cm} (8-1)
where \( VA \) is a delay parameter which controls the rate of release of \( AP \). \( AP \) is calculated by the following formula:

\[
AP(t + \Delta t) = AP(t) + L(t) - APR(t)
\]  
(8-2)

where \( L(t) \) is the training load from the previous time increment (the previous day in this case). The complete model is represented by Equation (8-3):

\[
p(t + \Delta t) = p(t) - (BF \times L(t)) + (AF \times BF \times \min(APR(t), APR_{\text{max}})) - AR
\]  
(8-3)

where \( p = \) performance, \( t = \) time \( t \), \( AF = \) adjustment factor, \( BF = \) fatigue factor, \( AR = \) atrophy rate, and \( APR = \) potential rate of growth.

The SimBEA model contains four parameters; an adjustment factor (\( AF \)), a fatigue factor (\( BF \)), a delay (\( VA \)) and an atrophy rate (\( AR \)). Adjustment of these parameters allows the model to be optimised for each individual athlete.

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjustment Factor (AF)</td>
<td>( AF ) determines the proportion of the fitness gain generated by a given training impulse.</td>
</tr>
<tr>
<td>Fatigue Factor (BF)</td>
<td>( BF ) determines the proportion of fatigue generated by a given training impulse.</td>
</tr>
<tr>
<td>Delay (VA)</td>
<td>As discussed previously, a given training impulse results in an immediate increase in fatigue, and a delayed increase in fitness as the recovery or adaptation process operates. The fitness component of training impulses is stored, until release, in what the SimBEA model terms the adaptive potential (( AP )). The delay parameter controls the release rate of the current stock of adaptive potential.</td>
</tr>
<tr>
<td>Atrophy Rate (AR)</td>
<td>As discussed previously, the principle of reversibility states that upon cessation or reduction in training, performance will gradually diminish. The parameter ( AR ) is the term in the SimBEA model which is responsible for controlling the rate of atrophy of</td>
</tr>
</tbody>
</table>
Model Fitting

The four parameters of the model are varied iteratively to determine the values resulting in the best fit for an individual dataset. The model is thus adjusted to suit the individual characteristics of each athlete in their response to training.

Table 8.1. Definition of parameters for the SimBEA model.

8.1.2. Artificial Neural Networks

Artificial Neural Networks (ANN) are modelling tools well suited to modelling complex nonlinear and linear relationships whose exact form is unknown; modelling the relationship between training and performance is just such a problem. Refer to Chapter 4 for a detailed overview of the characteristics of ANNs.

Multilayer Perceptrons (MLP) are the most common form of ANNs. They are particularly suitable for complex non-linear models (Tuffery, 2011). MLPs are made up of several layers: the input unit(s), the output units(s) and one or more hidden levels. Each unit at a layer is connected to the set of units of the preceding layer. Each unit \( j \) receives incoming signals from either the input variables or from every unit \( i \) in the previous layer. There is a weight \( (W_{ij} - \text{initially set randomly}) \), associated with each connection between units. The incoming signal \( (N_j) \) to the unit \( j \) is the weighted sum of all the incoming signals (Equation (8-4)).

\[
N_j = \sum_{i=1}^{m} x_i W_{i,j}
\]

(8-4)

\( N_j \) passes through an activation function \( (x_j) \) to produce the output signal \( (y_j) \) of the unit \( j \). A sigmoid function is the most popular activation function. It behaves linearly around the value of 0 (when weights are small) and non-linearly at the limits. Refer to Figure 8-4 for a diagram depicting the flow of signals through an ANN.
The weight associated with each connection is adjusted as the network learns. Gradient back-propagation is a common algorithm used for this adjustment process. The value delivered by the output unit is compared to the actual value, and an error value is calculated. The weights and bias of the network are then updated so as to minimise the mean squared error between the prediction and the actual value (Equation \((8-5)\)). The learning rate is a variable which controls the speed at which the weight and bias values change.

\[ W_{i,j}^{new} = W_{i,j}^{old} + (l \times err_j) \]

\[(8-5)\]

where \(l\) is the learning rate, and \(err_j\) is the error value for unit \(j\). For a unit \(j\) in the output layer, \(err_j\) is calculated by:

\[ err_j = O_j(1 - O_j)(T_j - O_j) \]
where \( O_j \) is the actual output of unit \( j \) and \( T_j \) is the known target value of the given training tuple. The error of a hidden layer unit \( j \) is:

\[
err_j = O_j (1 - O_j) \sum_k err_k w_{j,k}
\]

(8-7)

where \( w_{j,k} \) is the weight of the connection from unit \( j \) to a unit \( k \) in the next higher layer, and \( err_k \) is the error of unit \( k \) (Han & Kamber, 2006).

Researchers in the training modelling space have reported favourable results in the use of ANNs to model swimming performance (Edelmann-Nusser, Hohmann & Hennerberg, 2002; Hohmann, Edelmann-Nusser & Hennerberg, 2001; Silva et al., 2007). The research in this domain has, however, failed to properly address the overfitting issues involved with modelling small datasets using ANNs. Hohmann, Edelmann-Nusser and Hennerberg (2001) used data from a different athlete to pre-train their network, however the individual nature of the response to training makes this approach unlikely to be successful in all but exceptional cases.

8.1.3. Learning From Small Datasets

Learning from small data sets is fundamentally difficult; small data sets make models prone to overfitting, and increase the difficulty in discovering the exact relationship between input(s) and output (Yeh, 2007). High-noise, non-stationary time series increase the modelling difficulty even further. There are an infinite number of models which would fit the training data well in such situations, but few which generalise well (Giles, Lawrence & Tsoi, 2001).

In Chapter 4, the technique of bootstrapping (sampling with replacement) was introduced as a means of creating artificial data. Better performance can be achieved
when ANNs, as unstable predictors, are redundantly combined using an ensemble of models. This ensemble technique improves generalisation ability (Sharkey, 1996), and is particularly useful when learning from a small data set. Bagging is a related technique, referring to the construction of a family of models on $n$ bootstrap samples (Tuffery, 2011). The process of resampling with replacement increases the independence of the training sets, and hence the independence of the models, leading to improved ensemble results (Raviv & Intrator, 1996). Bagging produces predictions that are more robust to noisy data, and the technique has been proven to always have improved predictions over a single predictor (Han & Kamber, 2006).

Some researchers (Raviv & Intrator, 1996; Zur, Jiang & Metz, 2004; Zhang, 2007) have proposed ensembles of neural networks with noise or jitter added to the bagged training sets. Training with noise causes the data to appear smoother to the neural networks, improving the model’s ability to learn the true underlying pattern of the data and avoid overfitting (Zhang, 2007). The smoothed bagged approach means that larger datasets with greater independence are created, simulating the true noise in the data (Raviv & Intrator, 1996). An observed dataset can be thought of as a single instantiation of an infinite number of datasets which could have been realised from the underlying process. The particular instantaneous depends on the noise values at each point, or any other random shocks (Zhang, 2007).

There are no established principles reported in the literature for choosing the appropriate level of noise. The most effective level of noise is likely to be dependent on the individual dataset, and its inherent noise levels (Zhang, 2007).

8.1.4. Linear Models - ARIMA

Autoregressive integrated moving average (ARIMA) models are used for prediction of non-stationary time series where linearity between variables is assumed (Gomes et al.,
ARIMA models are linear in that predictions of future values are constrained to hold a linear relationship to past observations (Zhang, 2003). They are widely used in forecasting social, economic, engineering, foreign exchange and stock problems (Khashei, Bijari & Raissi Ardali, 2009).

The model formulation actually consists of three sub-models: the autoregressive (AR), the integrated (I), and the moving average (MA). ARIMA models require that the time series is stationary. A stationary time series does not show a trend; its mean and the autocorrelation structure are constant over time. Differencing is a commonly applied technique to remove a trend in the data (Zhang, 2003). The integrated component of the ARIMA model aims to render the data series stationary, while the MA and AR components aim to address the stochastic elements (Reddy, 2011).

ARIMA models are expressed as ARIMA \((p,q,d)\) where \(p\) is the order of the AR component, \(q\) is the order of the MA component, and \(d\) is the differencing number applied. The AR model captures the past behaviour of the system, expressing the residuals at the current time as a linear function of \(p\) past residuals. The MA model is a special type of low-pass filter, which captures the transient shocks or perturbations of the system with a linear function of \(q\) past white noise errors (Reddy, 2011).

The AR component or ARIMA \((p,0,0)\) method is represented as:

\[
Y_t = \theta_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + e_t
\]

where \(p\) is the number of the autoregressive terms, \(Y_t\) is the forecast output, \(Y_{t-p}\) is the observation at time \(t-p\), and \(\phi_1, \phi_2, \ldots, \phi_p\) is a finite set of parameters determined by linear regression. \(\theta_0\) is the intercept and \(e_t\) is the error associated with the regression.

The MA component, or ARIMA\((0,0,q)\) is represented as:

\[
Y_t = \mu - \phi_1 e_{t-1} - \phi_2 e_{t-2} - \cdots - \phi_q e_{t-q} + e_t
\]
where $q$ is the number of the moving average terms, $\phi_1, \phi_2, \ldots, \phi_q$ are the finite weights and $\mu$ is the mean of the series.

An ARIMA ($p,0,q$) model has the form:

\[
Y_t = \theta_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \mu - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q} + e_t
\]

An ARIMA ($p,d,q$) model is an ARIMA ($p,0,q$) model as per Equation (8-10), performed on a time series that has been differenced $d$ times.

ARIMA models can provide accurate forecasting over short periods of time. They are easy to implement, requiring no further information than the time series to be modelled. They do, however, require large amounts of historical data in order to yield desired results (Khashei, Bijari, Raissi Ardali, 2009). In the literature it is estimated that to be effective ARIMA models require at least 50 - or preferably greater than 100 - data points (Khashei, Bijari, Raissi Ardali, 2009; Tseng, Yu & Tzeng, 2002; Reddy, 2011).

Although ARIMA models have been used extensively in a variety of domains, only one athletic performance modelling study to date has applied an ARIMA model to a performance dataset. Ferger (2008) fit an ARIMA model to the performance dataset gained for a single subject in a lab study. The fit reported was moderate, with an Intraclass Correlation Coefficient (ICC) of 0.47. It is important to note, however, that the unrealistic conditions of data collection and the short (90 days) duration of the dataset are likely to result in enhanced performance of the model to what would be obtained under realistic conditions.

### 8.1.5. Hybrid Models

It is nearly always possible to build a more robust and precise model by combining a number of different models. The prediction obtained from the first model can be used as an independent variable in the second model (Tuffery, 2011). The idea of the
combined model is to use each model’s unique capabilities to capture different patterns in the data (Zhang, 2003).

Zhang (2003) notes that it is difficult for modellers to choose the right modelling technique for each unique situation. Modellers generally perform testing with a selection of the models which are most likely to fit their needs, and then choose the one that returns the best result. The final model selected is not necessarily the best for future predictive performance. Many studies suggest that by combining several models, forecasting performance can be improved over that of an individual model (e.g. Makridakis & Hibon, 2000; Khashei, Bijari & Raissi Ardali, 2009). This obviates to some extent the requirement to find the single best model (Zhang, 2003).

As noted in Chapter 4, real world time series are rarely purely linear or nonlinear, frequently containing both linear and nonlinear patterns (Zhang, 2003). As it is difficult to identify the nature of all the patterns in a dataset from a real-life problem, hybrid methodologies that have both linear and nonlinear modelling capabilities can be an effective modelling technique (Khasei & Bijari, 2010).

Hybrid ARIMA –ANN models have been used to model time series from a number of applied problem areas (Diaz-Robles et al., 2008). In such hybrids, the ARIMA model deals with the non-stationary linear component and the neural network deals with non-linear patterns in the data thus improving forecasting performance (Taskaya-Temizel & Casey, 2005). Zhang (2003) found that a hybrid ARIMA and neural network model resulted in improved forecasting accuracy compared to that achieved by either model in isolation.

Hybrid ARIMA-ANN models are generally constructed sequentially, by first applying an ARIMA model to the time series, and then using the residuals from the ARIMA model as the input for the ANN model (e.g. Zhang, 2003; Diaz-Robles et al., 2008). It is typically assumed that the residuals of a linear component will include valid non-linear patterns that can be modelled using a neural network (Zhang, 2003; Zhang & Qi, 2005). This
assumption that the relationship between the linear and non-linear components is additive may not, however, be a valid assumption (Taskaya-Temizel & Casey, 2005).

8.1.6. Proposed HANNEM Model

HANNEM has been designed and developed to overcome some of the limitations inherent in modelling small, noisy datasets, and to take advantage of the robustness and increased generalisation ability offered by new techniques such as the use of hybrid models, ensembles of models, and data processing techniques such as bagging. It has been designed to work with training and performance data collected from the field, and quantified using the novel techniques already outlined in this thesis. The proposed HANNEM approach incorporates the following components (refer to Figure 8-5 for a schematic of the components):

1. PTRIMP: input used to provide a summary of the athlete’s training load.

2. Performance: the HANNEM output.

3. Cluster analysis: used in a novel technique to identify and eliminate races where the athlete did not give 100% effort for strategic reasons.

4. Bruckner’s (2006) SimBEA model: the prediction from a SimBEA model is used as a model input.

5. Linear statistical models: a model input used to model the linear patterns in the data. An ARIMA model was used for two datasets, and least squares multiple linear regression for the third dataset.

6. An ensemble of ANNs which take as inputs the performance predications from a SimBEA model and the linear model as well as a summary of the recent training load (PTRIMP), to capture the nonlinear patterns in the data.
7. Data preprocessing: use of the technique of bagging and the addition of noise or jitter to the bagged training sets in order to create diversity within the ensemble component ANNs.
Figure 8-5. A schematic diagram of the component models of HANNEM, showing model inputs and outputs. A SimBEA model and either an ARIMA or linear regression model were fitted. The performance predictions from SimBEA and the linear model, along with a rolling average of PTRIMP (training load) were used as inputs into an ensemble of ANN models, after the technique of bagging and adding jitter or noise to the data was applied. The performance prediction of HANNEM is the mean of the component ANN predictions.
8.2. Method

8.2.1. Subjects

Two female and one male professional road cyclists provided data for the study over a period of 250, 870 and 1107 days for subjects B, A and C, respectively. An SRM power monitor (professional model, Schoeberer Rad Messtechnik, Germany) was fitted to each subject’s bike(s) over the data collection period. Power data from all rides (training and racing) was captured at 1Hz. In accordance with operator instructions, the SRM was zeroed prior to the start of each session. SRM data files were imported into a custom-built program for the calculation of training load (PTRIMP) and performance.

8.2.2. Quantifying Training Load

Table 8.2 describes the parameters that were derived to quantify training load.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTRIMP</td>
<td>Measure of training load, calculated as outlined in Chapter 5.</td>
</tr>
<tr>
<td>Chronic Training Load (CTL)</td>
<td>CTL provides a measure of training load over a longer time period by calculating an exponentially-weighted moving average of daily training load (PTRIMP) values. A time constant of 30 days has been used here.</td>
</tr>
<tr>
<td>Acute Training Load (ATL)</td>
<td>ATL provides a measure of short-term training load by calculating an exponentially-weighted moving average of daily training load (PTRIMP) values. A time constant of 5 days has been used here.</td>
</tr>
<tr>
<td>ATL3</td>
<td>Calculated as per ATL, but using a time constant of 3 days.</td>
</tr>
<tr>
<td>Training Stress Balance (TSB)</td>
<td>The difference between CTL and ATL. It conceptually represents the difference between 'fitness' and 'fatigue' and as such is used as a performance indicator.</td>
</tr>
</tbody>
</table>

Table 8.2. Training load parameters.
8.2.3. Quantifying Performance

The SRM download software (released by SRM to enable athletes to download data files from the device to their computer) enables the user to associate ride notes with each file. These notes, as well as the training diary kept by one athlete, were used to identify races. A performance measure (calculated as using the custom built program outlined in Chapter 7, Modelling the Taper) was calculated for each identified race. Performances were included in the dataset when training data existed for the three days prior to the performance.

The dataset for Subject C contained numerous stage races (multi-day races). It was noted that performance was highly variable within a stage race. It is common that not all stages within a stage race will be ridden with a maximal effort. Some stages may involve conserving energy for strategic reasons. Strategies to conserve energy can involve drafting, or riding in other rider’s slipstream to reduce drag and thus save energy. In order to identify which races involved a maximal effort, and which races involved conserving energy for strategic reasons, cluster analysis was used. The k-means cluster algorithm (JMP version 8 Cluster platform, SAS Institute, Cary, NC) was used to create 8 clusters on the variables performance and average speed. A further 8 clusters were also created on the variables average power and average speed (8 clusters were found to create the most cohesive groupings). Performances that were included in the high average speed and low performance or low average power clusters were identified as races in which energy was conserved rather than being maximal performances.

Twenty eight performances were identified for Subject A, twenty three for Subject B, and one hundred and forty two for Subject C. It is important to note that although performances are a time series, they occur at irregular intervals.
8.2.4. Additional Parameters

A number of additional parameters were derived and considered for inclusion in HANNEM. The following subsections provide details of these parameters.

Symbolic Aggregate Approximation

A time series consisting of the three days training load (PTRIMP) in the lead up to a performance was created. The Symbolic Aggregate approxiXimation (SAX) technique outlined in Chapter 7 (Modelling the Taper) was used to symbolise the time series of PTRIMP data. A window size of three days and an alphabet size of three were the settings used.

Taper Shapes

Chapter 7 (Section 2) outlined a grouping scheme consisting of four categories (low, high-tail, low-tail and high). These categories were again used for this study.

Orthostatic Test and HRV Analysis

The dataset for Subject A contains HRV data obtained from a daily orthostatic test. Details of the orthostatic test protocol and the resultant HRV analysis are contained in Chapter 5 (Modelling of the data). The variable Pnn50 was standardised (using a z-score) and included in the data set for Subject A. Pnn50 is the percentage of successive R-R interval differences larger than 50ms. It was calculated using custom-built software (HRV Athlete, FitSense Australia).

Race Type

The type of race from which a performance derived was determined, and categorised into one of three categories; road race, tour or criterium. Road race was the category used for single day races which were not criteriums. Criteriums are races held on a short circuit course, involving numerous laps. Multi-day stages races were categorised as tour.
8.2.5. Data Pre-processing

SRM files occasionally include short durations of spurious power readings caused by sensor interference. Such spurious data points were identified and removed. Each athlete’s dataset of performances was analysed for outliers. One outlier was identified for Subject A – as it was an early performance recorded before sufficient data had been captured to enable stabilisation of MMPs, the decision was made to cap this performance at -20,000 arbitrary units (au).

The performance time series of all subjects showed a significant downward trend. This was due to the nature of the performance measure. Performances were calculated in comparison to the athlete’s best performance to date. It takes some time before the database of personal bests for an athlete becomes stable. The trend in the performance series obscured the underlying performance data, and was thus pre-processed to remove the trend. Differencing of order 1 was used.

8.2.6. Model Fitting

This section describes the fitting process for the models which make up HANNEM: SimBEA, ARIMA or least squares multiple regression and an ensemble of ANNs.

SimBEA

JMP version 8 (SAS Institute, Cary, NC) was used to calculate and fit the SimBEA model as set forth by Bruckner (2006, 2008). Refer to Equation (8-3) for the equation used. The nonlinear platform in JMP was used to fit the model parameters, by minimising the residual sum of squares error. For Subject A and B, the model was fit to the entire dataset. For Subject C, whose dataset spanned 4 years, the model was fit to each year separately.
**ARIMA**

An ARIMA (1,0,1) model was fitted to the time series of differenced performances, using the JMP time series platform.

**Least Squares Multiple Linear Regression**

The dataset for Subject C contained the most noise. In order to improve the linear model for this dataset, a technique allowing the addition of additional information was required. To that end, a multiple linear regression (MLR) model was fitted using standard least squares, within the JMP fit model platform. The input parameters were: fitted ARIMA performance predictions; date of performance; duration and; race type.

**ANN Model Details**

After experimentation, the parameters in Table 8.3 were selected as the best model inputs for the ANN.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Parameter 1</th>
<th>Parameter 2</th>
<th>Parameter 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>SimBEA</td>
<td>CTL</td>
<td>ARIMA</td>
</tr>
<tr>
<td>B</td>
<td>SimBEA</td>
<td>CTL</td>
<td>ARIMA</td>
</tr>
<tr>
<td>C</td>
<td>SimBEA</td>
<td>ATL3</td>
<td>MLR</td>
</tr>
</tbody>
</table>

*Table 8.3. The input parameters for component ANN models by subject.*

The technique of bagging was used for each dataset. Random samples with replacement were taken of each original dataset. The size of the bags created were 42 for Subject A, 30 for Subject B and 350 for Subject C. In the absence of established guidelines, experimentation was used to determine appropriate bag sizes. Random noise was applied to the samples after bootstrapping. After experimentation, a level of noise resulting in good performance for each of the datasets was determined (refer to Table 8.4).
<table>
<thead>
<tr>
<th>Variable</th>
<th>Variation</th>
<th>Subject A &amp; B</th>
<th>Subject C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance</td>
<td>±8,000</td>
<td>±4,000</td>
<td></td>
</tr>
<tr>
<td>CTL</td>
<td>±0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATL3</td>
<td></td>
<td>±0.2</td>
<td></td>
</tr>
<tr>
<td>SimBEA</td>
<td>±0.3</td>
<td>±0.2</td>
<td></td>
</tr>
<tr>
<td>ARIMA</td>
<td>±8,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiple Linear Regression</td>
<td></td>
<td>±4,000</td>
<td></td>
</tr>
</tbody>
</table>

Table 8.4. The level of noise applied to bootstrapped sample by variable and dataset.

In order to identify the performances which resulted in the largest errors in model prediction, an ANN ensemble model was created on the performance dataset for Subject C prior to bootstrapping. The distribution of the difference between predicted and actual performances were examined for outliers. Six performances were consistently modelled poorly. In a simple form of boosting, these performances were added to the training sample with replacement, to increase the representation of these particular samples within a bootstrapped sample.

The neural net platform of JMP was used to create each component neural net model. A network with one hidden layer was created. An S-shaped (sigmoid) activation function (refer to Equation (8-11)) was used (refer to Chapter 4 for a discussion of activation functions for neural networks).

\[
S(x) = \frac{1}{1 + e^{-x}}
\]  

(8-11)

where \(x\) = the weighted sum of inputs.

Refer to Figure 8-6 for a sample diagram of the ANN architecture.
Figure 8-6. Diagram of ANN architecture. In this example there are 3 input units, a hidden layer with 3 units, and 1 output unit.

*K*-fold crossvalidation (with 5 groups) was used to judge model performance.

An ensemble of 8 neural networks was created. A script was used to automate the process within the JMP software (refer to Appendix C, Table C.1 for a listing of the algorithm used). Each individual neural network was trained on a training dataset with the addition of bootstrapped samples with random noise injected. Each model was thus trained on a slightly different training sample. *N* iterations were performed of the ensemble building process (where *n* = number of performances in the dataset minus 1). For each iteration, 1 performance was excluded from the training samples to enable ensemble performance to be validated using leave-one-out testing.

A sequence of fits was applied for each individual ANN model in order to determine the optimal ANN configuration. The number of hidden nodes was varied from 2 to 5, and the learning rate varied from 0.001 to 0.04. Learning rates typically have a value between 0.0 and 1.0 (Han & Kamber, 2006); rates in the range of 0.001 to 0.04 were found to be most effective with this data. The hidden node and learning rate parameters which resulted in the best performed model (judged by $R^2$ values for the fit of the trained model to the test partition created using $k$-fold cross-validation) was selected, and the resulting formula for the model was saved.

The predictions of each individual model were combined using a simple average. The averaged predicted performance was then compared to the actual performance.
8.2.7. Evaluation

Summary statistics such as the coefficient of determination \( R^2 \), adjusted coefficient of determination (adjusted \( R^2 \)), and root mean square error \( \text{RMSE} \) (defined in Equation (8-12)) were used as performance measures for HANNEM.

\[
    \text{RMSE} = \sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{N}}
\]

where \( y_i \) is actual performance at row \( i \), \( \hat{y}_i \) is the predicted performance at row \( i \) and \( N \) is the number of predictions.

8.3. Results

8.3.1. Identifying Sub-Maximal Performances: Cluster Analysis

Cluster analysis shows distinctive groupings of races based on the ratio between performance and average speed. A cluster of submaximal race performances were identified as Cluster 4 (Figure 8-7). This cluster was characterised by low performances and high average speeds, indicating flat stages with significant drafting.

Examining the ratio between average power and average speed also identified a cluster (Cluster 1) of submaximal race performances (Figure 8-8). This cluster was characterised by low performance and high average speeds, indicating a flat stage with significant drafting. There was some overlap with the sub-maximal performances identified in the previous cluster analysis.
Figure 8-7. Cluster analysis on the variables performance and average speed for Subject C. Cluster 4 shows a group of performances with high average speed and low performance – indicating a submaximal effort.
Figure 8-8. Cluster analysis on the variables average power and average speed for Subject C. Cluster 1 shows a group of performances with high average speed and low average power – indicating a submaximal effort.
8.3.2. Performance Time Series

Subject A

The performance time series for Subject A does not show a significant trend (Figure 8-9). Figure 8-10 shows the time series after a differencing of 1 was applied.

Subject B

The raw performance time series for Subject B shows a downward trend (Figure 8-11). Figure 8-12 shows the time series after a differencing of 1 was applied to remove the trend.

Subject C

The raw performance time series for Subject C shows a downward trend (Figure 8-13). The high variability in this time series is also apparent. Figure 8-14 shows the time series after a differencing of 1 was applied to remove the trend.

![Figure 8-9. Raw performance time series for Subject A – no significant trend apparent.](image)
Figure 8-10. The performance time series for Subject A after a differencing of 1 was applied.

Figure 8-11. Raw performance time series for Subject B showing downward trend.

Figure 8-12. De-trended performance time series for Subject B (differencing of 1 applied).
8.3.3. Individual Model Performance

The performance of the SimBEA model alone was poor for each subject’s dataset. The relationship between predicted and actual performance for Subject A and C was nonlinear, while for Subject B a weak linear relationship was apparent (refer to Table 8.5, Figure 8-15, Figure 8-16, Figure 8-17).

The ARIMA model had a stronger fit for each subject (Table 8.5, Figure 8-18, Figure 8-19). The fit of the ARIMA model for Subject C was still poor however, so a MLR model was created. The MLR model showed a substantial improvement over the ARIMA model (Table 8.5, Figure 8-20).
<table>
<thead>
<tr>
<th>Model details</th>
<th>Model performance (R²)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type</strong></td>
<td><strong>Model Input</strong></td>
</tr>
<tr>
<td>SimBEA</td>
<td>PTRIMP</td>
</tr>
<tr>
<td>ARIMA (1,0,1)</td>
<td>Performance (differenced)</td>
</tr>
<tr>
<td>MLR</td>
<td>ARIMA predictions, Performance Data, Duration, Race Type</td>
</tr>
</tbody>
</table>

Table 8.5. Performance of the individual models.
Figure 8-15. SimBEA v Performance for Subject A. A weak non-linear relationship is apparent.

Figure 8-16. SimBEA v Performance for Subject B. \( R^2 = 0.32 \)

Figure 8-17. SimBEA v Performance for Subject C. A weak nonlinear relationship is apparent.

Figure 8-18. Bivariate fit of Performance by ARIMA for Subject A. \( R^2 = 0.72 \).

Figure 8-19. Bivariate fit of Performance by ARIMA for Subject B. \( R^2 = 0.53 \).
8.3.4. ANN Model Input

The inputs for each of the subject’s ANN models are graphed in Figure 8-21, Figure 8-22 and Figure 8-23. It can be seen that each of the inputs pick up different parts of the overall pattern of performance. Slightly different parameters were found to be most effective for Subject C compared to the other two subjects.

Figure 8-21. Subject A ANN model inputs.
Figure 8-22. Subject B ANN model inputs.

Figure 8-23. Subject C ANN model inputs. Note the greater variability in performance for Subject C.
8.3.5. ANN Model Performance Prior to Bootstrapping

The subjects’ ANN model performances are summarised in Table 8.6. Each ANN model was generated three times, and the results reported. As neural networks are unstable estimators, reporting only the results from a single run could lead to misleadingly good or poor results. The performance of the ANN models for Subject B showed the greatest variability.

<table>
<thead>
<tr>
<th>Model details</th>
<th>Performance ($R^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model parameters</td>
<td>Hidden nodes</td>
</tr>
<tr>
<td>SimBEA, CTL, ARIMA</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>3-5</td>
</tr>
<tr>
<td>SimBEA, ATL(3), MLR</td>
<td>3</td>
</tr>
</tbody>
</table>

* Range of model performance over 3 runs.

8.3.6. Bootstrap and Noise

Section 8.2.6 Model Fitting detailed the steps taken in pre-processing the datasets to bootstrap the data and add random noise. The stepped effect created in the performance series by the bootstrapping with the addition of random noise is illustrated by Figure 8-24 - Figure 8-29.
The performance of the subjects’ ANN models for all datasets improved when they were constructed using datasets that had been bootstrapped with the addition of noise (refer to Table 8.7 for a summary of the performance improvement). The performance improvement was most marked for Subject B, the dataset which had displayed the most variability in the earlier ANN model.
### 8.3.7. HANNEM Performance

For Subjects B and C, the ANN ensemble performed better than the best single network. For Subject A, the best single network performed substantially better than the ensemble. In all cases, the ensemble performed substantially better than the worst single network (Table 8.8). The predicted performance tracks very closely to actual performance for Subject A and B (Figure 8-30 to Figure 8-33). The accuracy of the performance predictions is more variable for Subject C, but still tracks actual performance quite closely (Figure 8-34 and Figure 8-35).

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>Subject</th>
<th>Hidden Nodes</th>
<th>Overfit penalty</th>
<th>Performance ($R^2$)</th>
<th>Bootstrap Improvement ($R^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>SimBEA, CTL, ARIMA</td>
<td>A</td>
<td>5</td>
<td>0.001 – 0.004</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>4-5</td>
<td>0.002-0.004</td>
<td>0.94</td>
<td>0.96</td>
</tr>
<tr>
<td>SimBEA, ATL(3), MLR</td>
<td>C</td>
<td>3,5</td>
<td>0.004-0.002</td>
<td>0.61</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Table 8.7. Improvement of ANN model when bootstrapping and jitter are performed.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Ensemble Average</th>
<th>Best Single Network</th>
<th>Worst Single Network</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R^2$</td>
<td>$R^2$ Adjusted</td>
<td>RMSE</td>
</tr>
<tr>
<td>A</td>
<td>0.78</td>
<td>0.77</td>
<td>48690</td>
</tr>
<tr>
<td>B</td>
<td>0.62</td>
<td>0.60</td>
<td>19999</td>
</tr>
<tr>
<td>C</td>
<td>0.51</td>
<td>0.51</td>
<td>23824</td>
</tr>
</tbody>
</table>

Table 8.8. Performance of ANN ensemble predictions for each subject using leave on out testing.
Figure 8-30. Ensemble predicted performance compared to actual performance for Subject A.

Figure 8-31. Ensemble predicted performance versus actual performance for Subject A. $R^2 = 0.78$
Figure 8-32. Ensemble predicted performance compared to actual performance for Subject B.

Figure 8-33. Ensemble predicted performance versus actual performance for Subject B. $R^2 = 0.62$. 
Figure 8-34. Ensemble predicted performance compared to actual performance for Subject C.

Figure 8-35. Ensemble predicted performance versus actual performance for Subject C. $R^2 = 0.51$. 
8.4. Discussion

The performance quantification technique used for this research has been described in detail in Chapter 6. Briefly, performance is compared to an athlete's personal best maximal mean power (MMP) over a range of time durations. A record of an athlete's best MMP is kept for each time duration, and updated whenever a new personal best is achieved.

The personal bests of the athletes tended to improve over the duration of the data collection, resulting in the performance datasets exhibiting downward trends over time (refer to Figure 8-9, Figure 8-11 and Figure 8-13). These trends are functions of the data collection process, and were removed in the preprocessing phase by differencing.

The intent of this research is to design a performance modelling process which is practical in the real world of elite road cycling. As such, precise maximal mean power curves obtained from recent specific testing will frequently be unavailable, and so a database of MMPs needs to be built up over time. The models created will become increasingly accurate as the recorded MMPs become closer to the real MMP of the athlete.

An alternative option would be to calculate an absolute performance value by using the peak performance (for each duration) from the entire dataset as the performance benchmark. This approach could provide interesting additional information about the variation of performance over time, however in order to understand the relationship between training load and performance, performance needs to be calibrated to an athlete’s capabilities at that time. Using absolute performance would distort the relationship between training and performance in the early parts of the dataset, as what were - at the time - good performances for an athlete, would become poor performances in comparison to their later achievements.
Conversely, there may be a future need to develop a mechanism to reduce the MMP values, to cater for example to masters athletes whose maximal capabilities may be declining. This mechanism is not currently included in the modelling process.

Establishing a performance dataset for an athlete using real world training and racing data relies on being able to identify those sessions where the athlete made a maximal effort. For Athletes A and B, a training diary was available which recorded the athlete’s perception of effort for each session (e.g. ‘I gave everything I had’). This information was not available for Subject C.

All races were initially assigned as performances for Subject C. In initial modelling, the models constructed for only single day races performed well, while those constructed for only multi-day stage races performed extremely poorly. This was likely due to the tactical nature of multi-day stage races. Depending on a cyclist’s role within a team, certain stages are likely to be completed with the goal of conserving energy for later, tactically crucial, stages. For example, climbing specialists often attempt to conserve energy during flat stages which are predicted to end in a bunch sprint. They can do this by drafting – riding in the slipstream of other riders. Stages with power files which show a low average power output, and a high average speed, are indicative of such flat stages where drafting is occurring and a 100% effort is not made. To identify such stages, cluster analysis was performed on the variables performance and average speed, and average power and average speed (refer to Figure 8-7 and Figure 8-8). The identified performances were excluded from the study, substantially reducing the noise of the performance time series. This novel technique has the potential to increase the range of datasets that can be used for modelling, because additional information on the level of effort put in to each session is frequently not available.

The dataset for Subject C showed highly variable performances. To improve the performance of the model with the extreme performance values, a simple form of boosting was used. An ANN model was fitted to the dataset to identify the performances that resulted in the largest modelling errors. These performances were
re-sampled and added back to the dataset, thus increasing their influence within the dataset.

Boosting is a technique that is often used in modelling with decision trees. Boosting concentrates on data points that are difficult to model. The boosting algorithm involves an iterative process where a model is applied successively to different versions of the initial training sample – these samples are modified to place greater weight on the observations which were incorrectly modelled in the preceding iteration. A variant of boosting called ‘arcing’ (short for ‘adaptive resampling and combining’) draws a training sample with replacement in each iteration (i.e. bootstrapping) with a greater probability of drawing the observations that were incorrectly modelled in the preceding iteration (Tuffery, 2011). The simple boosting technique employed in this research was conceptually similar to the arcing technique. Future research might automate what was here a manual process by employing the arcing algorithm.

8.4.1. SimBEA Performance

The fitted SimBEA model on its own had a poor fit with the performance series for each athlete. The relationship between SimBEA and performance for Subject B was linear, with an $R^2$ value of 32%. The relationship between SimBEA and performance for Subject A and C was weak and appeared to be non-linear.

Bruckner and Wilhelm (2008) note that the SimBEA model had lower accuracy when it was fitted to a dataset of 68 weeks, compared to a dataset of 6 weeks, although they report an intraclass correlation coefficient (ICC) of 0.65 over 68 weeks which is still a moderate fit. Bruckner recommends that the calibration of the model be reviewed periodically to adjust the parameters as necessary, and therefore improve model accuracy. Calvert, Banister, Savage & Bach (1975) applied an impulse response model to the training and performance data of a swimmer. They found that the optimal parameters for the model varied slightly for each season.
The SimBEA model was fitted to the entire performance dataset for subjects A and B (consisting of 1 and 2 seasons of racing, respectively). These datasets did not contain enough performances to enable refitting of the model parameters for smaller time frames. The performance dataset for Subject C contained data from 4 seasons. Enough performances were contained within each season to refit the model parameters for each season.

Although the fits obtained with the SimBEA model were poor, including the SimBEA predicted performance as a parameter in an ANN model improved the model performance. It is concluded that the domain knowledge which is embedded within the SimBEA model, including information regarding physiological phenomena such as supercompensation and reversibility, added valuable information to the final ANN model.

8.4.2. ARIMA

A central task of fitting an ARIMA model is determining the model order. It is common to fit all plausible models, and choose on the basis of their performance (Venerables & Ripley, 2002). An ARIMA (1,0,1) model proved to have the best fit for these datasets.

The performance of the ARIMA models ranged from poor to good. The fit for subject A was $R^2 = 72\%$; for Subject B, $R^2 = 53\%$; and the fit for Subject C was poor, with $R^2 = 43\%$. These results are similar to those reported by Ferger (2008), who fitted an ARIMA (0,0,0) model to a performance time series. Ferger reported a model fit of ICC = 0.47.

It is important to note that the performance datasets are not evenly spaced in time. This has the effect of reducing the stability of the autocorrelation structure within the dataset. A performance is much more likely to be affected by the previous performance if it occurred the day before, rather than weeks before. This is most relevant to the ARIMA model for Subject C, whose dataset contained the highest proportion of multi-day stage races, and in fact the model for this dataset resulted in the poorest fit. In this
research, the function of the ARIMA model is to model the linear patterns contained in the performance dataset. As such, close model fits are not expected from the ARIMA models alone.

8.4.3. Least Squares Multiple Linear Regression

As discussed, the ARIMA model for the performance dataset of Subject C had the poorest fit. Multiple linear regression was identified as an alternative linear modelling technique that would be less affected by the varying time scales of the performance time series, as well as allow further information to be added to the model. The inputs which resulted in the best performing MLR model were: ARIMA performance predictions, date of performance, taper shape and race type.

The fact that the date of performance was a valid predictor suggests that successive performances are not independent, and this was confirmed during the ARIMA modelling process. The race type variable was also found to be a significant predictor in the multiple regression model. As discussed previously, performance within a multi-day stage race (categorised as ‘tour’) displayed some unique characteristics compared to single day races (‘road race’), most noticeably higher variation. Criteriums, raced over short circuits, are characterised by repeated short high-powered efforts. Distinctive power profiles are created by different race types - for example the repeated high-powered efforts of short duration which characterise criteriums and the long iso-powered efforts of time trials. This can also be observed with different types of terrain – flat races tend to be characterised by large sections of low power output, with relatively brief high power efforts for the intermediate sprint points, bunch positioning and the final sprint. Hilly races tend to be characterised by threshold power outputs for relatively long durations while climbing, and then very low power outputs for the descents. The performance algorithm was developed to work with a range of race types and terrain, and the resulting characteristics of their power files. There are inevitable simplifications involved, however, in designing an algorithm to calculate one number to
represent an entire performance. The multiple linear regression model showed that
tours tended to have lower performances compared to criterium and single-day races,
as would be expected due to the increased levels of fatigue experienced during a tour.
Similarly, performance duration was found to have an effect, with longer durations
resulting in slightly higher performances. Longer sessions are likely to provide more
opportunities for generating high MMP values over the range of time durations
recorded – 5 seconds to 20 minutes.

Linear regression is not a technique that has been commonly applied to the problem of
modelling training and performance, due to the established non-linear nature of the
problem. As with the ARIMA model, the intent is to model the linear patterns in the
dataset. The fit of the model was $R^2 = 54\%$, an improvement over the fit of the ARIMA
model alone for this dataset ($R^2 = 43\%$).

8.4.4. Bootstrap and Noise

No established guidelines exist in the literature for determining the appropriate amount
of bootstrapping and the appropriate level of noise to apply to a dataset to order to
achieve the most effective results. The optimum values were determined by
experimentation, and varied according to the dataset.

In each case, performance was improved when an ANN model was applied to a bag of
bootstrapped and jittered dataset rather than an original dataset. The small level of
performance improvement achieved (in $R^2$ values) ranged from an improvement of 7%
for Subject C, 9% for Subject A, and 42% for Subject B. The amount of improvement is
likely to be related to the size of the original dataset – Subject B had the smallest
dataset and Subject C the largest. The problems of overfitting and subsequent poor
performance on the validation sample is more severe in small datasets, and thus
techniques to reduce overfitting such as bootstrapping and adding noise are likely to be
more effective. This is consistent with Zhang (2007), who found that no significant
difference in performance was observed in bootstrapping and adding jitter with datasets of over 100 samples.

Zhang (2007) states that the bootstrapping and jitter process causes the data to appear smoother to a neural network. The visualisations of each performance series before and after the process (refer to Figure 8-25, Figure 8-27 & Figure 8-29) show that the data became more ‘stepped’ - with small clusters of values around each original value - rather than a true smoothing affect. Some papers – for example Yeh (2007) – suggest expanding the range of the dataset when creating artificial data to supplement small datasets. The difficulty lies in concomitantly expanding the range of the input parameters appropriately, when the underlying relationship between the model input and the output is unknown. I argue that it is better to add slight noise to bootstrapped data rather than expanding the range of parameters to create a smoothed dataset and risk distorting the underlying relationships present in the original dataset.

8.4.5. Component ANN Topology

Determining the topology of an ANN requires a trade off between model complexity and the amount of data provided. The greater the number of nodes (in the hidden layer as well as in the input layer), the more data is required. Edelmann-Nusser, Hohmann & Henneberg (2002) used the rule of thumb that the minimum number of observations required is about double the number of connections between nodes. In this research, three model inputs proved to be the optimal number, and the best parameters for inclusion in the model were selected after experimentation. The optimal number of hidden nodes was selected by fitting a sequence of models with the number of hidden nodes varying from 2 to 5.
8.4.6. **Hybrid Models**

The addition of the performance prediction from the SimBEA model as an input to the ANN model significantly improved the performance of the ANN model. The hybrid SimBEA-ANN model performed better than either model performed alone. Although the performance of the SimBEA model alone was poor, it appears that the domain knowledge embedded in the model provided additional information to the ANN model compared to using training load alone as an input.

A key limitation of the SimBEA model (used in isolation) is its lack of flexibility. It only accepts a single input - training load. Research indicates that the impulse response models (e.g. the Banister and Busso models, and also SimBEA) perform well in controlled conditions, such as in lab experiments, where the noise in the dataset is likely to be much less of an issue. Such models have frequently performed less well in real world conditions. In the current research, the modelling technique needs to be able to deal with noisy data. In these situations, adding further information to the model in the form of additional parameters is generally required to achieve adequate model performance.

The performance of the hybrid model (a combination of SimBEA, ARIMA and ANN models, or a combination of SimBEA, ARIMA, MLR and ANN models) was better than the performance of any single model for each of the datasets. The improvements for Subjects B and C, however, were slight. Zhang (2003) found that a hybrid ARIMA-ANN model performed better than either model alone. It is hypothesised that the slight improvement recorded for Subject B was likely due to the small size of the dataset, making overfitting with an ANN model inevitable unless further techniques were applied to reduce this. An overfitted model would explain the relatively poor performance of the model on the validation sample. The dataset for Subject C was much larger, and overfitting is not likely to have been much of a problem. It is likely that the ANN in this
case had difficulty in modelling the extremes of performance seen in this dataset, and consistently underestimated both poor performances and good performances.

The literature around using hybrid ARIMA and ANN models contends that the ARIMA model models the linear patterns in the dataset, and the ANN model models the non-linear patterns (e.g. Zhang, 2003; Khashei & Bijari, 2010; Hippert, Pedreira, & Souza, 2000). Some practitioners use the residuals from the ARIMA model as the input to the ANN model (e.g. Zhang, 2003; Khashei & Bijari, 2010). Hippert, Pedreira, & Souza (2000) successfully composed a hybrid model using the predictions from an ARIMA model rather than the residuals as the input to an ANN model.

Modelling the residuals from an ARIMA model assumes that the relationship between the linear and non-linear components of a dataset is additive (Taskaya-Temizel & Casey, 2005). The validity of this assumption has not been established. Initial trials of using ARIMA residuals as an input into the ANN model resulted in extremely poorly performing models. The approach followed in this research was therefore to use the predictions from the ARIMA models as an input – and this resulted in better performing ANN models.

Choosing the appropriate input parameters for an ANN model can be difficult, particularly if parameters are correlated. One technique commonly used to reduce the number of inputs while still retaining vital information from the all parameters is to use Principle Components Analysis (PCA) or factor analysis. PCA uses an orthogonal transformation to convert a number of potentially correlated variables into a generally reduced set of linearly uncorrelated variables. The trials in this research of using PCA to reduce two or three different correlated parameters into one value resulted in poorer model performance compared to including the three best parameters in the ANN model. The hybrid MLR-ANN model used for Subject C allowed valuable information from additional parameters to be derived, and combined into a single input - the MLR prediction. In this manner the use of the MLR model as an input to the ANN model
performed a similar function to PCA, but resulted in significantly better model performance.

8.4.7. ANN Ensemble

The literature offers no definitive suggestions for choosing the optimal number of models to include in an ensemble. Zhang (2007) found that between 5 and 10 models offered a reasonable compromise between parsimony and modelling accuracy. Tuffery (2011) suggests using around 10 models. Perrone & Cooper (1992) found that ensemble performance ceased to improve after inclusion of more than 6 to 8 component ANNs, pointing out that the number of distinct local minima in functional space is quite small. In this research experimental trials showed that an ensemble of 8 models performed better with these datasets than an ensemble of 5 models, while the improvement using 10 models was minimal, and it was judged to be not worth the extra computational expense.

The use of bootstrapping and jitter in generating the training sets for each of the component ANN models meant that each model was trained on slightly different data. Sharkey (1996) considers that varying the data on which a set of ANNs is trained is more likely to create an effective ensemble, and Zhang (2007) found this to be the case experimentally. Raviv and Intrator (1996) found that noisy bootstrapping performed best in conjunction with ensemble averaging. The noise helped to push different component models to different local minima, producing a more independent ensemble (Raviv and Intrator, 1996). Networks which have fallen into different local minima will have difficulty in modelling different areas of feature space, and thus their error terms will not be strongly correlated (Perrone & Cooper, 1992).
8.4.8. Testing Methodology

For small datasets holdout validation is not feasible, as partitioning an already small dataset into training and testing sets reduces the available training information. Instead, jacknifing or leave-one-out validation was used for testing the performance of the ensemble in this research. This is where one data point at a time is removed from the dataset before the model is constructed on the remaining data points. The model output was tested on the removed data point, before repeating for each data point in the dataset. Jacknifing is not commonly used in neural network training, due to the large computational overhead (Perrone & Cooper, 1992), but it is an appropriate technique for small datasets.

8.4.9. Combining the Component ANNs

Some authors (e.g. Sharkey, 1996; Zhou, Wu & Tang, 2002) have found that selecting ANNs for inclusion in an ensemble, rather than including all, results in better performance. Zhou, Wu & Tang (2002) used genetic algorithms to select the most appropriate ANNs. Perrone and Cooper (1992) created a large number of component models then used a heuristic selection method to select the ANNs with the least errors. The presence of collinearity between the errors made by component ANNs can be an indicator that averaging may not be the most effective method of combining ensemble output – however if the majority of the components are correct averaging will perform well (Sharkey, 1996). Zhang (2007) found that averaging component outputs was a simple and effective technique. Simple averaging was used in this research, however future research could investigate whether model performance could be improved by implementing a model selection algorithm.

For Subjects B and C, the ensemble performed better than the best single network. For Subject A, the best single network performed substantially better than the ensemble. In
all cases, the ensemble performed substantially better than the worst single network. This is to be expected, as an ensemble works as a smoothing operator over the predictions of the individual ANN models.

HANNEM explained 77% of the variance of performance for Subject A. 60% of the variance was explained for Subject B, and 51% for Subject C. Bruckner (2006) points out that his SimBEA model can predict performance direction or trends rather than absolute performance. The performance quantification technique developed in this research uses arbitrary units as its unit of measurement. This is a function of the fact that for road cycling, there is no performance standard against which an individual performance can be measured. There is no time, distance or average power that can be used as a standard; unlike sports such as swimming, running and many track cycling events. This means that performance measurements cannot be converted back into concrete units like time, distance or watts. For this reason, the focus of the final model developed in this research does not aim to predict absolute performance (which would be meaningless) - rather it aims to predict the direction and rate of performance trends.

The large amount of data needed for both the impulse-response models and particularly the neural network models has always been a significant problem affecting the practical usefulness of training and performance models. Athletes and coaches wish to receive feedback from such a model as soon as possible, rather than waiting while a dataset of sufficient size is gathered. Overtraining is a particular issue when using neural networks to model small datasets. The smallest dataset in this research contained just 30 performances; yet achieved a good level of generalisation to the validation data (using jackknife validation). This result supports the use of the techniques implemented to reduce overfitting; namely bootstrapping (bagging), adding noise, and the use of an ensemble of ANNs.

At the other end of the spectrum there is a potential issue with having data that extends over a long time period. The largest dataset in this research included data from 4 seasons. The relationship between training and performance changes over time, and
including data from such large time horizons in the training sample might reduce the accuracy of the modelling for the most recent data. One potential avenue of investigation for dealing with this issue is to use boosting to increase the weight or relative importance of the most recent data points compared to older data points.

A useful model needs to be able to be used to calculate predicted performance given a proposed training load structure (Pfeiffer & Hohmann, 2011). Busso and Thomas (2006) caution that the quality of model predictions are likely to suffer greatly from the simplifications made in aggregating training load into a single number, as well as the simplifications inherent in modelling complex physiological processes in a small number of entities. To be useful in the real world, the model needs to be as parsimonious as possible, require as few inputs as possible, while maintaining high levels of accuracy in performance prediction.

An important consideration is to determine how far in time the model can predict performance and retain an appropriate level of accuracy. Bruckner (2006) found that prediction accuracy dropped significantly for time horizons longer than one week. The data used in the Bruckner study contained evenly spaced performance data, which makes investigating prediction accuracy over different time frames a straightforward process.

The time horizon over which performance predictions maintain accuracy is related to the stability of model parameters over time. If the perfect model was developed - one that accurately captured the underlying relationship between training and performance, then performance predictions would remain accurate for any time horizon until the underlying relationship changes. The question of how far in time predictions can remain accurate is therefore to some extent dependent on the stability of the training-performance relationship in an individual athlete.

One relatively simple way to potentially improve model performance would be to trial additional inputs to the model. The inputs for the models developed in this research are based on information derived from a single sensor – the power monitor on the subject’s
bike. This was done to ensure that the requirements for subjects participating in this research were as minimal as possible.

Lamberts, Swart, Noakes & Lambert (2011) believe that the combination of an objective data source (measurement of power), a subjective data source (for example a rating of perceived exertion) and alterations in the functioning of the autonomic nervous system (HRV for example) shows advantages over interpreting a single variable in creating a performance test. Similarly, a subjective measurement of training load (e.g. RPE) could bring important information to the model, and information that is less highly correlated than the power sensor derived variables. RPE would also assist in determining which races involved sub-maximal efforts and thus should not be included as performances.

The model developed was designed to function without requiring regular performance testing, unlike many of the impulse-response models. Further work to examine the long-term reliability of the performance metric could use submaximal testing protocols, such as the one proposed by Lamberts, Swart, Noakes & Lambert (2011), as an method of verifying observed performance trends. An appropriate sub-maximal performance testing protocol also has the potential to reduce the amount of data an athlete requires before a realistic optimal performance profile is developed – which is essential before a workable model can be developed.

The current model assumes that poor performances are functions of the training leading in to the performance, rather than a decline due to aging for example. Further work could be done on developing a technique to identify situations where the optimal performance profile of an athlete needs to be reduced.

### 8.5. Summary

This chapter outlined the development of a hybrid linear artificial neural network ensemble model to model the relationship between training and performance. Training load was quantified using the PTRIMP algorithm (described in Chapter 5) and
performance was quantified using the technique described in Chapter 6. The SimBEA model developed by Bruckner (2006) was used to capture important domain knowledge regarding the physiological process underlying training and its impact on performance. A linear model (either an ARIMA or Multiple Linear Regression model) was created to model the linear patterns in the data. A neural network model, which took as inputs the performance predictions from the SimBEA model and the linear model, as well as a summary of recent training load, was employed to capture the nonlinear patterns in the data. The performance of the hybrid model (a combination of SimBEA, ARIMA and ANN models, or a combination of SimBEA, ARIMA, MLR and ANN models) was better than the performance of any single model for each of the datasets.

The small datasets inherent in the elite sport domain mean that overfitting with a neural network model is a potential problem. To reduce overfitting and increase the generalisability of the model the techniques of bootstrapping, adding noise or jitter to the dataset, and training an ensemble of models on slightly different datasets (bagging) were used.

Three datasets containing power data captured from the training and racing of three elite athletes were used to test the performance of the model on real world datasets. Moderate to good fits were obtained for the model predictions using jacknife validation ($R^2 = 77\%$; $R^2 = 60\%$ and; $R^2 = 51\%$ for Subjects A, B and C respectively).

One of the key questions surrounding these results is whether the level of accuracy reported is sufficient to enable the model to be used as a training planning tool for elite athletes. Measurement of performance in road cycling cannot be absolute, only relative to an individual’s previously recorded capabilities. The aim of the model is to correctly identify the direction and rate of performance trends, rather than predict an exact performance level.
CHAPTER 9 – VALIDATION OF HANNEM

9.1. Introduction

The development and testing of HANNEM - the model developed in this work to describe the relationship between training and performance in elite cyclists – was described in Chapter 8. HANNEM uses an ensemble of neural networks to generate the final model output. Ensembles are a popular method of increasing the performance and robustness of single ANN models; however questions remain about how they should be constructed. In addition, their use with very small datasets has not received much attention in the research literature.

Learning with very small datasets is a fundamentally difficult problem. If a dataset is noisy as well as small - as is the case with the training and performance datasets detailed in this work - the difficulty is compounded as the number of data points reflecting the underlying relationship is reduced even further. The ability of a model, trained on such data, to generalise to new data can be questioned. A validation dataset that is also necessarily small compounds this problem, meaning it may not be possible to estimate the true generalisation ability of the model with sufficient accuracy. The question of how, therefore, to validate a model developed with a very small dataset arises.

This chapter details a set of experiments designed to test key points of the HANNEM modelling methodology and to address the identified risks of small noisy datasets. A number of datasets with known characteristics were generated for these experiments. In particular, the relationship between overfitting individual networks and ensemble generalisation performance will be investigated, along with the impact of noisy data and dataset size. The investigations in this chapter aim to answer the following questions: Does an ensemble mitigate overfitting issues with small datasets? Can overtraining improve performance of an ANN ensemble? How many ANNs should form an ensemble?
9.1.1. Neural Network Ensembles

Neural networks are unstable classifiers (Tuffery, 2011). In general ANNs have many local minima and frequently fail to converge to the global optimum solution. Combining the output of several ANNs into an ensemble has a smoothing, stabilising affect (Carney & Cunningham, 1999). Each component model has errors, but they are combined in such a way as to minimise the effect of the errors of individual nets (Sharkey, 1996). Perrone and Cooper (2003) show that the existence of differing local minima in component nets actually benefits the ensemble. Ensemble techniques can yield dramatic improvements in performance compared to single ANNs (Carney & Cunningham, 1999).

The error of a predictor such as an ANN ensemble can be expressed in terms of the bias squared plus the variance\(^1\). The bias and variance of a predictor can be estimated when the predictor is trained on different random data samples taken from the entire set. The bias can be characterised as a measure of the predictor’s ability to generalise to new data - it corresponds to the performance of the ensemble on the validation set. Variance can be characterised as the extent to which the predictions vary according to the particular sample the predictor was trained on. A well-performing estimator has both low bias and low variance, but these two elements have conflicting requirements and thus involve some element of tradeoff (Sharkey, 1996).

The general principle for creating good ensembles is to include diverse models. Diverse models disagree in their predictions and thus have high variance (Tuffery, 2011;

---

\(^1\) An ANN constructs a function \(f(x;D)\) based on a particular dataset \(D\) with training set \((x_1, y_1), \ldots, (x_n, y_n)\). The mean squared error (MSE) of \(f\) may be written

\[
E_D[(f(x;D) - E[y|x])^2]
\]

where \(E_D\) is the expectation with respect to the training set \(D\), and \(E[y|x]\) is the target function. This equation can be decomposed into bias and variance components which gives us,

\[
MSE = E_D((f(x;D) - E[y|x])^2) \text{ (bias)}
+ E_D \left[ (f(x;D) - E_D f(x;D))^2 \right] \text{ (variance)}
\]
Kuncheva & Whitaker, 2003). Ensemble members that exhibit high variance should also display a low correlation of errors (Sharkey, 1996). The usual effect of combining the output of ensemble members is to reduce the variance in the final estimator. Good ensemble members therefore need to be both accurate (low bias) and diverse (high variance) (Granitto, Verdes & Ceccatto, 2005). Component members exhibiting low bias and high variance can be combined to produce an ensemble estimator with low bias and low variance.

Several measures of diversity have been suggested for estimating the diversity of classifiers – including ANNs. There are, however, few accepted formal definitions of diversity to date (Fu et al., 2013). Kuncheva and Whitaker (2003) give a good review of a number of methods - they found that the ten methods investigated tended to be highly correlated with each other. There appears to be no widely accepted best method for measuring diversity in ensembles (Melville & Mooney, 2005), and much of the literature is concerned with techniques for measuring diversity in regression estimators. This research will use Kohavi-Wolpert variance (Kohavi & Wolpert, 1996) in the modified form outlined by Kuncheva and Whitaker (2003).

Creating a diverse ensemble can be done in a number of ways; by varying the training data, using differing network architecture, or conducting training using differing algorithms. Perhaps the most popular approach to the creation of a diverse set of ANNs is bagging, a technique which varies the training data. In bagging a training set (t) of size n is perturbed by randomly sampling from t with replacement until a bag with n data points is created. The bag may thus contain repeated data points and may exclude some points from the original set. On average, 63.2% of the data points from the original set will be contained in each bag, with the remainder consisting of duplicates, some of which will appear multiple times (Melville & Mooney, 2005). Each ensemble member is trained on a different bag, and the variations in the training sets create diverse estimators.
Bagging is particularly useful for reversing the lack of robustness of unstable classifiers such as ANNs. Some of the effectiveness of bagging may be due to the fact that any outliers in the initial sample will only be included in some bags - thus reducing their effect on predictions (Tuffery, 2011). Bagging is particularly useful where there is a shortage of data (Sharkey, 1996), however, with small training datasets there is a limit to the amount of diversity that can be created using this method (Melville & Mooney, 2005).

9.1.2. Combining Component Output

Once a set of nets has been created, a way of combining their outputs must be found (Sharkey, 1996). Taking a simple average, as was done in HANNEM, is a popular approach for combining the output of predictors. Majority voting is a simple and popular strategy for combining the output of classifiers. It is a strategy where each component classifier votes for a category, and the category with the largest number of votes is taken as the ensemble prediction (Hansen & Salamon, 1990). More complex schemes have been proposed, such as ranking the output of each component model (e.g. Perrone & Cooper, 1993) and slight improvements have been reported. This investigation, however, utilised the simple and computationally efficient technique of majority voting.

9.1.3. Choosing the Number of Component Networks

There is no established technique or rule of thumb in the literature for selecting an optimal number of component networks. Some recommendations range from 5 (Zhang, 2007) to 40 (Raviv & Intrator, 1996). Perrone and Cooper (1993) suggest that choosing the number of models included in the ensemble in dependent on the number of distinct models that can be created from the dataset. This implies that smaller ensembles are appropriate for smaller datasets. This investigation examined the performance of differing numbers of component networks in an ensemble, from 1 to 99 networks. Each
of these ensemble sizes were tested on two different dataset sizes, to investigate the
effect the size of the training set has on the optimal number of networks.

9.1.4. Training of Component ANNs

Deciding when to stop training is an important consideration in creating an ANN model. Early exit techniques are widely used and include simple techniques for stopping training of a neural network when certain criteria are met. This is done with the aim of avoiding overtraining and increasing generalisation ability of the network. Early exit techniques are among the most popular for optimising ANN generalisation performance, particularly for ensemble learning (Carney & Cunningham, 1999). Carney and Cunningham (1999) argue that early-stopping is particularly suitable for bagged ensembles. Caruana, Lawrence, & Giles (2001) note that early-stopping is critical for all nets, not only those prone to overfitting (for example due to over-large nets or small datasets).

The basic concept of early-exit techniques is to terminate neural networking training as soon as some estimate of generalisation error begins to rise. The generalisation error is generally calculated against a validation set, commonly using cross-validation or hold-out validation techniques (Carney & Cunningham, 1999).

Early exit techniques aim to avoid overtraining of the network. Overtraining or overfitting of a network is considered to have happened when a link or relationship between input and output variables that appears in the training sample – and as such is learned by the model - does not occur in the wider population (Tuffery, 2011). Overtraining is occurring when the performance of a model on a training set continues to improve but the generalisation performance on unseen data (the validation set) begins to fall. The overtrained network conforms accurately to the individual quirks in the training set which prevents it from generalising to unseen data which does not contain the same individual quirks.
Neural networks are widely recognised as prone to overtraining. Overtraining can occur when datasets are small, or when the number of hidden nodes is too high. The general consensus is that it is important to avoid overtraining and early exit techniques are common for this reason. Common wisdom states that machine learning techniques, such as neural networks, (which are prone to overtraining) should be avoided for modelling with small databases for this reason (e.g. Tuffery, 2011).

9.1.5. Creating Diversity in Component ANNs

Advice in the literature on techniques for producing diverse ANN estimators generally suggests varying one or more of the following: the initial random weights; the topology; the algorithm employed; or the data. We have discussed how training each ensemble member on slightly different sets of data (using the technique of bagging) creates diversity, and can lessen the deleterious effects of noise and outliers in the original dataset.

Sharkey (1996) notes that it makes sense to take steps to reduce the bias of individual ensemble members (i.e. by taking more account of the training data) since the increased variance that results will be removed by combining ensemble member outputs. Early exit training increases the generalisation capacity of individual networks to unseen data – in other words it reduces the account that model takes of the training data. Conversely, overtraining generally decreases an individual network's ability to generalise to new data by taking more account of the training data. Thus it is proposed that overtraining component ANNs may result in a decrease in the overall bias of an ensemble, and the resulting increase in variance will be removed by the smoothing affect of the ensemble; giving a net effect of a performance increase.

It is proposed that the concerns of overfitting individual neural network models when datasets are very small - and the consequent compromising of the model’s ability to generalise to new data - are obviated by the generation of diverse ensemble
components (using the technique of bagging) and the subsequent regularising effect of ensemble combining. In fact, it is proposed that in many cases deliberately overtraining ensemble members will increase diversity and thus the eventual performance of the ensemble. The following sections outline an investigation to determine whether creating an ensemble of ANNs mitigates any overfitting issues with small datasets, and in fact whether deliberate overtraining can improve ANN ensemble performance. Finally, the optimal number of component nets in an ensemble is investigated in relation to the size and level of error in training datasets.

9.2. Method

Experiments were conducted on two different dataset sizes. These experiments will be referred to as Experiment 1 and Experiment 2. The experiments were designed to test the questions mentioned in the introduction, namely: Does an ensemble mitigate overfitting issues with small datasets? Can overtraining improve performance of an ANN ensemble? How many ANNs should be in an ensemble?

9.2.1. Data Generation

The function used to generate the training and validation datasets is depicted in Equation (9-1). This function was chosen because it exhibited visual similarities to the expected inverted parabolic shape (and complexity) of the relationship between training and performance (Busso, 2003). Refer to Figure 9-1 for a visualisation of the function.

\[ y = 0.2 \left(1 + \cos(7\pi x)\right) + 0.65x^2 \]  \hspace{1cm} (9-1)

Datasets of 300 (Experiment 1) and 60 (Experiment 2) data points were generated, and then divided into equal partitions of training and testing sets. Figure 9-1 shows a visualisation of the dataset generated for Experiment 1, and Figure 9-2 shows a
visualisation of the dataset for Experiment 2. The datasets were divided into two classes (represented by the red and green dots in the figures) that are separated by the function generated with Equation (9-1). A validation set of 1,000 data points was generated in each experiment. Each component ANN in an ensemble was trained using a bag of data (randomly selected from the training partition with replacement). For Experiment 1, bags contained 150 data points, and for Experiment 2, 30 data points. Each dataset was created with a specified level of error, between 10% and 50%. Refer to Figure 9-3 for a visualisation of a dataset with 50% error. The points shown in blue in the graphs represent data points that are erroneous or incorrectly labelled. It can be seen that the high level of error in the dataset obscures the underlying function of the dataset. The ability of the model to accurately assess the correct class of a given point is represented by the error rate of the model.
Figure 9-1. Visualisation of the data function. A sample training set of 150 points belonging to one of two classes is visualised here. The error rate is 0% for this visualisation.

Figure 9-2. The problem of correctly identifying the underlying pattern in the dataset becomes more difficult with sparse data. This visualisation shows 30 points in the training set, as used in Experiment 2. The error rate is 0% for this visualisation.

Figure 9-3. Visualisation of training dataset with 50% data errors. Points with an erroneous class label are shown in blue.
9.2.2. Network Architecture and Parameters

A multilayer perceptron (MLP) with 1 hidden layer was the ANN used. The MLP incorporated self training logic. The number of nodes in the hidden layer, the optimal number of training epochs and the training constant were all determined by the self training algorithm using a trial and error approach. An early exit rule was used, whereby training ends provided that either:

a) A prescribed minimum number of epochs has elapsed (300 in this case) and the last peak in testing accuracy was within a given percentage of the start of the run (45% in this case); or

b) The prescribed minimum number of epochs has elapsed and the last peak in testing accuracy was more than a given number of epochs previously (2,000 in this case).

The number of nodes was given an upper limit of 1 input node, 9 hidden layer nodes, and 1 output layer node. Tuffery (2011) suggests that the optimal number of hidden layer nodes lies between $\frac{n}{2}$ and $2n$, where $n$ equals the number of input nodes, but reports that some authors suggest extending this to $3n$. An upper limit of 9 hidden layer nodes was chosen as a value that follows these suggested rules of thumb plus allows a margin.

9.2.3. Overtraining Factor

The OverTrained ensemble members were trained for $n$ epochs where $n =$ optimal number of training epochs, multiplied by the overtraining factor. The overtraining factor was set to 10 for these experiments. This factor was chosen to ensure a significant level of overtraining was achieved.
9.2.4. **Combining Component Output**

Majority voting was employed. The ensemble made a decision if \( \frac{n+1}{2} \) component models agreed, otherwise no decision was made.

9.2.5. **Confusion Matrix**

A confusion matrix was calculated for each ensemble to evaluate its prediction performance. The confusion matrix contained the rates of True Positives, False Positives, True Negatives and False Negatives.

The True Positive rate of the classifier was estimated as:

\[
\text{TP rate} = \frac{\text{positives correctly classified}}{\text{total positives}}
\]

The False Positive rate of a classifier was estimated as:

\[
\text{FP rate} = \frac{\text{negatives incorrectly classified}}{\text{total negatives}}
\]

9.2.6. **Calculating Variance**

Kohavi and Wolpert (1996) gave an expression for the variability of a predicted class label \( y \) for data point \( x \). Kuncheva and Whitaker (2003) used this general idea and adapted it for binary class labels and a correct or incorrect decision (oracle output) as shown in Equation (9-2).

\[
\frac{1}{2} \left( 1 - \hat{P}(y = 1|x)^2 - \hat{P}(y = 0|x)^2 \right)
\]

(9-2)

Averaged over the whole set of networks, Equation (9-3) was used to derive Kohavi-Wolpert variance.
\[
\text{variance}_x = \frac{1}{n^2} \left( 1 - \left( \frac{\sum_{i=1}^{n} \hat{P}(y = 1|x)}{n} \right)^2 - \left( \frac{\sum_{i=1}^{n} \hat{P}(y = 0|x)}{n} \right)^2 \right)
\]

(9-3)

Where \( n \) = the number of networks in the ensemble.

9.2.7. Algorithm for Conducting Experiments

Table 9.1 describes the algorithm used to conduct the experiments.

<table>
<thead>
<tr>
<th>For networks 1 to 99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generate a dataset (with specified data function, data distribution and error rate)</td>
</tr>
<tr>
<td>Partition into two equal datasets: a training and a testing dataset</td>
</tr>
<tr>
<td>For ( i = 0 ) to ( i &lt; \text{numNetworks} )</td>
</tr>
<tr>
<td>Create a bag of data of size (sizeData) by sampling with replacement</td>
</tr>
<tr>
<td>Partition into two equal datasets: a training and a testing dataset</td>
</tr>
<tr>
<td>Determine the optimal number of nodes, epochs and training constant by varying these parameters iteratively.</td>
</tr>
<tr>
<td>Train an early stopping network using the optimal number of nodes, epochs and training constant.</td>
</tr>
<tr>
<td>Train an overtrained network using the optimal number of nodes, the optimal number of epochs ( \times ) overtraining factor, and the optimal training constant.</td>
</tr>
<tr>
<td>Generate a confusion matrix for both the early stopping network and the overtrained network.</td>
</tr>
<tr>
<td>End for loop</td>
</tr>
<tr>
<td>Perform majority voting for each ensemble member for early stopping networks. Generate a confusion matrix.</td>
</tr>
<tr>
<td>Perform majority voting for each ensemble member for overtrained networks. Generate a confusion matrix.</td>
</tr>
<tr>
<td>End for loop</td>
</tr>
</tbody>
</table>

Table 9.1. The algorithm used for Experiment 1 and 2.
9.3. Results

The performance of the overtrained and early exit ensembles with varying error rates and ensemble sizes were assessed using two criteria: the percentage of correct classifications (performance), and the variance of the ensemble components.

9.3.1. Experiment 1

The mean performance for OverTrained and EarlyExit ensembles was calculated over the range of ensemble sizes tested, for each data error rate. OverTrained Ensembles performed better than EarlyExit ensembles for data error rates of 10, 30 and 40%. EarlyExit ensembles performed slightly better for data error rates of 20 and better for data error rates of 50%. All differences in means between the performance of EarlyExit and OverTrained ensembles were statistically significant ($p = 0.05$). Refer to Table 9.2 for a comparison of the means of the performance of OverTrained and EarlyExit ensembles, and to Figure 9-4 for a visualisation of the performance of each type of ensemble by number of networks and error rate.

<table>
<thead>
<tr>
<th>Data Error Rate</th>
<th>OverTrained Mean -Performance</th>
<th>EarlyExit Mean -Performance</th>
<th>Mean Difference - Performance</th>
<th>Significance (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>88.52929</td>
<td>88.04747</td>
<td>0.481818</td>
<td>.000</td>
</tr>
<tr>
<td>20</td>
<td>87.76667</td>
<td>87.99899</td>
<td>-0.23232</td>
<td>.000</td>
</tr>
<tr>
<td>30</td>
<td>86.00909</td>
<td>83.41818</td>
<td>2.590909</td>
<td>.000</td>
</tr>
<tr>
<td>40</td>
<td>83.89293</td>
<td>83.13838</td>
<td>0.754545</td>
<td>.011</td>
</tr>
<tr>
<td>50</td>
<td>43.19091</td>
<td>52.61414</td>
<td>-9.42323</td>
<td>.000</td>
</tr>
</tbody>
</table>

Table 9.2. Experiment 1: Mean performance of OverTrained and EarlyExit ensembles, across ensemble sizes of 1 to 99, by data error rates. Significance testing used paired samples methodology.
Figure 9-4. Experiment 1: Performance of EarlyExit and OverTrained ensembles by number of networks and error rate. The decline in performance with increasing data error rates is relatively steady, until the 50% error rate.

9.3.2. Experiment 2

OverTrained Ensembles performed better than EarlyExit ensembles for data error rates of 10%, 20%, 30% and 40%. EarlyExit ensembles performed slightly better on average for a data error rate of 50% with the exception of an error rate of 30%, all differences in means between the performance of EarlyExit and OverTrained ensembles were statistically significant \( (p = 0.05) \). Refer to Table 9.3 for a comparison of the means for OverTrained and EarlyExit ensembles, and to Figure 9-5 for a visualisation of the performance of each type of ensemble by number of networks and error rate.
Table 9.3. Experiment 2: Mean performance of OverTrained and EarlyExit ensembles, across ensemble sizes of 1 to 99, by data error rates. Significance testing used paired samples methodology.

<table>
<thead>
<tr>
<th>Data Error Rate</th>
<th>OverTrained Mean – Performance</th>
<th>EarlyExit Mean - Performance</th>
<th>Mean Difference - Performance</th>
<th>Significance (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>84.69394</td>
<td>82.10303</td>
<td>2.590909</td>
<td>.000</td>
</tr>
<tr>
<td>20</td>
<td>82.68788</td>
<td>80.53131</td>
<td>2.156566</td>
<td>.000</td>
</tr>
<tr>
<td>30</td>
<td>80.61212</td>
<td>80.04646</td>
<td>0.565657</td>
<td>.056</td>
</tr>
<tr>
<td>40</td>
<td>68.94141</td>
<td>65.32525</td>
<td>3.616162</td>
<td>.000</td>
</tr>
<tr>
<td>50</td>
<td>54.10101</td>
<td>60.40101</td>
<td>-6.3</td>
<td>.000</td>
</tr>
</tbody>
</table>

Figure 9-5. Experiment 2: EarlyExit and OverTrained ensembles by number of networks and error rate. The performance of both ensembles is much poorer on data with a 40% error rate than it was for the larger dataset in Experiment 1.
9.3.3. Effect of Dataset Size - Comparison of Experiment 1 and 2

EarlyExit ensembles performed better in both experiments when the error rate was 50%, and when the error rate was 20% in Experiment 1. In all other levels of data error - OverTrained ensembles were better performed in both experiments. Refer to Appendix D, Figure D-1 for a detailed visualisation allowing a comparison to be made of the performance of EarlyExit and OverTrainined ensemble models, for both experiments.

9.3.4. Variance

**Experiment 1**

OverTrained ensembles had the highest mean variance for error rates of 10%, 20% and 30%. EarlyExit ensembles had the highest variance for error rates of 40% and 50%. The difference in means between EarlyExit and OverTrained ensembles were statistically significant ($p = 0.05$) for each of the error rate levels. Refer to Table 9.4 for a comparison of the means of variance for the OverTrained and EarlyExit ensembles by data error rate. Figure 9-6 shows a comparison of the difference in the variance of EarlyExit and OverTrained ensemble models by error rate. The OverTrained ensembles had higher variance for error rates from 10% to 30%.

<table>
<thead>
<tr>
<th>Data Error Rate (%)</th>
<th>OverTrained Mean - variance</th>
<th>EarlyExit Mean - variance</th>
<th>Mean Diff - variance</th>
<th>Significance (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.001692</td>
<td>0.001671</td>
<td>2.1E-05</td>
<td>.000</td>
</tr>
<tr>
<td>20</td>
<td>0.001815</td>
<td>0.001726</td>
<td>8.96E-05</td>
<td>.000</td>
</tr>
<tr>
<td>30</td>
<td>0.001792</td>
<td>0.001435</td>
<td>0.000357</td>
<td>.000</td>
</tr>
<tr>
<td>40</td>
<td>0.001756</td>
<td>0.002465</td>
<td>-0.00071</td>
<td>.000</td>
</tr>
<tr>
<td>50</td>
<td>0.001808</td>
<td>0.002119</td>
<td>-0.00031</td>
<td>.000</td>
</tr>
</tbody>
</table>

Table 9.4. Experiment 1: mean variance for OverTrained and EarlyExit ensembles across ensemble sizes of 5-99, by error rate.
**Experiment 2**

OverTrained ensembles had the highest mean variance for error rates of 10%, 20% and 30% and 40%. EarlyExit ensembles had the highest variance for error rates of 50%. The difference in means between EarlyExit and OverTrained ensembles were statistically significant at the $\theta = 0.05$ level for each of the error rate levels. Refer to Table 9.5 for a comparison of the means of variance for OverTrained and EarlyExit ensembles by data error rate. Refer to Figure 9.7 for a visualisation of the difference in the variance of EarlyExit and OverTrained ensemble models by error rate. It can be seen the OverTrained ensembles have higher variance for error rates from 10% to 40%.

<table>
<thead>
<tr>
<th>Error Rate (%)</th>
<th>OverTrained Mean</th>
<th>EarlyExit Mean</th>
<th>Mean Diff</th>
<th>Significance (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.001174</td>
<td>0.000916</td>
<td>0.000258</td>
<td>.000</td>
</tr>
<tr>
<td>20</td>
<td>0.000981</td>
<td>0.000494</td>
<td>0.000487</td>
<td>.000</td>
</tr>
<tr>
<td>30</td>
<td>0.001174</td>
<td>0.000916</td>
<td>0.000258</td>
<td>.000</td>
</tr>
<tr>
<td>40</td>
<td>0.000705</td>
<td>0.000294</td>
<td>0.000411</td>
<td>.000</td>
</tr>
<tr>
<td>50</td>
<td>0.000795</td>
<td>0.001102</td>
<td>-0.00031</td>
<td>.0099</td>
</tr>
</tbody>
</table>

Table 9.5. Experiment 2: mean variance for OverTrained and EarlyExit ensembles across ensemble sizes of 5-99, by error rate.

Figure 9.6. Experiment 1: Comparison of the difference in the variance of EarlyExit and OverTrained ensemble models by error rate. The x axis shows the number of networks included in the ensemble (from 20 – 50 for this chart). If the difference is above 0, the OverTrained ensemble has greater variance than the EarlyExit ensemble, and conversely if the difference is below 0 the EarlyExit ensemble has the greater variance. The OverTrained ensembles have higher variance for error rates from 10% to 30%.
Figure 9-7. Experiment 2: Comparison of the difference in the variance of EarlyExit and OverTrained ensemble models by error rate. The x axis shows the number of networks included in the ensemble (from 20 – 50 for this chart). If the difference is above 0, the OverTrained ensemble has greater variance than the EarlyExit ensemble, and conversely if the difference is below 0 the EarlyExit ensemble has the greater variance. The OverTrained ensembles have higher variance for error rates from 10% to 40%.

OverTrained ensembles tend to out-perform EarlyExit ensembles for low to medium (10% – 40%) rates of data error. This behaviour is consistent across the differing sizes of training dataset for Experiment 1 and 2 (150 and 30 training data points respectively). The level of variance in an ensemble tracks with its overall performance. OverTrained ensembles tended to have higher variance for low to medium (10% – 40%) rates of data error.

9.3.5. Number of Networks

It can be seen from Figure 9-4 that the optimal number of networks for peak performance for each combination of data error rates and type of ensemble (OverTraining and EarlyExit) is highly variable. In general for lower to moderate data error rates (10% - 40%) ensembles with number of networks in the range of 20-40 appear to be a good trade-off between model parsimony and performance. The level of variance among component networks in an ensemble drops in a predictable exponential decay curve as the number of networks in the ensemble increases (refer to Figure 9-8 and Figure 9-9). Ensembles consisting of networks in the range of 20 to 40 appear to offer the best compromise between bias (or how correct the predictions of the ensemble are) and variance (the level of disagreement among ensemble members) and model parsimony.
Figure 9-8. Experiment 1: EarlyExit and OverTrained variance by number of networks in the ensemble (6-99 shown) and data error rate. The graph shows that the level of variance among component networks in an ensemble drops in a predictable exponential decay curve as the number of networks in the ensemble increases. It can also be seen that the variance for the EarlyExit ensemble shows more variation between the different error levels than that of the OverTrained ensembles.
Figure 9-9. Experiment 2: EarlyExit and OverTrained variance by number of networks in the ensemble (6-99 shown) and data error rate. The graph shows that the level of variance among component networks in an ensemble drops in a predictable exponential decay curve as the number of networks in the ensemble increases.
9.4. Discussion

For both datasets sizes tested (150 and 30 data points) the ensembles consisting of overtrained nets tended to perform slightly better for data error rates from 10% to 40%. The ensembles consisting of nets which were trained using the EarlyExit algorithm performed substantially better with high data error rates of 50%.

These findings suggest that the regularisation effect of combining the output of an ensemble of models does indeed remove the deleterious effects of overtraining under most data conditions. For high levels of noise in the data, nets trained using an early exit strategy were the most effective. Variance among nets is likely to be high with noisy datasets, due to the different bags of data containing different outliers. In this case reducing the bias of the estimators by taking less account of the data (achieved by the early exit training algorithm) is likely to be the most effective way of increasing ensemble performance. This idea is lent support by Krogh (1996), who found that when there are large amounts of noise in the training data it is unnecessary to increase the diversity of component networks.

For both dataset sizes tested, ensembles where the component nets were overtrained had the highest mean variance for error rates of 10%, 20% and 30%. For error rates of 40% with the larger dataset (Experiment 1), the early exit ensemble had greater variance; while with the smaller dataset (Experiment 2), the overtrained ensemble had greater variance. Early Exit ensembles had the highest mean variance for error rates of 50% for both dataset sizes.

These findings indicate that when date error levels are low to moderate, overtraining component networks does indeed increase their diversity. This is in accordance with Carney and Cunningham (1999), who believe that overtraining the networks in an ensemble generates more variance. It appears that the effect of overtraining on variance was slightly more pronounced with the smaller dataset (Experiment 2). In both experiments the overtraining was achieved by multiplying the optimal number of training epochs by an overtraining factor of 10. The level of overfitting achieved by the
net trained in this manner, however, would vary slightly according to the size of the dataset. The level of overfitting achieved for the smaller dataset is likely to be greater. We have seen that the increase in variance for these low to moderate levels of error in the data was accompanied by an increase in ensemble performance, indicating that bias was not unduly increased by the overtraining. Grannitto, Verdes and Ceccato (2005) agree that some controlled degree of overtraining in component networks improves ensemble performance.

It is interesting to note that when noise in the dataset is high, overtraining the component nets actually reduces the diversity. This is surprising, as it was expected that overtraining would continue to increase variance for datasets with high levels of error in the data, along with the bias.

The mean performance of the EarlyExit ensembles for the smaller dataset (Experiment 2) and a 50% data error rate is 60%. This stands out as anomalous. Given that the level of error in the dataset is known to be 50%, a performance above this level is suggestive of overfitting. The mean performance for OverTrained ensembles for this experiment was 54%, which is more realistic. It thus appears that deliberately overtraining component nets trained on a very small, very noisy dataset actually reduced the level of overfitting in the resultant ensemble compared to an ensemble whose component nets weren’t deliberately overtrained. It is widely accepted that the estimation of error used to determine when to stop training in an early-exit network can fluctuate during training in the presence of noise in the data (Carney & Cunningham, 1999). It is therefore likely that there may have been more variation in the early exit point chosen by the self training algorithm when trained on datasets with high noise levels.

The EarlyExit ensemble for the larger dataset and a data error level of 50% didn’t appear to suffer from the same overfitting issues, with a realistic mean performance of 52.6%. The OverTrained ensembles performed poorly on this experiment, with a mean performance of 43%. Again, it is unexpected that overtrained ensembles had a lower mean variance for this experiment than that of the early exit ensembles. Interestingly,
however, the overtrained ensembles have higher variances than the early exit ensembles towards the larger end (from around 40 onwards) of ensemble sizes tested. It is important to note, in these discussions of overfitting, that overfitting is not necessarily a global phenomenon. Overfitting can vary significantly in different regions of input space (Caruana, Lawrence, & Giles, 2001). This effect is particularly noticeable with sparse data, with the sparse regions tending to suffer from overfitting before the more populous regions. Figure 9-2 demonstrates that with 30 data points, some regions are more sparsely populated than others. The training and performance datasets used throughout this research have sparse data, and as such it is likely that the component nets of the HANNEM model contained some regions of input space that were overtrained. The findings of this investigation show that the regularisation effect of combining the outputs of an ensemble mitigates the adverse effects of overtraining.

Performance was highly variable across different ensemble sizes, particularly with low numbers of ensemble members. This is to be expected with unstable predictors. Ensembles with around 20 to 40 networks appeared to perform the best, particularly with low to moderate data error rates. Slightly larger ensemble sizes appear to be optimal with the higher levels of error in the data. The diversity of component nets drops rapidly from 1 to around 20 component nets. The inflection point for the variance curves occurs around 20 to 40 net mark – the same region where performance tends to be greatest.

Findings on the optimal number of component networks in an ensemble have been variable in the literature. Carney and Cunningham (1999) found that there was little to be gained by using more than 30 networks and improvements seemed to level out at about 100 networks. Raviv and Intrator (1996) found that a 40-net ensemble gave superior results to a 5-net ensemble. Zhang (2007) suggests that ensemble sizes of 5 to 10 are a reasonable balance between ensemble performance and computational efficiency. Tuffery (2011) considers that about 10 models is sufficient. Perrone and Cooper (1993) suggest that the optimal number of models for an ensemble is
dependent on the number of distinct models that can be created from the dataset. The results from these experiments suggest that the optimal number of networks doesn’t vary largely with variations in the size of the dataset, and the level of noise in the data. The results do suggest, however, that highly noisy datasets benefit from a slightly larger number of component models. This finding suggests that the HANNEM model could be improved by increasing the number of component ANNs to between 20 and 40.

9.5. Summary

This chapter is concerned with investigating the use of neural network ensemble models on very small datasets, and indentifying the subsequent generalisation ability of such models. This novel investigation examined the relationship between overfitting individual networks and ensemble generalisation performance, as well as the effect of different levels of noise in the data, and the inter-relationship of these variables with the size of the dataset.

The results show that using a bootstrapped ensemble of ANNs mitigates the affect of overtraining component nets, and in fact for low to moderate levels of noise in the data an overtraining strategy increases model diversity and provides performance benefits. The results also suggest that early exit rules become unstable in situations of high data error and very small datasets. It may be useful in these situations to increase the minimum number of epochs performed.

Although the HANNEM model did not use deliberately overtrained component nets, with sparse data it is likely that some regions of input space would be overtrained and other regions undertrained. The findings of this study show that the regularisation effect of an ensemble means that the ability of the ensemble to generalise to new data is not adversely affected.
The findings suggest that the HANNEM model could be improved by increasing the number of component nets. It was shown that the optimal number of nets increases slightly for datasets with large rates of error. Although the rate of error in the three case study datasets used to validate the HANNEM model is unknown, given the performance of the models reported in Chapter 8, it is likely that the rate of error ranges from around 30% - 48% (it is a reasonable assumption that the variance of performance unexplained by HANNEM is largely due to noise in the data). Estimating the level of variance in the ensemble provides a technique that gives an insight into the number of distinct models that can be created from a dataset, and can be used in establishing an appropriate number of component models for a particular dataset.
CHAPTER 10 – CONCLUSION AND FUTURE WORK

Researchers have been working on the problem of how to model athletic training and performance for at least the last 30 years. A model that enables prediction of performance and hence offers a practical solution to the problem of how to optimise training for an individual athlete has remained the "holy grail" of sport science. Early approaches used statistical and mathematical techniques, in conjunction with a systems modelling approach. As machine learning techniques continue to be developed, some researchers have experimented with applying these techniques to the problem. Machine learning techniques, however, are dependent on the quality of the data available. Athletic training and performance datasets are usually incomplete, noisy, and very small. In addition, training load is only one of a myriad of factors which affects performance.

The work reported in this thesis began with the development of a novel training load quantification technique called PTRIMP. Researchers have found great difficulty in finding a way to effectively quantify training load using a single term. I propose that two commonly used approaches - TRIMPS and TSS - place too much emphasis on the duration of a session, while not weighting high intensity training enough. PTRIMP was developed to address this limitation, and to more accurately reflect the physiological cost of high intensity efforts.

The first investigation undertaken in this thesis was a case study to examine the relationship between PTRIMP and a Recovery Index derived from HRV data. The aim of this investigation was to provide validation of the PTRIMP algorithm, by determining whether PTRIMP tracked with the fatigue measure in a manner consistent with known physiological responses to training. Wavelet-based semblance analysis, in a technique developed by Cooper and Cowan (2007), was used to compare the PTRIMP and the RI datasets on the basis of their phase, as a function of frequency.
The results of this investigation showed that PTRIMP and Recovery Index were strongly negatively correlated on larger time scales of 30 days or above. This is in accordance with known physiological responses to training; when training levels over the past seven days have been high, it is expected that fatigue levels would be high, and thus the RI should be low. At smaller time scales of around seven days, regular periods of moderate to strong positive correlation were interspersed with periods of moderate to strong negative correlation. It was hypothesised that this phenomenon may have been due to the delay that is known to exist between increased training and the onset of long-term fatigue. This investigation provided evidence that the novel training load quantification technique developed (PTRIMP) performed as expected in a real world situations - tracking with a marker of fatigue in a physiologically realistic manner.

The focus of this work next turned to developing a method of quantifying performance for road cyclists. Many researchers have used a method whereby performances from a standardised exercise test are converted into a percentage of the subject's personal best performance for that test. This approach requires subjects to complete regular performance tests. A key point of difference in this research is that it required no change in routine or additional work in order to achieve compliance with the requirements of data collection. This becomes vital when working with elite athletes for whom their performance is their livelihood. It was therefore necessary to develop a technique for quantifying performance that works with field-derived data.

An athlete's best performances are likely to occur during an actual competition as training induced fatigue will have been minimised by a taper. Field-derived performance metrics for some sports - such as swimming, or to a lesser extent sprinting - are relatively easily developed because performance is a function of time (over a certain distance) and conditions are relatively stable between competitions. Time cannot be used as a proxy for performance in road cycling, however, due to the variation in distances, terrain, environmental conditions, drafting and tactics which are integral to road cycling races.
The results of a cycling road race are typically determined by one or more critical periods during the race. At its simplest, to improve performance the athlete needs to increase the amount of power they can generate for these critical periods - the durations of which will be specific to the race. By recording Maximal Mean Powers over a range of durations, the power outputs during these critical periods can be captured. MMP profiles can then theoretically be used to track performance improvements.

The proposed performance quantification technique generates a MMP profile for each athlete, recording their best output to date over regular intervals from 5s to 20min. These durations represent a spectrum of energy system contributions; from primarily anaerobic to primarily aerobic systems. A second MMP profile, created with the same durations from 5s to 20min, is generated from the race for which performance is to be calculated. The area under the curve of each of the MMP profiles is then calculated, and the difference between the MMP curve for the performance and the MMP curve for the athlete's best output to date is taken as performance.

The second investigation undertaken in this work examined the correlation of PTRIMP, RI and the novel performance metric developed. PTRIMP data was smoothed over a number of time windows (3, 7, 14 and 30 days). Correlation analysis indicated that PTRIMP data smoothed over 3 days (PTRIMP(3)) was most strongly correlated to performance. A multiple linear regression model with PTRIMP(3) and RI as inputs, and performance as the output was fitted using linear least squares to minimise the residual sum of squares. The inclusion of an interaction term between PTRIMP(3) and RI, as well as a quadratic tern for PTRIMP(3) provided the best fit. The fit for the model was weak, explaining only 40% of the variation in performance measured. The explanatory power of the model did, however, suggest that there is a non-linear positive relationship between training load as measured by PTRIMP, and performance, as quantified by the proposed technique.

The third investigation undertaken in this work built upon the previous investigations by taking the novel training load and performance quantification techniques developed,
and performing an experiment to determine whether this information could be used to provide relevant feedback on optimal training strategies. This investigation focused on the taper period – the period of reduced training undertaken prior to a competition.

The aim of a taper is to reduce accumulated fatigue caused by training, while maintaining or enhancing fitness. The optimal design of a taper is still unknown, and in any case is likely to be specific to an individual athlete in accordance with the principle of specificity. The three key elements that can be manipulated in designing a taper are the magnitude of the reduction in training volume, the intensity of the training completed during the taper, and the pattern of the taper. This investigation focused on the pattern of the taper.

A number of simple taper patterns have been investigated by researchers. Existing evidence suggests that a progressive reduction in training load (using either a constant linear slope or an exponential decay) is to be preferred over reductions in the form of a simple step – where training load is suddenly reduced and then maintained at the same level. Little work has been done on more complex patterns, although some early modelling work by Thomas, Mujika and Busso (2009) suggests that such work could be beneficial. Few studies have used data from real world training and performance data. As the relationship between performance in tests and performance in events is questionable, such real world modelling can potentially provide very valuable information.

Modelling an athlete’s response to training gives the opportunity to examine the impact of taper shape on performance. The aim of this investigation was to firstly identify the effect the shape of a taper has on an elite cyclists’ performance; and secondly, to determine if individual differences in optimal taper shape exist between athletes.

The datasets used for this investigation consisted of power data from the training and racing of two female elite cyclists over a period of 250 and 870 days. PTRIMP and performance were calculated as per the previous investigations.
It was necessary to find a pre-processing technique to enable an understanding and subsequent classification of the shape of the training load in the final three days of a taper. In a novel application, Symbolic Aggregate approxImation (SAX) was used to discretise the taper time-series (the training load or PTRIMP for the three days prior to a competition) to a symbolic representation using an alphabet size of three. The discretisation resulted in a sequence of three strings for each three day taper period, with the string representing that shape of the taper. A string of ‘321’, for example, represented a taper that started with a high training load on day \( p_3 \), then a medium load on day \( p_2 \) and a low training load on day \( p_1 \) (where \( p = \) the day of the performance or competition). The strings of taper shapes were grouped into four categories to describe their shape (low, high-tail, low-tail and high). Analysis showed that the main effect of taper shape on performance was statistically significant for both athletes a 5% level of significance. Considerable variance existed between individuals and their response to a particular taper shape. The interaction effects between taper shape and subject were statistically significant.

These results suggest that tapering advice, which in the literature is frequently generalised across genders, between different sports, and between athletes of differing levels of training, cannot be usefully provided using a generalist model.

The effect size of the different taper shape treatments was statistically significant for both subjects. The evidence that training load has a predictable and significant effect on performance provides validation for the integrity of the techniques used to quantify training and performance.

The most effective taper shape for each athlete represented a 6.1% and 4.4% improvement respectively, compared with the least effective taper shape. While drafting and team tactics preclude the estimation of the smallest worthwhile improvement in road cycling, Paton and Hopkins (2006) estimate that the smallest worthwhile improvement for top-level athletes in individual road cycling time trials is approximately 0.6%. It is therefore likely that the small improvements in performance...
observed in this investigation as the result of employing an optimal taper shape could have a significant effect on the outcome of elite road races. These findings point to the usefulness of modelling training and performance and the ability of such models to provide practical feedback for athletes and coaches to assist them in the planning of training programs. The relatively minimal data collection requirements for this study - made possible by the development of the novel training load and performance quantification techniques which require only the provision of power data from all training rides and performances – make it feasible for elite athletes to provide data without interfering with their normal training and racing schedules.

The investigations already described set up the foundation and framework for the development of a model to describe the relationship between training and performance for elite road cyclists. The fourth investigation in this work described the development of a novel hybrid artificial network ensemble model (HANNEM).

For HANNEM to be effective, it needed to overcome a number of limitations in the data associated with the training and performance domain, namely limited amounts of noisy and incomplete data. Data collected from elite athletes needs to make minimal changes to their normal training and racing routines, and this limits the number of independent parameters available for modelling.

The modelling relies on collecting data on performances that represent maximal efforts – it is thus crucial to be able to identify such sessions. A training diary recording the athlete’s perception of effort for each session can be useful here, but is not always available. A novel technique was therefore developed to identify races where, due to tactical (or other) considerations, 100% effort was not made by the athlete. Power files which show a low average power output, and a high average speed are indicative of flat stages in a multi-day stage race where drafting is occurring and 100% effort is not made. Cluster analysis was performed on the variables performance and average speed, and average power and average speed. Clusters with high speed and low performance or power were identified as stages not involving 100% effort, and excluded from the
dataset. This substantially reduced the noise of the performance dataset. This novel technique increases the range of datasets that can be used for modelling.

HANNEM combined three key modelling elements: a SimBEA model (Bruckner, 2006); a linear model (either an ARIMA or multiple linear regression model); and an ensemble of artificial neural networks. PTRIMP was used as the model input and performance was the output.

The SimBEA model developed by Bruckner (2006) was used to capture important domain knowledge about the underlying physiological processes governing the response to training, and their impact on performance. A linear model was incorporated to model the linear patterns in the data. A neural network model, taking as inputs a summary of recent training load quantified using the PTRIMP algorithm, as well as the performance predictions from the SimBEA model and the linear model, was used to capture the nonlinear patterns in the data. The performance of the hybrid model (a combination of SimBEA, ARIMA and ANN models, or a combination of SimBEA, ARIMA, MLR and ANN models) was better than the performance of any single model for each of the datasets.

The use of neural networks with small, noisy datasets is potentially problematic. Neural networks created on such datasets are prone to overfitting issues, resulting in models with poor generalisation abilities. To reduce overfitting problems, the techniques of bagging, and adding noise or jitter to each bag was used. An ensemble of ANNs were trained on the bagged sets, which all varied slightly.

Three datasets captured from the training and racing of three elite road cyclists were used to test the performance of HANNEM. Moderate to good fits were obtained for the HANNEM predictions using jackknife validation ($R^2 = 78\%$; $R^2 = 60\%$ and; $R^2 = 51\%$ for Subjects A, B and C respectively).

These results are in contrast to the fits obtained for an impulse-response model alone. Fits for SimBEA (described by Bruckner, 2006) were poor – an $R^2$ of 32\% for Subject B,
and weak not linear relationships between actual performance and predicted performance for Subjects A and C.

A key question when evaluating these results is whether the level of accuracy reported is such that HANNEM can be used as a training planning tool for coaches and elite athletes. The results from Chapter 7 indicate that the techniques developed throughout this research are indeed able to determine appropriate taper strategies that result in worthwhile levels of performance improvement. Measurement of performance in road cycling cannot be absolute but rather is made up of a number of critical moments. Performance is best measured relative to an individual’s previously recorded capabilities during critical moments. The aim of HANNEM is to correctly identify the direction and rate of change in performance trends, rather than predict an exact performance level, and this it achieved.

The sparse datasets used in the modelling make it likely that some regions of input space would be overtrained by component ANN models, and other regions undertrained. The final investigation of this work addressed this issue of potential overfitting and resultant lack of generalisation with small noisy datasets. The investigation aimed to determine whether creating an ensemble of ANNs mitigates the effect of overfitting with small and potentially noisy datasets, and in fact whether deliberate overtraining can actually improve the performance of an ensemble of ANNs. The optimal number of component ANNs in an ensemble was also investigated in relation to the size and level of noise in training datasets.

The results showed that by combining a diverse collection of ANN models (created using bagging) into an ensemble, the smoothing effect of the ensemble removed any adverse effects of overtraining on individual nets. In fact, for low to moderate levels of noise in the data, a strategy of overtraining was shown to increase the diversity of the component models and consequently provide performance benefits to the ensemble. The findings suggest that HANNEM is not likely to be compromised in its ability to
generalise to new data due to overtrained regions in the sparse dataset - as a result of the regularisation effect of the ANN ensemble.

The investigation showed the optimal size of the ensemble was variable, as you would expect with unstable estimators. Twenty to forty nets tended to be a good range for an ensemble, with the optimal number of nets increasing slightly for datasets with large rates of error. These findings suggested that HANNEM could be improved by increasing the number of component nets in the ensemble to between 20 and 40 nets. The findings also suggest that estimation of the level of variance in the ensemble for differing ensemble sizes can give insight into the number of distinct models that can be created from a particular dataset. Examining variance and performance over a range of ensemble sizes, and choosing a number of nets somewhere around the point of inflection in the level of variance was shown to be an effective strategy.

In summary, the results (as shown in Chapter 8) showed that the application of HANNEM in predicting performance in elite cyclists is promising. The method presented in this thesis provides the basis for real world performance prediction using data obtained in the field. The model has been designed to:

1. Provide a technique for quantifying training load in a physiologically accurate manner from power data file collected in the field.
2. Quantify performance from race power data collected in the field, using the insight that performance in a race consists of maximal performance in a number of critical moments in the race.
3. Provide a technique for symbolising taper shape and subsequently identifying the optimal taper shape for an athlete.
4. Identify and eliminate races or stages where the athlete did not give 100% effort for strategic reasons.
5. Incorporate Bruckner’s (2006) SimBEA model to capture domain knowledge about the physiological processed underlying training and performance.

6. Construct linear statistical models to model the linear patterns in the data and take advantage of the strengths of such statistical models.

7. Estimate performance using an ensemble of ANNs to capture the nonlinear patterns in the data, using as inputs the performance predications from the SimBEA model and the linear model as well as a summary of the recent training load (PTRIMP).

8. Use the technique of bagging and the addition of noise or jitter to the bagged training sets in order to create diversity in the component ANNs. These techniques are particularly appropriate for small, noisy datasets.

A coach or elite athlete may be able to use the HANNEM model to:

- Identify the direction and rate of change in performance trends;

- Identify the optimal shape of the taper;

- Present the model with proposed changes in training load (in the form of estimated PTRIMP values) and observe the resultant estimated change in performance; and

- Ultimately use the above information to provide decision support when developing a training plan to enable the athlete to deliver optimal performance at the desired time.

The research conducted in this thesis advances current training and performance modelling techniques to allow prediction of road cycling performance training and racing data collected from the field. New techniques were developed to allow the field-derived data to be quantified for modelling. Modelling techniques have been
developed to extract patterns from these small, noisy and incomplete datasets, and build robust prediction models with sufficient accuracy to be used in assisting the planning of training. The new approach shows great promise for assisting elite athletes and coaches in planning and monitoring training, and shows potential for both future research and commercialisation. The approach also provides a modelling framework that can potentially be utilised in other domains where datasets are small, noisy and incomplete.

The research conducted has raised a number of issues worthy of further research. The major future extensions to this work which have been identified include the following:

1. **Add more component ANNs to the ensemble of ANNs used in HANNEM.** The investigation reported in Chapter 9 suggested that 20-40 nets might be appropriate. Creating a test harness to trial a range of nets sizes, and then examining the resultant estimated performance and variance should allow the optimal ensemble size to be estimated.

2. **Investigate the effect of deliberate overtraining on component ANNs in HANNEM.** The investigations reported in this work indicate that early stopping algorithms may become unstable with very high levels of error exist in the dataset. The evidence suggests that overtrained ensembles perform better than early exit ensembles when data error levels are low to moderate.

3. **Investigations (undertaken by the author and a collaborator) are currently underway into a new type of neural network architecture and training algorithm, which is designed to reduce the adverse impact of regions of sparse data and unsupported data points on modelling.** The network, nicknamed SANATY, uses mapping in input space to identify the \( k \) nearest neighbours who are best placed to provide a prediction for any unseen new data point. An intelligent arbiter then takes the input from the nearest neighbours and uses fusion techniques to synthesis the final prediction.
SANATY has the potential to be of use for all datasets that suffer from regions of sparse data.

4. Develop an automated tool which uses a technique such as genetic algorithms to solve the optimisation problem of sequencing training loads.

5. HANNEM has the potential to be useful in identifying abnormal patterns in performance. These abnormal patterns could be the result of illness or doping. HANNEM establishes a relationship between training and performance, and when this relationship becomes disrupted, it indicates that a variable unaccounted for by the model warrants identification and investigation.
APPENDIX A – LIST OF PUBLICATIONS

This appendix displays the list of refereed publications that have been achieved to date from the thesis work.


## APPENDIX B - GLOSSARY

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANN</td>
<td>Artificial Neural Network</td>
</tr>
<tr>
<td>ARIMA</td>
<td>Autoregressive Integrated Moving Average</td>
</tr>
<tr>
<td>ATL</td>
<td>Acute Training Load</td>
</tr>
<tr>
<td>CTL</td>
<td>Chronic Training Load</td>
</tr>
<tr>
<td>HR</td>
<td>Heart rate</td>
</tr>
<tr>
<td>HRV</td>
<td>Heart Rate Variability</td>
</tr>
<tr>
<td>ICC</td>
<td>Intraclass Correlation Coefficient</td>
</tr>
<tr>
<td>MMP</td>
<td>Maximum Mean Power</td>
</tr>
<tr>
<td>PNS</td>
<td>Parasympathetic Nervous System</td>
</tr>
<tr>
<td>PTRIMP</td>
<td>Power Training Impulse</td>
</tr>
<tr>
<td>RI</td>
<td>Recovery Index</td>
</tr>
<tr>
<td>RPE</td>
<td>Rate of Perceived Exertion</td>
</tr>
<tr>
<td>RR interval</td>
<td>The interval between successive R waves</td>
</tr>
<tr>
<td>SAX</td>
<td>Symbolic Aggregate approXimation</td>
</tr>
<tr>
<td>SimBEA</td>
<td>An impulse-response model</td>
</tr>
<tr>
<td>SNS</td>
<td>Sympathetic Nervous System</td>
</tr>
<tr>
<td>SRM</td>
<td>Schoberer Rad Messtechnik. Device to measure power output of cyclists</td>
</tr>
<tr>
<td>SVM</td>
<td>Support Vector Machine</td>
</tr>
<tr>
<td>TSB</td>
<td>Training Stress Balance</td>
</tr>
</tbody>
</table>
APPENDIX C – ALGORITHM USED IN HANNEM

The algorithm used in HANNEM to generate and validate an ensemble of ANN models.

For $i = 0; i < \text{numberOfRows}; i++$
{
    Go to Row($i$);
    Exclude;
    For($i = 0; i < 8; i++$)
    {
        CreateBagOfData(sampleSize);
        AddNoise();
        With bag
        Neural Net(
            Y(:Performance),
            X(:SimBEA, :LinearModelPrediction, :TrainingLoad),
            CrossValidation("K-Fold", 5),
            SequenceOfFits (
                OverfitPenalty(Vary from 0.001 to 0.04),
                HiddenNodes(Vary from 2 to 5)
            )
        )
        Save Formula for best performing model
    }
    Average prediction of component nets for excluded row
}

Table C.1. Algorithm used to generate and validate an ensemble of ANN model.
APPENDIX D – EXPERIMENT 1-2

<table>
<thead>
<tr>
<th>Experiment 1</th>
<th>Experiment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Overlay Plot errRate=10</strong></td>
<td><strong>Overlay Plot errRate=10</strong></td>
</tr>
<tr>
<td><img src="image1" alt="Performance graph" /></td>
<td><img src="image2" alt="Performance graph" /></td>
</tr>
<tr>
<td>Y ○ EarlyEx + OverT</td>
<td>Y ○ EarlyEx + OverT</td>
</tr>
<tr>
<td><strong>Overlay Plot errRate=20</strong></td>
<td><strong>Overlay Plot errRate=20</strong></td>
</tr>
<tr>
<td><img src="image3" alt="Performance graph" /></td>
<td><img src="image4" alt="Performance graph" /></td>
</tr>
<tr>
<td>Y ○ EarlyEx + OverT</td>
<td>Y ○ EarlyEx + OverT</td>
</tr>
<tr>
<td><strong>Overlay Plot errRate=30</strong></td>
<td><strong>Overlay Plot errRate=30</strong></td>
</tr>
<tr>
<td><img src="image5" alt="Performance graph" /></td>
<td><img src="image6" alt="Performance graph" /></td>
</tr>
<tr>
<td>Y ○ EarlyEx + OverT</td>
<td>Y ○ EarlyEx + OverT</td>
</tr>
</tbody>
</table>
Figure D-1. Comparison of performance of EarlyExit and OverTrained ensemble models by error rate. The x axis shows the number of networks included in the ensemble (from 1 – 99). The difference in behaviour of the networks between Experiment 1 and 2 can be compared for each data error level.
REFERENCE LIST


230


237
doi:10.1016/j.ics.2004.03.238