AN INVESTIGATION OF PRIMARY TEACHERS’ MATHEMATICAL PEDAGOGICAL CONTENT KNOWLEDGE

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Abstract

In an era of educational reform, investigating teachers’ pedagogical content knowledge has implications for many involved in education, from policy makers and curriculum designers to those in teacher education. This thesis proposed a model, designed by the researcher, used to examine Shulman’s (1986) theory of pedagogical content knowledge. In particular, it addressed primary teachers’ pedagogical content knowledge required for teaching measurement.

By examining teachers’ mathematics pedagogical content knowledge a greater understanding of teachers’ professional knowledge was gained enabling improvement of teacher quality, by being able to identify more clearly individual teacher’s needs for professional development. This study addressed four specific research questions. How evident is the teacher’s depth of mathematical knowledge of measurement within their teaching? How do teachers show that they understand and address the needs of students when teaching? How to teachers demonstrate their general pedagogical knowledge when teaching? How is a teacher’s knowledge and practice impacted by other factors when teaching and what are these major factors?

A qualitative research model was used in which four teachers of Years Three and Four participated, providing four individual case studies. Each teacher was interviewed at the commencement of the study, was observed and recorded throughout their teaching of a sequence of measurement lessons, interviewed prior to and following each lesson, and finally responded to a reflective questionnaire two weeks after the sequence of lessons had concluded.

Due to the extensive nature of the data, a series of vignettes was written, based upon
identified teaching episodes, significant to addressing the research questions. These vignettes contributed to the cross case analysis (Yin, 2010), along with the other data. The study found that the teachers’ knowledge varied considerably in each of the areas of knowledge of teaching, knowledge of students and knowledge of mathematics. Consequently, the teachers were rated differently in relation to their pedagogical content knowledge, ranging from very weak to strong. These differences were examined in terms of the model, providing evidence that the model effectively explained variations in teachers’ pedagogical content knowledge. Factors such as self-efficacy, teacher beliefs and the culture of the school were also shown to influence each teacher’s pedagogical content knowledge. The model was shown to be dynamic and it clearly identified how and why pedagogical content knowledge varied from one teacher to another.

This study has shown that the model used to represent pedagogical content knowledge demonstrated theoretical, methodological and diagnostic value. This study concludes with a discussion of implications for policy and practice at system level and for teacher education courses for preservice teachers. The findings of this study provide further understanding of teacher pedagogical content knowledge, which is an essential step towards improving teacher quality and teaching practice. The evidence suggests that this model could be used for further research into pedagogical content knowledge beyond the teaching of measurement.
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Chapter One: Setting the Scene—Mathematics Teaching in an Era of Educational Reform

The teaching of mathematics has been a key focus for policy reform in education in recent decades. Changes in teaching approaches and curricula have been influenced by a greater emphasis on computing technology, a greater emphasis on problem solving and process skills, a greater awareness of the importance of developing mathematical language and a greater emphasis on the use of manipulative materials. Yet, with all the focus on reform, it appears any real effect on changing the quality of classroom instruction has largely been unsuccessful (Handal & Herrington, 2003; Yates, 2006). This lack of success is attributable to ‘a lack of congruence between the intent of the curriculum innovations and teachers’ pedagogical knowledge’ (Yates, 2006, p. 2).

Policy makers have responded to perceived problems with measures to improve teacher quality and in specifying professional standards (Invargson, 2010). Yet, this current approach to reform will not be effective unless educators make pedagogical content knowledge a central concern for policy and practice.

This thesis will argue that gaining a better understanding of teachers’ pedagogical content knowledge in the area of mathematics is essential if teacher quality levels are to be raised and if achieving teacher standards is to be effective. To achieve this, a number of premises will be argued that both contextualise and justify the current research.

Mathematics teaching standards are of concern both nationally and globally. A brief review of recent curriculum reforms and policy changes within Australia and internationally, dealing with the teaching of mathematics will be presented to set the
context for this study. It will be argued that educational research is needed to address
the recognised problem of teacher quality and standards and that the specific focus of
this research needs to be teacher knowledge.

While considerable research has focused upon the learner and learning, this
study will suggest that it is now time to examine teacher knowledge. Research is
needed to examine how teachers understand and how they teach mathematics.
Importantly, both aspects need to be examined. How teachers understand
mathematics will provide essential in comprehending the personally constructed
mathematical knowledge teachers bring to the classroom. Examining how they teach
mathematics will enable better comprehension of how teachers represent these
personally constructed understandings to students.

If time is devoted to investigating teaching and teacher knowledge, the
outcomes of such research will have multiple benefits. The educational research
community will be rewarded with greater appreciation and understanding of a
complex phenomenon. It will be argued that teacher knowledge is complex, as it
involves the combining of different types of knowledge in appropriate ways to result
in quality teaching.

Good theoretical resources have been available for some time and theories
regarding child learning and representational theory have been advanced in
educational literature. One such contribution that has had considerable effect on
educational theory is Shulman’s (1986b) theory of pedagogical content knowledge.
This thesis will develop a model providing an interpretation of Shulman’s theory,
operationalising the work to investigate teacher knowledge with specific reference to
measurement. The model will incorporate three types of knowledge required by
teachers: knowledge of the subject (in this case, mathematics), knowledge of students (their learning styles and individual needs) and knowledge of teaching.

This thesis will demonstrate how a close-grained analysis of teacher knowledge of four teachers, using classroom observation and teacher interview data, can be used to construct four different teacher case studies. The thesis will firstly show the nature of each teacher’s classroom practice and understandings. It will then furnish a further analysis based on cross-case comparisons that focus on aspects of representation in the teaching of measurement. It will provide evidence to show how the model of pedagogical content knowledge can predict weaknesses and account for variations in the quality of teacher performance. This thesis will thereby demonstrate how this approach has significance for the field of mathematics education in both policy and practice and specifically, how it has significance in elaborating on Shulman’s theory of pedagogical content knowledge.

The Problem

This thesis argues that if research is to contribute to the reform agenda of improving teaching standards and quality, it must direct greater attention to a deeper analysis of teacher knowledge as it is applied to classroom practice. The research problem concerns how teachers’ mathematical knowledge is applied in the context of their teaching, the classroom and the treatment of learners. To examine teacher knowledge requires a robust conceptual framework that provides a way to investigate the relationship between teacher expertise and subject knowledge.

This thesis sets out to build on the ideas of Shulman and develop a comprehensive model for examining what was termed ‘pedagogical content
knowledge’; focusing on how teachers represent mathematical information to students, rather than simply on student learning or teacher pedagogy.

Shulman formulated this concept of pedagogical content knowledge in an important 1986 paper, questioning the distinction between content knowledge and pedagogical knowledge:

Why this sharp distinction between content and pedagogical process? Whether in the spirit of the 1870s, when pedagogy was essentially ignored, or in the 1980s when content is conspicuously absent, has there always been a cleavage between the two? Has it always been asserted that one either knows content and pedagogy is secondary and unimportant, or that one knows pedagogy and is not held accountable for content? (Shulman, 1986b, p. 6)

Shulman suggested that within the knowledge base required by teachers there was a ‘missing paradigm’ and claimed that much of the literature about pedagogy dealt with issues such as classroom management and other general, pedagogical issues. However, there was a severe lack of research that dealt with more specific pedagogy regarding the content of the lessons being taught. Shulman suggested that a number of important questions were not being addressed. These included: ‘Where do teacher explanations come from? How do teachers decide what to teach, how to represent it, how to question students about it and how to deal with problems of misunderstanding?’ (Shulman, 1986b, p. 8).

Shulman argued that it is necessary to learn much more about this specific pedagogical content knowledge required by teachers to teach a particular subject. In essence, there is a need for better understanding of the knowledge required by
teachers that enable them to use ‘ways of representing and formulating the subject that make it comprehensible to others’ (Shulman, 1986b, p. 9).

Since Shulman’s address, many researchers have taken this idea of ‘a missing paradigm’ and conducted research into teachers’ specific pedagogical content knowledge (Ball, Lubienski & Mewborn, 2001; Huckstep, Rowland & Thwaites, 2003; Jones & Moreland, 2004; Turnuklu & Yesildere, 2007). Given the interest Shulman created in this aspect of teachers’ knowledge, and the number of research studies that have since been conducted, the collection and organisation of new understandings of pedagogical content knowledge is still in its early stages. Although it is now 25 years since Shulman’s address, the nature of the task of documenting this new kind of teacher knowledge remains a daunting challenge to educational research.

Researchers have responded to Shulman’s challenge to investigate pedagogical content knowledge and highlighted the need for further development of the theoretical framework (Ball, Thames & Phelps, 2008; Magnusson, Krajcik & Borko, 2002; Mishra & Koehler, 2006). Ball, Thames and Phelps (2008) assert that after two decades, Shulman’s theoretical framework remains underdeveloped. Hill, Ball and Schilling (2008) report a lack of studies concerning pedagogical content knowledge that investigate the character of this knowledge, its dimensions and the factors that explain strength and weakness in teaching effectiveness.

There is a need to account for teaching quality in terms of subject knowledge in a key learning area, such as mathematics. The potential theoretical and didactic value has not been exploited to the extent that the research has a level of understanding of teacher knowledge that will enable achievement of higher teacher
quality. This thesis seeks to remedy this deficiency and contribute to policy and practice in mathematics education.

**Purposes of the Study**

There were four purposes involved in this research project. Firstly, to build on Shulman’s original theory and develop a robust model that could guide the investigation of teachers’ pedagogical content knowledge. Secondly, to understand how teachers represent mathematical information and to address the challenge of reducing its mystique:

One of the biggest problems of mathematics is to explain to everyone else what it is all about. The technical trappings of the subject, its symbolism and formality, its baffling terminology, its apparent delight in lengthy calculations: these tend to obscure its real nature. A musician would be horrified if his art were to be summed up as ‘a lot of tadpoles drawn on a row of lines’; but that’s all that the untrained eye can see in a page of sheet music … In the same way, the symbolism of mathematics is merely its coded form, not its substance. (Stewart, 1996, p. 1)

One major problem for teachers, as Stewart describes, is explaining the ‘substance’ of mathematics to everyone else. The success of teachers explaining ‘the symbolism of mathematics’ is largely dependent upon their own knowledge base. This is the key role of teachers. Teachers need to help their students create meaning from mathematics to ensure that ‘the technical trappings of the subject, its symbolism and formality, its baffling terminology, its apparent delight in lengthy calculations’, enable their students to understand and appreciate the real nature of mathematics. Students need to understand the patterns and relationships that form the basis of
Thirdly, this research sets out to explore what needs to be undertaken to enhance relational understanding as opposed to instrumental understanding in teacher approaches to mathematics teaching. In exploring representation, there is a key union of ideas in Shulman’s pedagogical content knowledge and the notion of teaching ‘relationally’ (Skemp, 1976). Skemp makes a distinction between two different types of mathematical knowledge, proposing that the teaching of mathematics results in either instrumental knowledge or relational understanding. Instrumental knowledge is largely rote and mechanical without any real understanding being acquired. Relational understanding, by contrast, is established by learning through connections and networks of relationships between concepts. These relationships are developed by students experiencing appropriate representations of mathematical ideas.

The thesis will also explore the teaching of measurement by primary school teachers. Any investigation of teachers’ pedagogical content knowledge must be given a specific focus within these larger concerns. Therefore, this study focused its inquiry on the pedagogical content knowledge of primary school teachers and their teaching of measurement lessons to students in Years Three and Four. As in all mathematics, measurement is the study of significant patterns and relationships that enable the quantification of the varied aspects of the surrounding world. Measurement is arguably a key focus for primary teachers of mathematics; it is an everyday life skill that everyone engages in or is affected by, on a daily basis.

Given the central importance of measurement and its application to everyday life, it is unsurprising that considerable research has been conducted into this subject. However, while it has received attention in an extensive body of emerging research,
O’Keefe and Bobis (2008) identify the majority of this research as focusing upon key measurement concepts contained within curricular expectations that students need to learn. Particular emphasis has been placed upon the learning of measurement, especially in terms of students’ conservation of measurement attributes (Piaget & Inhelder, 1967; Piaget, Inhelder & Szeminska, 1960) and their learning of the measurement process (Battista, 2003; Bragg & Outhred, 2004; Reece & Kamii, 2001). Despite this importance, research dealing with required teacher knowledge of measurement is still relatively limited and has long escaped attention.

Therefore this study investigated the general research question: How do teachers demonstrate the depth of their pedagogical content knowledge when teaching measurement lessons to Years Three and Four students? This general question can be expressed in terms of four specific inquiries:

1. How evident is the teacher’s depth of mathematical knowledge of measurement in their teaching?
2. How do teachers show that they understand and address the needs of students when teaching?
3. How do teachers demonstrate their general pedagogical knowledge when teaching?
4. How is a teacher’s knowledge and practice impacted by other factors when teaching and what are these major factors?

**Context for the Research**

It is necessary to briefly review the context for the research to appreciate how policy reform is driving the search for improvement of mathematics teaching. Policy
reform in education has targeted the inadequacy of mathematics teaching, arguing that standards are low and large numbers of students in Australian classrooms are underperforming (Balfanz & Byrnes, 2006; Cohen & Ball, 1990a; Schoenfeld, 1989). So widespread is the belief that mathematics teaching has failed for large numbers of students that there has been considerable effort devoted to addressing the problem (Cates & Rhymer, 2003; Skemp, 1989; Midkiff & Thomasson, 1993). Policy documents have attributed this inadequacy to two major factors, the impact of technology and high levels of mathematics anxiety.

Constant developments in technology have been observed as key contributing factors for teachers failing to deliver quality teaching. Mathematics continues to mystify many students and is perceived to be a subject with little relevance to real life. Therefore, some students leave school without the mathematical knowledge ‘that they will need to function smoothly in our increasingly technological society’ (Middleton & Spanias, 1999, p. 65). The National Review into Teacher Education in Mathematics and Science (1989) claimed that while teachers convey mathematics to students, the preparation of teachers has not kept pace with the changes in technology or the subsequent demands of society.

The National Statement on Mathematics for Australian Schools (Australian Education Council, 1991) recognised the changing nature of mathematics due to rapid advances in technology and asserted that mathematics curricula in Australia must respond to these changes. The design of the National Statement deliberately attempted to move from the traditional view of mathematics as just a body of knowledge, comprising facts, concepts and generalisations and emphasised mathematical processes as well as ways of knowing and thinking skills.
There are demonstrably high levels of mathematics anxiety among large numbers of students (Jennison & Beswick, 2010; Khatoon & Mahmood, 2010) and this anxiety is prevalent among pre-service primary teachers (McCulloch Vinson, 2001). Mathematics anxiety is most frequently attributed to situations when students experience frequent failure (Karimi & Venkatesan, 2009). High levels of student failure must lead to suspicions of poor quality of teaching. Teachers need to re-examine traditional teaching methods, which often do not match the needs of students or the levels of skill needed in society. A change in teaching methods requires a greater understanding of required content knowledge.

**Policy directions within the Australian context.**


The *National Review into Teacher Education in Mathematics and Science* (1989) argued that improvement of teaching could be effected by studying exemplary teachers, since there is insufficient detailed knowledge as to the making of a ‘good’ mathematics teacher. This view assumes, rather simplistically, that those who are not ‘exemplary’ teachers can change their performance by learning from those who know how to perform at a high level. This approach overlooks the effect of mathematics

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* In 2008, the National Assessment Program—Literacy and Numeracy (NAPLAN) commenced in Australian schools. Every year, all students in Years 3, 5, 7 and 9 are assessed at the same time, using national tests in Reading, Writing, Language Conventions (Spelling, Grammar and Punctuation) and Numeracy.
anxiety that many teachers entering primary teaching have never managed to overcome. This study suggests that there is value in examining teacher knowledge from teachers who may not be considered exemplary and focusing on individual areas of weakness and deficiencies to assist teachers to develop the knowledge required to make effective changes in their teaching practice.

The pressures for greater accountability of teachers have increased with national testing in numeracy as part of the NAPLAN process, which teachers are required to implement. With NAPLAN, there is a new sense of accountability in the Australian context. With results published on the My School† website, teachers and schools are now under scrutiny as to their level of performance and comparisons are made with the performances of similar schools. While NAPLAN and My School have brought with them a certain degree of controversy, one of the results of these policy initiatives is to place teachers under the spotlight and to reveal where further development is needed. However, NAPLAN testing is a blunt instrument for securing specific improvements in mathematics teaching and does not reveal the specific areas that teachers may need to develop.

**Policy reform and teacher quality.**

The reform activity within Australia over the last three decades was clearly a reaction to perceived needs locally, yet it mirrored similar activity occurring in other countries. Major reforms in mathematics education have also taken place in other countries, particularly Great Britain and the United States of America, influencing developments in Australia. Ingvarson (2010) claims that the new *Australian National Curricula* illustrated the ‘complexity of what we expect our teachers to know’ (p. 1).

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† My School is a website that allows the public to search the profiles of almost 10,000 Australian schools. NAPLAN results for each school are published on this site.
Yet, Ingvarson confirms that research has shown that curriculum documents alone rarely bring about change in the quality of teaching because they do not focus sufficiently on the complexities involved. Teaching practice will only change as teachers are able to make improvements to pedagogical content knowledge. A greater understanding of teachers’ pedagogical content knowledge is essential if improvements are to be made:

The real educational challenge in implementing Australia’s National Curriculum is capacity building in every teacher and setting strong and clear standards to articulate what teachers need to know and be able to do to bring the curriculum vision to life. (Ingvarson, 2010)

**International developments.**

In England and Wales, *Mathematics Counts* (1982) challenged teachers to maintain in children the enthusiasm and curiosity that they exhibited towards mathematics when they first entered school. Understanding of higher levels of mathematics was more likely if teachers continued to nurture this natural enthusiasm through the strategies they used and their classroom organisation.

The Cockcroft report was influential and Britain has since seen considerable activity in mathematics education. In 1985, the document *Mathematics from 5 to 16* (Department of Education and Science, 1985) expanded on the learning outcomes from Cockcroft and particularly focused on issues such as the ‘uses of mathematics (as a language and a tool), appreciation of mathematical relationships, in depth study, creativity in mathematics, and most of all, personal qualities (working systematically, independently, cooperatively, and developing confidence)’ (Ernest, 1991, p. 222).
In recent years, there has been a succession of reports in England, including the *National Numeracy Strategy* (Department for Education and Employment, 1999); the *Primary National Strategy* (2003); *Every Child Matters* (HM Government, 2005); the renewed *Primary Framework for Literacy and Mathematics* (2006); the *Children’s Plan: Building Brighter Futures* (Department for Children, Schools and Families, 2007) and *Every Child Counts* (Edgehill University, 2007). Kyriacou (2005) suggests that classroom practice is still lagging behind the goals of new policy.

The United States has also recognised an apparent failure of mathematics education and made it the subject of several major initiatives. As early as 1980, *An Agenda for Action* (National Council of Teachers of Mathematics, 1980) projected a ten-year plan for revising school mathematics and changing the focus of both content and teaching strategies for the subject. It also recognised that for the *Agenda* recommendations to be effective, teachers would need the knowledge base to allow them to transfer the recommendations into classroom practice.

In 1983, the National Commission on Excellence in Education released *A Nation at Risk* (National Commission on Excellence in Education, 1983), echoed in later reports that emphasised declining mathematical achievement levels and an over-emphasis on mastery of computational skills that were too narrow in scope to prepare students for contemporary society. The National Research Council produced *Everybody Counts* (National Research Council, 1989) that restated claims that mathematics education was in a poor state and that major changes in curricula and pedagogy were not achieved easily. The changes required in teachers’ classroom practice were considerable and without substantial professional development, were
unlikely to eventuate. *Everybody Counts* reported that large numbers of Americans stop studying mathematics before completing career prerequisites and most students leave school with insufficient preparation in mathematics for on-the-job demands or college expectations.

Other major efforts have been made in an attempt to prompt a reform of mathematics education in the United States. In 1989, the National Council of Teachers of Mathematics (NCTM) published the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and in 1991, a second document, *Professional Standards for Teaching Mathematics* (NCTM, 1991), which suggested innovative and practical teaching methods and recommendations for those involved in teacher education. The new goals, as set down in these two documents, clearly go beyond the mastery of computational skills and expect that students should learn to value mathematics, develop confidence in their ability to do mathematics, become mathematical problem solvers, learn to communicate mathematically and learn to reason mathematically.

In 1995, a third NCTM document, *Assessment Standards for School Mathematics* (NCTM, 1995) urged teachers to provide classes that are creative, incorporate the appropriate use of manipulative materials, introduce students to the use of calculators and computers and emphasise understanding and problem solving. The *Standards* acknowledge the importance of the role of the teacher if their implementation is to be effective: ‘The kind of instruction needed to implement the NCTM Standards requires a high degree of individual responsibility, authority, and autonomy—in short, professionalism on the part of the teacher’ (NCTM, 1991, p. 4). Then in 2000, *Principles and Standards for School Mathematics* was released
The purpose of this document was to acknowledge the implications of the rapidly changing world for the teaching of mathematics and highlight the importance of the accessibility of mathematics for all students, and to provide a resource for teachers, leaders and policy makers that would achieve equity and excellence.

This brief survey of global developments demonstrates that mathematics education has entered a new era and highlights the changes in theories, policies and curricula that are influencing practising teachers. Burghes (1989) aptly described the current situation in mathematics education as one of ‘so much activity in mathematics education post-Cockcroft that it is difficult to know where to begin’ (p. 83).

Such perplexity underlines the need to set a clear research agenda, one in which the adequacy of teachers’ subject knowledge in mathematics must have a high priority. Much of the present research agenda has arisen from recent reform in mathematics education (Silver, Kilpatrick & Schlesinger, 1990). This thesis seeks to make a contribution to this goal.

**Definition of Terms**

It is important to provide key definitions of terms as they are used throughout this study to avoid confusion. Ball, Thames and Phelps (2008) observe that while there has been considerable attention to pedagogical content knowledge in recent literature, ‘nearly one-third of the articles that cite pedagogical content knowledge do so without direct attention to a specific content area’ (p 3). Therefore, this has resulted in researchers placing different emphases and even different meanings onto pedagogical content knowledge, meanings that have moved away from Shulman’s original use of the term. The usage of the term within this thesis follows Shulman’s
original definition.

‘Instrumental knowledge’ refers to knowledge learnt by rote. It may be difficult for students to apply this to new situations as they have no genuine understanding of the content. It lacks coherence and relationships to other concepts (Skemp, 1978).

‘Relational understanding’ is often considered the only real form of understanding. It is an understanding that is built upon relationships between concepts. It involves understanding the ‘why’ and the ‘how’. Understanding concepts in a relational way enables the learner to more readily apply their understanding to new situations (Skemp, 1978).

‘Content knowledge’ is knowledge specifically about a discipline area. Within this research, this will signify the knowledge of mathematics. Ball, Thames and Phelps (2008) suggest that this is ‘pure’ content knowledge of the discipline of mathematics (Freiberg, 2002).

‘Pedagogical knowledge’ refers to the knowledge a teacher has about teaching that is not specific to a particular discipline. Knowledge of classroom management, grouping students, questioning techniques, assessment principles and reporting are all examples of this kind of knowledge.

‘Pedagogical content knowledge’ refers to the knowledge a teacher needs to teach a particular discipline or body of content in such a way that it can be understood by a specific group of learners. More precisely, it will be understood as the intersection of the three types of knowledge about mathematics, students and teaching (Shulman, 1986).

‘Representation’ refers to the examples, models and analogies that teachers
use to enable students to learn a concept. ‘Internal representation’ means the internal mental models that students construct as a result of learning. Internal representations will be influenced heavily by the kinds of representations teachers use to help their students understand new content (Post & Cramer, 1989).

Conclusion

Framed in this way, inquiries into the pedagogical content knowledge primary teachers demonstrate when teaching measurement holds significance at four key levels. Firstly, it has the potential to contribute to our understanding of teachers’ professional knowledge and mathematics pedagogy, particularly in the area of teaching measurement. This research addresses the lack of research dealing with mathematics pedagogical content knowledge and attempts to redress the problem that much of the research into teaching measurement has focused upon the measurement process itself and how students come learn it. This study changes that focus and examines the role of the teacher in terms of the knowledge they demonstrate and use when teaching measurement lessons. This study has developed a model that, while based upon the work of Shulman and others, broadens and further develops the initial theory.

Secondly, the present study contributes to our understanding of and recommendations for the improvement of teaching practice. By examining types of teacher knowledge and how these interact with beliefs, self-perceptions, school cultures and systemic constraints, a clearer understanding may emerge for recommendations to improve practices, particularly in the teaching of measurement. The findings have implications for teacher education programs in mathematics education, professional development programs for in-service teachers and for staff
Thirdly, this study will explore and contribute to the application of Shulman’s theory of pedagogical content knowledge. This study highlights the importance methodologically of studying pedagogical content knowledge within a relatively narrow focus on discipline content. The study will develop Shulman’s pedagogical content knowledge, theorising it as the intersection of three types of knowledge—knowledge of mathematics, knowledge of students and knowledge of teaching. It will show how specific teaching practices in the teaching of measurement can be understood as dynamic interactions of these ‘knowledges’. In so doing, the research will contribute to the scholarly understanding of pedagogical content knowledge.

Lastly, the study will also contribute to policy on reform of mathematics education by developing a clearer understanding of pedagogical content knowledge required for identifying the needs of teachers and the nature of appropriate professional development for teachers as they are confronted with multiple policy changes.

Currently, much attention is being given to new policy directions in teacher registration, teacher quality, and in the implementation of national assessment and national curriculum. This study will contribute to a better understanding of the knowledge required by teachers who are expected to implement each of these policy initiatives.

Organisation of the thesis.

Chapter One defines the research problem and its significance, referring to Shulman’s concept of pedagogical content knowledge and exploring its significance for research on the teaching of mathematics. Chapter Two develops the conceptual
model for the research, reviewing the relevant structure and illustrating how Shulman’s theory may be extended and operationalised in respect of primary school teachers’ practices in teaching mathematics.

Chapter Three examines the methodological issues of the study. The use of qualitative methodology is explained and justified and the design of a multiple case study approach with a cross-case study analysis and discussion is presented. The research procedures employed are outlined and their adequacy by using multiple data sources for investigating pedagogical content knowledge—a complex phenomenon—is argued. Profiles of the four teachers are provided to give context to the analytical chapter that follows.

Chapter Four provides documentation of the observations and results of the interviews and questionnaires with each of the teachers. Each teacher is presented individually using critical teaching incidents as well as responses to interview questions and the questionnaire to address the research questions as the basis of documenting the findings.

Chapter Five provides analysis and further discussion of the results. The model used throughout this study as a representation of Shulman’s theory on pedagogical content knowledge is analysed. It is argued with detailed reference to the case studies that the model has both conceptual power and interpretive adequacy. The model has a dynamic structure and it is demonstrated how the structural elements of the model may interact and how external factors impact on the structural elements to bring about further change. It will account for disequilibrium and potential conflict within the teachers’ pedagogical content knowledge.
Chapter Six provides a conclusion and recommendations based upon the findings of this study. In particular, the value of the present research, its contribution to the field and recommendations for future policy directions and for teacher education are presented.
Chapter Two: Conceptualising the Research

Introduction

It is widely accepted that any study investigating teacher knowledge is engaged with a complex phenomenon that has many interrelated dimensions (Even & Tirosh, 2002; Mishra & Koehler, 2006; Moloney & Clarke, 2010). The purpose of this chapter is to review pertinent literature dealing with research into teachers’ required knowledge when teaching mathematics, with particular focus on the teaching of measurement.

The chapter will initially examine what constitutes subject content knowledge—an essential component of Shulman’s pedagogical content knowledge—as it relates to the teaching of measurement. The chapter will then examine the literature dealing with the additional knowledge base necessary for teachers of mathematics. This includes teaching mathematics for understanding (Skemp, 1978, 1989); pedagogical content knowledge (Ball & Bass, 2000; Shulman, 1987); the importance of representation (Hiebert & Carpenter, 1992; Post & Cramer, 1989) and multiple modes of representation (Akkus & Cakiroglu, 2009; Barmby, Bolden & Harries, 2011; Lesh, Post & Behr, 1987).

The chapter examines three other issues said to impact on teachers’ knowledge. They are teacher beliefs (Bingimlas & Hanrahan, 2010; OECD, 2009), teachers’ self-efficacy (Perry, Lewis, Friedkin & Baker, 2009) and the cultural context of the school environment (Cooper, 1998; Yackel & Cobb, 1993). The chapter will establish a conceptual framework for researching teachers’ knowledge, based upon Shulman’s theory of pedagogical content knowledge. The researcher will meet the challenge of operationalising Shulman’s theory of pedagogical content
knowledge by presenting a model that provides a framework for the research involving the four teachers examined in this study.

**Measurement as Subject Knowledge**

Research on measurement in mathematics education has tended to focus on learners and the problems they encounter rather than on teachers and the methods they use to teach measurement ideas (Clements, 1999; Lehrer, Jenkins & Osana, 1998; Wilson, 1990). Over the last two decades there has been an increase in research conducted focusing on measurement, although this has largely investigated children’s understanding of the measurement process (Bragg & Outhred, 2001; Wilson & Osborne, 1992), or their understanding of particular attributes of measurement, such as length, area or volume (Barrett, Jones, Thornton & Dickson, 2003; Bragg & Outhred, 2000). However, this work still markedly ignores teachers and their required knowledge base for teaching measurement.

There is no doubt that research on student understanding of measurement is invaluable in understanding how teachers should successfully approach teaching the topic. Much work has focused on students’ learning of particular attributes and has been driven by the mathematics curriculum requirements that students learn many measurement attributes in their primary school years. Researchers have aimed to ‘unpack’ each of these attributes and examine them in more detail. Examples of this include the examination of students’ knowledge of units for length and their knowledge of rulers (Bragg & Outhred, 2000; Levine, Kwon, Huttenlocher, Ratliff & Deitz 2009; Nührenbörger, 2004) and investigation of area and perimeter (Cass, Cates, Smith & Jackson, 2003; Kellogg, 2010; Reinke, 1997; Yeo, 2008). Other research not specifically focused upon a single attribute has focused upon very
specific stages of schooling, such as MacDonald (2010), which examined young children’s knowledge of measurement upon commencing school.

**Understandings of measurement.**

To appreciate the nature of subject knowledge, it is necessary to consider what teachers understand by ‘measurement’. Measurement is more than memorising the procedure involved in arriving at a number. It involves developing ‘measurement sense’, which Shaw and Puckett Cliatt (1989) suggest involves three components.

Firstly, teachers who have ‘measurement sense’ have knowledge of the appropriate units for any given task, but have also developed mental models representing measurement units. These mental models then allow them to select the most efficient units, as well as those most effective for communication, even though other units may have been used (such as selecting ‘kilometre’ rather than ‘centimetre’ to measure the distance between home and school).

Secondly, measurement sense requires learners to have an understanding of the measurement process (Shaw & Puckett Cliatt, 1989), recognising that measurement requires the making of comparisons using standard or nonstandard units. Knowing how to attach numbers and how to obtain the appropriate numbers often involves understanding a range of instruments and selecting the appropriate instrument for the task. Proficiency using these instruments (such as clocks, thermometers and trundle wheels) needs to be developed and only arises with practice. Reys, Lindquist, Lambdin and Smith (2012) support these notions in their description of the measurement process, suggesting the steps needed for measuring each attribute, as shown in Figure 2.1 below. These steps bring a certain degree of
coherency to the measurement strand, as this ‘process of measuring is identical for any attribute’ (NCTM, 1989, p. 52).

Figure 2.1. The measuring process portraying the five steps listed by Reys et al. (2012).

Thirdly, Shaw and Puckett Cliatt (1989) argue measurement sense also involves the use of estimation skills, as in everyday life adults use estimation on a regular basis and, given its importance, develop estimation strategies (Joram, Bertheau, Gabriele, Gelman & Subrahmanyam, 2005; Shaw & Puckett Cliatt, 1989). Estimation does not come naturally to most learners but requires practice and exposure to explicit strategies. Skilled estimation needs to be encouraged by teachers, and children need to be able to see how useful estimation is in everyday life, as well as in the classroom. However, Lang (1999) warns that estimation is a difficult skill for teachers to impart to their students and doing so involves a range of strategies that the teacher needs to know and understand. Lang (1999) suggests that teachers need to use referents, ‘chunking’ and unitising as strategies for teaching estimation.
The measurement process.

It is important to recognise the complexity of the measurement process portrayed in Figure 2.1. This knowledge of the measurement process is a significant component of the mathematical knowledge of measurement that teachers need in order to have sound pedagogical content knowledge. The analysis of the measurement process demonstrated by Reys et al. (2012) indicates how teachers must grasp complexity. The final step of the measurement process is particularly important, as it considerably influences the kinds of activities teachers select when teaching measurement in a way that is authentic and meaningful. Teachers need to understand that measurement requires a purpose:

Yet many students could be forgiven for believing that the lessons are on arithmetic, even though their teachers are addressing content from the measurement strand. Students often view questions about volume, area and perimeter as poorly disguised multiplication and addition questions. (NSWDET, 2004, p. 1)

It is this final step of the measurement process that demonstrates whether teachers recognise the need for authenticity. Further, it provides evidence of this authenticity by the kinds of activities teachers select. As O’Keefe and Bobis (2008) claim, teachers often use inappropriate activities due to the lack of appropriate teacher knowledge about the measurement process. They state:

There was a reluctance to teach measurement concepts by primary teachers and that this may stem from contextual constraints including teacher knowledge. Such findings build support for the view that student
misconceptions may simply reflect teachers’ inadequate understandings of measurement concepts. (p. 392)

A lack of knowledge results in an emphasis on procedure rather than process and teacher reliance on worksheets and other inappropriate activities to teach measurement (Bragg & Outhred, 2000; O’Keefe & Bobis, 2008).

**Estimation and Measurement**

The kind of estimation evident in classrooms is dependent upon the teacher’s personal knowledge of measurement and their recognition of the interdependence of estimation with measurement. Knowledge of measurement involves more than knowledge of the measurement process, it includes dealing with the process mentally, as well as using instruments and performing actual measurements. Estimation can be meaningful and contribute to helping children understand the measurement process or, alternatively, it can be associated with ‘vagueness, ambiguity and guessing’ (Forrester & Pike, 1998, p. 334).

Selection of appropriate units not only involves understanding of the various attributes, but also requires the development of clear mental models of available units. Estimation assists this development of mental models or mental referents. Bright (1976) described estimation of measurements as ‘the use of units in a strictly mental way, without the aid of measuring tools’ (p. 88).

Thus, the development of estimation skills is an important aspect of a teacher’s pedagogical content knowledge. Teachers need to have the knowledge of appropriate strategies to enable their students to accomplish the creation of these estimation skills. If students are to develop meaningful mental models of specific measurement units, then considerable practice needs to be given to this skill.
Estimation needs to be a learning experience. Every estimation a child makes needs to help clarify and construct accurate mental models that he or she can use. If the estimate is not close to the actual measurement, the child needs to adjust the mental model accordingly.

Therefore, students need to develop their estimation skills in a meaningful way by estimating, measuring and checking (Coburn & Shulte, 1986). In the early stages of developing estimation skills, students need to frequently estimate a measurement and follow through by measuring and checking the accuracy of their estimation, since it is only through this kind of association that estimation in itself can become meaningful and mental adjustments can be made as necessary.

Teachers also need to understand overestimation and the appropriate associated strategies for its correction. When a child overestimates, their mental model of the unit is too small. To counter this, it is of value to have a physical model of the unit available while making a series of estimates and measurements. Conversely, when a child underestimates, their mental model of the unit is too large. Once again, the presence of a physical model of the unit will assist them to make the necessary adjustments to their mental model (Barmby, Harries, Higgins & Suggate, 2007; Hiebert & Carpenter, 1992; Sierpinska, 1994).

As well as using physical models to help focus attention on the size of the unit when estimating, children should be encouraged to use newly measured objects as referents for future estimates, thereby increasing the accuracy of their new estimates. Comparing objects to be measured with referents is an important strategy for teachers to incorporate when teaching estimation in measurement. Coburn and Shulte (1986) suggest that accuracy of estimation can be improved by ‘range estimation’, when
‘students attempt to estimate within a given amount or a given percentage of the actual measurement’ (1986, p. 195). For example, when measuring length, students may be encouraged to estimate the length of various objects with a range of plus or minus five centimetres, or to estimate the length to an accuracy of at least 80 per cent. As the students’ estimation skills improve, the range can be narrowed.

Teachers also need to be clear how estimating can aid clarity and understanding. Provided the estimate is an accurate one, it is often more desirable to work with the estimate than a more precise figure (Usiskin, 1986). Usiskin acknowledges that the decision of whether to seek clarity or precision can cause some conflict:

Clarity and precision are often in conflict; the more precise value is often not as easily comprehended. For example, it is easier to remember that a kilometer is a little over 1600 yards than to recall or memorize that 1 km = 1609·344 yd. Precision is an important idea that should not be underrated, but on many occasions sacrificing precision for clarity is beneficial. (Usiskin, 1986, p. 6)

Usiskin provides an example: ‘A house on a lot whose width was surveyed as 40·13 feet would almost certainly be said to be on a '40 foot' lot’ (p. 6). The estimate, Usiskin argues, provides greater clarity and ease of understanding.

Nevertheless, estimating for clarity often only achieves ease of understanding; it is not always necessary that the estimate be used in any practical manner. Usiskin (1986) also stresses the need to estimate for facility. This is described as the ‘active use of estimation for the specific purpose of simplifying later work or making it more efficient or economical’ (p. 9). Estimation for facility could also be argued to include
the notion of estimation for developing more precise mental models of measurement units.

This brief discussion of the knowledge required by teachers to assist their students to develop estimation skills and an understanding of the measurement process illustrates the complexity of pedagogical content knowledge.

**Teaching Mathematics for Understanding**

An important aspect that needs to be considered when examining teacher’s pedagogical content knowledge is the meaning attached to ‘teaching for understanding of mathematics’. It will be obvious that a weakness here will impact on the decisions and judgments teachers make when teaching, which will consequently affect the understanding their students are able to develop. Recent reforms in mathematics education over the last two decades have emphasised teaching mathematics for understanding, rather than rote memorisation. It is important to examine what is meant by ‘teaching for understanding’.

There has been considerable recent discussion of the term ‘understanding’ (Barmby et al., 2007; Hiebert & Carpenter, 1992; Mayer, 1989; Mousley, 2005; Pritchard, 2010; Schroeder & Lester, 1989; Skemp, 1976, 1989; Stylianides & Stylianides, 2007; Warren & Cooper, 2009). Skemp (1976, 1989) suggests that there are two kinds of understanding, distinguishing ‘relational’ and ‘instrumental understanding’. Relational understanding is ‘knowing both what to do and why’, while instrumental understanding is ‘rules without reasons’ (Skemp, 1989, p. 2).

Cohen and Ball (1990a) suggest that ‘mathematics teaching in most elementary classrooms emphasises rules, procedures, memorisation, and right answers’ (p. 348); an approach consistent with Skemp's idea of instrumental
mathematics rather than relational mathematics. Others have argued that there are two different kinds of mathematical learning that result from mathematics teaching (Hiebert & Carpenter, 1992; Leinhardt, 1990; Post & Cramer, 1989): ‘conceptual learning’ and ‘procedural’, or ‘rote learning’. Post and Cramer (1989) define conceptual knowledge as ‘knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as important as the discrete pieces of information’ (p. 222). Recognition of these relationships is an essential component of conceptual knowledge.

These relationships can be established at two levels, both of which are important for the development of mathematical thinking. Firstly, understanding can develop from the ‘ideas embedded within the context in which they are presented, that is context-specific understanding’ (Post & Cramer, 1989, p. 222). On another level, it is important that relationships are understood in a ‘context free’ environment where the learner is required to make appropriate abstractions. Post and Cramer (1989) suggest that ‘this latter level is the sine qua non of the professional mathematician, but too often ignored or unrecognised at the school level’ (p. 222).

In contrast to conceptual knowledge, procedural knowledge implies only an awareness of superficial features and leaves the learner without an awareness or understanding of meaning or underlying structure (Post & Cramer, 1989). Procedural knowledge consists of step-by-step instructions that enable a learner to complete a mathematical task without necessarily understanding why that procedure ‘works’. In essence, it is mechanical or technical knowledge that is applied in a predetermined, linear sequence (Post & Cramer, 1989).
It will be accepted here that such distinctions as ‘instrumental understanding’, ‘relational understanding’, what others call ‘procedural knowledge’ and ‘conceptual knowledge’ are similar, in that both pairs of terms differentiate understanding on the basis that one has meaning and the other involves minimal meaning. The two sets of terms are closely related (Hiebert & Carpenter, 1992).

**Representation.**

An important related issue is the role of ‘representation’ in discussing ‘understanding’. Potentially, a key focus of teachers’ pedagogical content knowledge of mathematics is the widely accepted view that a mathematical idea is understood if its internal representation is part of a network of representations (Barmby et al., 2007; Bruner, 1966, 1973; Davis, 1984; Hiebert & Carpenter, 1992; Skemp, 1978, 1989). Further, the degree of understanding is determined by the number of representations making up the network. As Hiebert and Carpenter (1992) suggest, ‘A mathematical idea, procedure, or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections’ (p. 67).

Therefore, although understanding is complex, two points are generally accepted. Firstly, that multiple representations need to be part of the internal representations if understanding is to be relational. In other words, there needs to be a developing network of connected representations. Secondly, that internal representations are influenced by, and able to be developed because of, external representations to which one is exposed. Helping students become aware of the wide range of external representations for the many mathematical ideas they will encounter is an important aspect of teaching. Shulman (1986a, 1986b, 1987, 1989) argues this is
what teachers do, and by implication, teachers themselves must have a sound knowledge of such representational systems.

Part of a teacher's knowledge base is the knowledge of such multiple representations. This is consistent with Shulman’s theory of pedagogical content knowledge. Teachers need to have this knowledge of how to represent mathematical concepts in ways that assist students in constructing relational understanding.

**What is Pedagogical Content Knowledge?**

Against this background analysis of relevant subject content knowledge of mathematics, a more detailed examination of Shulman’s pedagogical content knowledge can now be made. Shulman (1986b) identified a significant factor that researchers and practitioners had previously overlooked, that of content, and more specifically, the pedagogical content knowledge of teachers. This includes:

- for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others…[Also,] an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (Shulman, 1986b, p. 9)

As can be observed from Shulman's description, pedagogical content knowledge is subject-specific and needs to be distinguished from general pedagogical knowledge that may range over various situations and types of knowledge. Its significance is that while it is important for teachers to understand the subject matter
they teach, they must also have a thorough knowledge of how to represent it in such a way that students will understand it. This lead Shulman to state: ‘To think properly about content knowledge requires going beyond knowledge of the facts or concepts of a domain. It requires understanding the structures of the subject matter’ (Shulman, 1986b, p. 9).

This connects with the earlier distinction made between relational and instrumental understanding. Some years earlier, Smith, Cohen and Pearl (1969) made the point that understanding structures of the subject matter was important for teachers if they were to provide appropriate explanations. They wrote, ‘a teacher cannot handle with skill a form of knowledge he [sic] cannot identify and whose structure he does not understand’ (p. 127).

Based on Skemp’s assertion that these two different types of understanding represent two different bodies of knowledge, then teachers have at least two sets of pedagogical content knowledge that are available to them. Firstly, there is the collection of explanations that provide students with rules and formulas memorised without any real meaning, which leads to instrumental understanding. Secondly, there is the collection of explanations and representations that develops relational understanding.

Bringing together these ideas, it is helpful to give a more precise form to Shulman’s theory as applied to mathematics teaching. Figure 2.2 provides a model designed by the researcher that conceptualises pedagogical content knowledge as intersections of the three components of a teacher's knowledge. The top circle assumes that a teacher of mathematics has a good understanding of the discipline itself. The bottom left circle represents the knowledge a teacher has of the learning
process or the understanding of learners and the conditions under which learning is most likely to take place. Finally, the bottom right circle represents the knowledge a teacher has about teaching. This not only includes knowledge of how to represent information so that others can understand, it also includes general pedagogical knowledge. General pedagogical knowledge is the knowledge a teacher uses for classroom management and organisation. This knowledge of teaching also includes that which teachers have of the school as an organisation and as part of a system, and the knowledge of their role within the school and the system.

The conceptualisation of Figure 2.2 also identifies several key intersections between types of knowledge that can take place and which are worthy of discussion.

*Figure 2.2.* Researcher’s model of the three essential components of a teacher's knowledge base when teaching mathematics.
Intersection A identifies the teacher’s knowledge of the informal learning of mathematics by students without formal teaching. Examples of this are mathematical knowledge that children learn from educational television programmes, basic counting skills often learnt at home, awareness of money and its value (also learnt at home), familiarity with time and clocks and an understanding of some of the applications of percentages, such as credit card and mortgage interests. It is true that not all students will come to school with the same experiences and some of these mathematical ideas may still form part of formal teaching. Nevertheless, it is essential that teachers have as part of their knowledge base an awareness of the kinds of mathematical experiences children are exposed to outside of the school setting. Teachers need to have knowledge of children’s prior knowledge.

Intersection B represents teachers’ understanding of mathematics teaching in the abstract—knowledge possessed by teachers about teaching mathematics that does not take the learner into account. Examples of this kind of knowledge include such things as teaching mathematics as a series of shortcuts, teaching mathematics where a final examination is the major focus rather than student understanding, teaching mathematics in a manner where, for convenience, the whole class progresses at the same rate, working through a set text page-by-page and teaching mathematics from only one representational mode.

Intersection C represents the knowledge a teacher possesses about teaching and learning that is not specifically related to the teaching of mathematics. This could be described as general pedagogical knowledge and includes important aspects such as grouping and cooperative learning, the importance of questioning, matching assessment to instruction and catering for individual differences.
The strength of the model is that it predicts inadequate understandings of different kinds, as shown in the intersections A, B and C. The most critical intersection for the teaching of any given topic lies at the intersection of all three circles and it is not by accident that this intersection can be considered central to all teaching, or the ‘hub of learning’. Thus, pedagogical content knowledge can be understood in a dynamic way, as an interaction of component knowledge, an amalgam of knowledge that results when teachers combine what they know about mathematics, teaching and learning. This knowledge is unique to the specific content being taught. It assumes an awareness of students’ common misconceptions (Even & Tirosh, 2002), an understanding of the structure of the content being taught and its relationship to other mathematical ideas, and awareness of a variety of models, analogies or representations that can be used to help the student make sense of the content.

Clearly, this concept of teachers’ subject knowledge goes beyond general pedagogical issues such as classroom organisation, cooperative learning and asking questions. It suggests the teacher knows exactly what questions to ask and why they should be asked. It implies the teacher is familiar with aspects of the particular topic that are likely to cause problems for students and they have the ability to deal with these situations. Therefore, Figure 2.2 provides a model that represents Shulman's notion of pedagogical content knowledge and demonstrates how it differs from other forms of teacher knowledge also represented within the model. When teachers take into account the learners, carefully consider the subject and give thought as to how to best represent it, relational understanding is the most likely outcome for students.
The model also suggests conditions when instrumental understanding is likely to result from the kind of instruction described in the intersection labelled ‘B’. As can be observed from the examples provided above, when teachers ignore the knowledge of learning/learners, the kind of teaching that results is likely to more heavily involve the memorisation of procedures and shortcuts to keep the class moving at the same rate, leading to restriction of relational understanding.

The model is also useful in clarifying the importance of ‘pedagogical reasoning’ (Shulman, 1987) in effective teaching. When a teacher takes an aspect of content knowledge and prepares it for instruction, taking into account the characteristics of the learners, there is much to consider. Clearly, there are general pedagogical principles to reflect on, such as whether students will work in groups, taking into account students’ prior experiences and accommodating special needs within the class. There are also many considerations that need to be given that are particular to the specific content to be taught at that point in time. Shulman (1987) suggests that this consideration needs to be logical and well-reasoned, referring to the process as pedagogical reasoning. Shulman advances a model of pedagogical reasoning and action as illustrated by the researcher in Figure 2.3, which involves ‘a cycle through the activities of comprehension, transformation, instruction, evaluation, and reflection’ (p. 14).
Shulman argues that each of these activities is critical if a teacher is to take what he or she already understands and prepares that information or skill for effective instruction. Teachers must initially understand what it is they plan to accomplish. There must be some substance to a lesson, some subject matter that the learner will be introduced to or have consolidated. Ideally, teachers will understand the subject matter to the extent where they know how the information or skill relates to other ideas, both within and outside of mathematics.

Representation is fundamental to teaching mathematics. In terms of the model presented in Figure 2.2, teachers can represent mathematical concepts in a variety of ways. Each of the intersections A, B and C involve representations of a limited usefulness on their own, as it is when all three components of teacher knowledge interact that meaningful representations will result. Thus, representation is a key focus for exploring the model and its importance to pedagogical content knowledge needs to be discussed in some detail.
Representation is an important transformation that requires skills of analysis of structure and interpretation. Teachers need to take into account pedagogical issues such as teaching methods, use of materials and characteristics of the learners, if they are to successfully move from personal comprehension to prepare for the comprehension of others. Shulman states:

These forms of transformation, these aspects of the process where one moves from personal comprehension to preparing for the comprehension of others, are the essence of the act of pedagogical reasoning, of teaching as thinking, and of planning—whether explicitly or implicitly—the performance of teaching. (Shulman, 1987, p. 16)

It is only if these ideas are understood that a teacher is able to transform them in some way. Instruction then includes implementing selected strategies and representations. Part of instruction also concerns more general pedagogical issues such as classroom management, assigning and checking students' work, questioning and discussion, among others.

Following instruction, teachers need to engage in the process of evaluation. Not only is there the need to evaluate students’ learning, but the teacher should also evaluate other aspects of the teaching process, such as appropriate use of materials, adequate representations and variety of representations, that is, evaluation directed at one’s own teaching. This evaluative process leads directly to reflection, ‘that set of processes through which a professional learns from experience’ (Shulman, 1987, p. 19). Shulman completes this discussion of the model of pedagogical reasoning by stressing that it is not intended to be a sequence that represents fixed stages. Some of these stages may not always occur, but every teacher at some time should
demonstrate the capacity to engage in the processes described. Shulman (1987) concludes:

Thus we arrive at the new beginning, the expectation that through acts of teaching that are ‘reasoned’ and ‘reasonable’ the teacher achieves new comprehension, both of the purposes and of the subjects to be taught, and also of the students and of the processes of pedagogy themselves. (p. 19)

**Representation and Pedagogical Content Knowledge**

Having outlined Shulman’s theory of pedagogical content knowledge and introduced the researcher’s model that operationalises this theory, it is now necessary to examine the question of representation in detail, since it is argued as a key area of pedagogical content knowledge in teaching mathematics. Shulman (1989) stresses the value and importance of examining the ways in which particular topics are represented by teachers. The knowledge gained from such activity will help to provide a more elaborated account of a ‘pedagogy of substance’. Shulman particularly emphasises the need for documentation of teachers’ use of alternative representations. Alternative representations ‘enrich the discourse, layering the accounts’, they ‘illuminate via multiple sources what could not be understood through the light shown from a single source alone’ (1989, p. 3).

Shulman (1989) further suggests neither teachers nor students make direct sense of the world. Instead, they construct representations of various forms including visual images, metaphors, stories, analogies and key cases that are the essence of explanation. Shulman (1989) argues that representations need to be examined in content-specific ways. Information collected from ‘empirical studies of the uses of representation in the pedagogy of school subjects at different levels of the curriculum
and contexts of schooling’ (p. 8) needs to be carefully analysed. Particularly, from the perspective of the representational potential of the domain itself and ‘the ways in which wise teaching practitioners stretch the representational repertoire for pedagogy while adapting instruction to variations in students and contexts’ (p. 8). These studies should include an examination of the limitations of representations as well as their appropriateness. Teachers’ representational repertoires are worthy of investigation. In terms of Skemp’s relational understanding, it is the representations a teacher uses that influences the kind of understanding that their students develop.

Shulman’s (1989) emphasis on the importance of representation is appropriate, given the widespread scholarly attention to the topic. In the act of teaching, especially teaching for understanding, representation is considered to be of importance, particularly in the role of student development of internal representations (Arcavi, 2003; Ball et al., 2001; Hiebert & Carpenter, 1992; Indiogine, 2010).

Hiebert and Carpenter (1992) suggest that understanding can be considered:

based on the assumption that knowledge is represented internally, and that these internal representations are structured. A useful way of describing understanding is in terms of the way an individual’s internal representations are structured. (p. 66)

They suggest that internal (mental) representations are necessary to think about mathematical ideas, arguing that these internal representations are stimulated and connected into a network by external activity. Thus, external representations of mathematical ideas assist in developing internal representations. Due to the fact that internal representations are not observable, Hiebert and Carpenter (1992) are careful not to make any claims about their specific nature. For example, they state:
We do not presume, for example, that if second graders work with bundled sticks when dealing with two digit numbers, they represent internally all quantities more than nine as mental images of sticks. We do presume, though, that students who interact with bundled sticks represent those quantities differently for themselves than students who work only with written symbols. (p. 66)

The suggestion that mathematical understanding involves the development of internal representations and that these internal representations are influenced by external representations, though not new, is highly significant in terms of the researcher’s model for pedagogical content knowledge.

Bruner (1966) considerably influenced the development of the notion that representation is an important means of developing mathematical understanding. Bruner (1973) suggested that ‘there are three kinds of representational systems that are operative during the growth of human intellect and whose interaction is central to growth’ (p. 316). These are enactive representation, iconic representation and symbolic representation, ‘knowing something through doing it, through a picture or image of it, and through some such symbolic means as language’ (Bruner, 1973, p. 316).

Bruner introduced another important aspect of representations, that the three different systems, enactive, iconic and symbolic, are all parallel and unique. That is, each system may be used to represent the same mathematical idea, but each may emphasise different features of the idea:

A representation of an event is selective. In constructing a model of something, we do not include everything about it. The principle of selectivity
is usually determined by the needs to which a representation is put—what we are going to do with what has been retained in this ordered way. (Bruner, 1973, p. 316)

The representation of place value provides a good illustration of the significance of this approach to understanding pedagogical content knowledge in mathematics teaching. When introducing place value to young children, teachers may use materials such as paddle pop sticks and place value charts to represent the number 16. This representation is largely ‘enactive’ as it requires the child to bundle ten of the paddle pop sticks together and then to position the bundle appropriately in the ‘tens’ column, and the remaining six in the ‘ones’ column.

Another representation could be made using an abacus. Here, the ‘principle of selectivity’ can be seen at work. No longer is the grouping into ‘tens’ a deliberate action that is represented. Rather, it is assumed the child, when using an abacus, understands that a bead in the second column has the value of ten by virtue of its position. Clearly, there are numerous ways that the number 16 can be represented to demonstrate correct place value understanding. Perhaps the most abstract is simply ‘16’—the symbolic representation.

While it is important to recognise that each different representation of a mathematical idea is unique, it is equally important to note that ‘all are also capable of partial translation, one into the other’ (Bruner, 1966, p. 11):

And here lies one very important ‘impulsion’ to cognitive growth. For as we shall see, there is serious disequilibrium when two systems of representation do not correspond—what one sees with how one says it, or how one must act overtly and how the world appears. Indeed, we shall see that it is usually
when systems of representation come into conflict or contradiction that the child makes sharp revisions in his [sic] way of solving problems. (Bruner, 1966, p. 11)

The need for teachers to grasp the importance of representation if they are to be effective teachers has major pedagogical implications. Post and Cramer (1989) argue that representation is a crucial component in the development of mathematical understanding. They suggest that without it, ‘mathematics would be totally abstract, largely philosophical, and probably inaccessible to the majority of the populace’ (p. 223). They continue their position by stating that children’s understanding of mathematics becomes meaningful only as they translate between different representations of mathematical ideas. This is consistent with Bruner's idea of translation mentioned previously.

Research examining the importance of representations has contributed considerable insight into what is meant by ‘relational understanding’. The suggestion by Lesh (1979) that it is the transformations within and the translations between modes of representation that make ideas meaningful for children, helps clarify the distinction made earlier between instrumental understanding and relational understanding. The more transformations and translations an individual can make within and between modes of representation, the richer their relational understanding.

The scholarly literature highlights how internal representations are linked to external representations (Bruner, 1973; Hiebert & Carpenter, 1992) implying that selection of appropriate external representations may be a critical feature of effective teaching. Teachers need a repertoire of external representations if this selection is to occur. Leinhardt (1989) stresses the importance of familiarity with the representation
being used. Once a representation has been selected, the teacher needs to be aware of the strengths and limitations of that representation for the maximum benefit to occur.

In the researcher’s model proposed within this study, the concept of representation will be central to analysing how teachers attempt to teach for understanding—a key area in which Shulman’s pedagogical content knowledge can be operationalised in terms of the teaching of mathematics. Understanding has been discussed briefly only because it is believed all teachers attempt to teach for understanding.

In terms of this study, relational understanding is demonstrated by the transformations within translations between the modes of representation, as suggested by Bruner (1973), Lesh et al. (1987) and Post and Cramer (1989). The translations are the basis of ‘connected networks’ (Hiebert & Carpenter, 1992). Therefore, relational understanding is rich in relationships and must be part of such a network.

Instrumental understanding is best described as being locked into one mode of representation and not having the ability to represent beyond that mode. Instrumental understanding requires a minimum number of connections to be made, mere ‘connections between succeeding actions in the procedure’ (Hiebert & Carpenter, 1992, p. 78). While it need not necessarily be the case, the instrumental mode is more commonly that of symbols: ‘Often, in school mathematics, procedures prescribe the manipulation of written symbols in a step-by-step sequence’ (Hiebert & Carpenter, 1992, p. 78).

**Teachers' Beliefs and School Context**

The model proposed in Figure 2.2 can be strengthened by recognising that teacher knowledge is situated within a broader matrix. All teachers work within
schools that have a localised school culture. Teachers, influenced by their experiences, develop their own personal philosophies and their own pedagogical beliefs—what teachers do is influenced by their beliefs (Barkatsas & Malone, 2005; Beswick, 2003; Peterson, Fennema, Carpenter & Loef, 1989; Stipek, Givvin, Salmon & MacGyvers, 2001). Therefore, the knowledge required by a teacher to specifically teach mathematics cannot be examined in isolation. Teacher beliefs include what the teacher believes mathematics is about (Wilkins, 2008; Wu, 2007), how they believe it should be taught (Leder, Pehkonen & Törner, 2002; Maass, 2009; OECD, 2009), and the role mathematics serves as a discipline (Kilpatrick, Swafford & Findell, 2001).

Research has begun to investigate how and why teachers represent mathematical ideas the way they do (Fennell & Rowan, 2001; Goldin & Shteingold, 2001; Harries & Barmby, 2007). Thompson (1984) suggested that research in the area of teacher knowledge and beliefs is vital, arguing:

> There is strong reason to believe that in mathematics, teachers' conceptions (their beliefs, views, and preferences) about the subject matter and its teaching play an important role in affecting their effectiveness as the primary mediators between the subject and the learners. (p. 105)

Thompson suggests that understanding how teachers’ knowledge of mathematics influences instructional practice and how their conception of mathematics affects teaching have largely been ignored. Ball (1990a) reported that one of the teachers studied stressed how important student participation was, yet all classroom discourse was very structured: ‘Mathematical speculation, conjecture, and invention are not a part of the discourse’ (p. 268).
Although the relationships between teachers’ beliefs and their instructional practices are complex, such beliefs play a significant role in shaping their instructional behaviour:

In particular, the observed consistency between the teachers' professed conceptions of mathematics and the manner in which they typically presented the content strongly suggests that the teachers' views, beliefs, and preferences about mathematics do influence their instructional practice. (Thompson, 1984, p. 125)

However, Thompson emphasises that to understand these relationships more fully, more research is needed, particularly involving detailed case studies, ‘in order to gain access to the thoughts and mental processes that accompany the teachers' actions’ (p. 126).

Other research suggests it is necessary to focus on teachers’ specific beliefs about mathematics in exploring pedagogical content knowledge. Peterson et al. (1989) examined relationships among teacher's pedagogical content beliefs, their pedagogical content knowledge and students' achievement in mathematics. While other researchers have examined teachers' beliefs about mathematics curricula in the past, Peterson et al. (1989) distinguish their research from this previous work by analysing teachers' beliefs within a specific topic area and grade level. Rather than examining general beliefs about mathematics, Peterson et al.(1989) commenced by developing a conceptual framework for analysing teachers' pedagogical content beliefs in addition and subtraction.

From their study of the literature, they derived four constructs about children’s learning:
1. Children construct their own mathematical knowledge.

2. Mathematics instruction should be organised to facilitate children's construction of knowledge.

3. Children's development of mathematical ideas should provide the basis for sequencing topics for instruction.

4. Mathematical skills should be taught in relation to understanding and problem solving. (p. 4)

In contrast to this precision in examining teacher knowledge, the language of policy statements directed towards teachers is vague and ambiguous, bringing into question the effectiveness of new reforms in mathematics education (Cohen & Ball, 1990b):

For example, few people would disagree with terms such as understanding and problem solving. That students need experiences with concrete materials or that they should be able to apply mathematical ideas will not generate debate … Yet what people mean by these terms can—and does—differ widely … As one traditional teacher commented, 'What do they think we've been doing—teaching for misunderstanding?' (p. 249)

Peterson et al. (1989) make other important inferences. In studying teachers' pedagogical content knowledge only on addition and subtraction, the teachers involved provided no information regarding how they teach other mathematical topics. Although teaching for relational understanding in one area of mathematics, given a different topic (perhaps one with which the teacher does not feel as comfortable), a more instrumental approach may well be taken.
The authors agree that their study ‘represents only a beginning attempt to examine teachers' pedagogical content beliefs in mathematics’ (p. 36). They state:

In future, researchers need to conceptualise further teachers' pedagogical content knowledge and beliefs and to gather empirical data to explore the interconnections between teachers' pedagogical content beliefs and pedagogical content knowledge. (p. 38)

Darling-Hammond (1990) underlines the important role that teacher knowledge and beliefs play in implementing new policies, stating: ‘All the cases make clear that as teachers interpret the thin guidance they've received, they fill the gaps in their understanding of the policy with what is already familiar to them’ (p. 236), thus creating a 'melange' of practices.

**Teacher Self-Efficacy**

The discussion in this chapter has focused upon the kinds of knowledge that teachers require to be effective teachers of mathematics, with particular emphasis on teaching measurement and the importance of teachers’ beliefs. There is also emerging research that examines the relationship between teacher performance and teacher self-efficacy. Evans (2010) claims that both teacher knowledge and teacher self-efficacy are important to teaching and learning, suggesting: ‘Teachers with higher levels of content knowledge and self-efficacy are better able to produce high student achievement than are teachers with lower levels’ (Evans, 2010, p. 4).

Self-efficacy has been defined as ‘a social-psychological construct’ (Zehir Topkaya & Yavuz, 2011, p. 35), and refers to ‘people’s judgments of their capabilities to organise and execute courses of action required to attain desired types of performances’ (Bandura, 1986, p. 391). These judgments are affected by a
person’s previous successes and failures, their beliefs about others’ views of them, and the successes and failures of others. Each of these factors greatly influences human action as they have the power to determine people’s choices, goals, effort and persistence (Bandura, 1989, 1995; Ormrod, 2006). Bandura (1986) suggests that teacher self-efficacy incorporates both a teacher’s belief in his or her ability to teach effectively, and his or her ability to affect student learning outcomes.

Research dealing with teachers’ efficacy has examined their behaviour and performance in the classroom (Tschannen-Moran, Woolfolk Hoy & Hoy, 1998) with high levels of efficacy resulting in higher levels of performance. Coladarci (1992) reports high efficacy results in a greater commitment to teaching, while others have reported more positive attitudes to using innovative teaching strategies (Eslami & Fatahi, 2008; Swars, 2005).

**Cultural Context of the School**

Studies dealing with the culture of schools and classrooms have suggested that how students relate to mathematics and perform mathematically strongly correlates to the view of mathematics projected by the teacher (Nickson, 1992). Teachers who have a strong relational understanding of mathematics tend to plan lessons that engage their students in conceptual aspects of mathematics (Fennema & Franke, 1992). This view is supported by Henningsen and Stein (1997), who claim that student engagement is high when teachers select appropriate tasks and push their students to make meaningful mathematical connections. Darling-Hammond (1990) concludes that teachers' creativity and ability to take initiative is related to their knowledge base: ‘teachers who lack a “mathematical and pedagogical infrastructure” must go by the book—but only the parts of the book they can understand’ (p. 239).
Skott (2001) demonstrated that teachers’ beliefs about non-mathematical aspects were responsible for student performance in mathematics. In particular, teachers’ concerns about student self-esteem and the social context of the mathematics classroom can be linked to higher student motivation and higher levels of performance.

Several teacher actions that lead to a classroom culture that supports student learning and understanding have been identified. Among these were the setting of clear expectations of the students (Marzano, 2003; Yackel & Cobb, 1993), insisting that students solve personally challenging problems and that students are provided the opportunity to explain their personal solutions to their peers.

**Conclusion**

As a result of the recent suggestions for reform in mathematics education, there has emerged a renewed push to change the way in which mathematics is taught. Emphasis has been placed on the role of the teacher and in developing a framework for examining teachers' pedagogical content knowledge. If a change to genuine relational understanding is to be achieved, then a greater investment in teacher knowledge is needed. Investment is necessary both in terms of research and in the professional development of teachers to assist them to change from their traditional mathematics knowledge base to a more progressive knowledge base.

Sykes (1990) agrees that to teach mathematics effectively requires ‘a major reorientation for many teachers, a journey into unfamiliar territory beyond the bounds of their present knowledge and competence’ (p. 244). Ashton (1990) suggests that considerably more work needs to be carried out if the teaching/learning process is to
be more fully understood and to make significant improvements in student understanding of subject matter, stating:

To be effective, this work requires the collaboration of specialists in subject matter and pedagogy, developmental and cognitive psychologists, and classroom teachers. Without systematic programs of research and development of pedagogical content knowledge, significant improvements in student understanding of subject matter are unlikely. (p. 2)

This chapter has presented a discussion of the literature that emphasises the need for more teachers to utilise strategies that will result in their students developing relational understanding rather than instrumental understanding. Shulman's (1987) notion of pedagogical content knowledge has been discussed and the importance of the need for research in this area has been stressed. Issues central to pedagogical content knowledge, including the researchers’ model of pedagogical content knowledge, representation as a framework for understanding the model and the implications for teachers, pedagogical reasoning and teachers' pedagogical content beliefs, have been presented. Finally, areas of teacher efficacy and the culture of the classroom were briefly presented, as the present research connects these areas to teacher knowledge.

It is from within this wider scholarly framework that this thesis emerges. As Shulman (1986) and Ashton (1990) both suggest, a great deal of further work is needed for the teaching and learning process to be more fully understood. The following chapter describes the research method used for this study and, in particular, provides a detailed profile of each of the participating teachers involved in the study.
Chapter Three: Methodological Issues

Introduction

There is potential to learn a great deal from teachers if the time is taken to study what they do, how they do it and why they do it in the manner that they choose (Shulman, 1987). Ashton (1990), referring to Shulman’s notion of pedagogical content knowledge, suggests, ‘Without systematic programs of research and development of pedagogical content knowledge, significant improvements in student understanding of subject matter are unlikely’ (p. 2).

Chapter Two explored the application of the concept of pedagogical content knowledge to the teaching of mathematics. This chapter will argue the methodology for its investigation through the study of four teachers. It will be argued that research needs to provide detailed accounts of pedagogical content knowledge across a range of content-specific domains of a particular subject. As outlined in Chapter Two, this study has focused on the specific domain of the teaching of measurement.

This chapter will firstly address this study’s approach and argue the merits of qualitative research as an appropriate methodology. It will address the specific research design utilised and describe the techniques used for gathering and analysing data. It will discuss a range of methodological issues to establish the appropriateness and credibility of the procedures and the strategic use of vignettes in data analysis will be outlined. Finally, a profile of each of the participants will be provided.

The Research Issues

The task of investigating pedagogical content knowledge as identified by Shulman is an ambitious program. While considerable research has been conducted since Shulman first introduced the concept, there is still much to be done to develop
the framework further, adding to the understanding of this important area of teacher knowledge (Ball et al., 2008). The discussion of Shulman’s theory in Chapter Two identified two key parameters for the study of pedagogical content knowledge. Firstly, little research has examined pedagogical content knowledge with an emphasis on teaching measurement. Hence, it was determined that the focus of this study would be on the teaching of measurement in the primary years. Middle primary, Years Three and Four, were deemed appropriate, as by this stage children have progressed from arbitrary units of measurement to learning formal measurement.

Secondly, it was determined that the focus of inquiry should be to gain an understanding of how teachers plan, teach and assess measurement lessons, through accessing classrooms and both observing and interviewing teachers about their teaching. In doing so, teachers should be asked questions about their beliefs and self-efficacy, areas that current research lays claim to have influence on teaching practice and on how teachers use knowledge.

These parameters serve to frame the present research and delineate the research issues. Thus, the general research question is: How do teachers demonstrate the depth of their pedagogical content knowledge when teaching measurement lessons to Years Three and Four students? This general question can be expressed in terms of four specific inquiries:

5. How evident is the teacher’s depth of mathematical knowledge of measurement in their teaching?

6. How do teachers show that they understand and address the needs of students when teaching?
7. How to teachers demonstrate their general pedagogical knowledge when teaching?

8. How is a teacher’s knowledge and practice impacted by other factors when teaching and what are these major factors?

**Qualitative research methodology.**

A key methodological assumption is the need for a close study of teacher understanding. The framing of the inquiry into pedagogical content knowledge adopts a qualitative approach. Qualitative research is established in education as a legitimate mode of research where the purposes of research in educational contexts are interpretive in nature (Glesne & Peshkin, 1992; Le Compte & Preissle, 1993; Merriam, 1998; Merriam & Simpson, 2000; Yin, 2009). A key decision researchers need to make when planning their research study is to determine whether the research warrants a quantitative approach, a qualitative approach or, in fact, whether a mixed method would be best suited (Creswell, 2003; Gay, Mills & Airasian, 2012).

A quantitative methodology rests on positivist assumptions about knowledge of educational realities (Creswell, 2003; Grbich, 2007) and assumes that there is an objective reality (Denzin & Lincoln, 2011; Sale, Lohfeld & Brazil, 2002), one that can be measured and investigated without the researcher influencing the phenomenon under investigation and without being influenced by it (Denzin & Lincoln, 2011). It is said that ‘epistemologically, the investigator and investigated are independent entities’ (Sale et al., 2002, p 44).

Conversely, qualitative research does not primarily seek to measure attributes or variables but emphasises the socially constructed nature of reality and the investigation of social phenomena (Wiersma, 1995). ‘Qualitative researchers study
things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them’, argue Denzin & Lincoln (2011, p 3).

Much fruitless debate has focused on the justification of competing ‘paradigms’ (Lincoln & Guba, 1985), when the real questions concern the purposes of research and which research techniques are appropriate for undertaking specific research. Glesne and Peshkin (1992) state:

The research methods we choose say something about our views on what qualifies as valuable knowledge and our perspective on the nature of reality … qualitative methods are generally supported by the interpretivist paradigm, which portrays a world in which reality is socially constructed, complex, and ever changing. (pp. 5–6)

An approach is needed that allows for the exploration of teachers working in context. Proponents of qualitative research agree that while the focus of such research is generally to investigate real-world settings, there are many approaches from which to choose when engaging in qualitative research. The emphasis placed on natural settings is stressed by many researchers as a significant feature of qualitative research (Denzin & Lincoln, 2011; Holliday, 2007; Robson, 2002; Yin, 2011).

The approach adopted emphasises the need to be interpretive and seeks to understand how individuals experience their world, the interactions they have with others and the meaning they attach to these experiences (Merriam, 2002). All qualitative researchers engage in interpretive studies with the intention of deriving meaning from investigating phenomena within real-world settings (Ospina, 2004).

Yin (2011) suggests that five key features can help to frame a qualitative research approach. The first is that qualitative research involves ‘studying the
meaning of people’s lives, under real-world conditions’ (Yin, 2011, p. 7). This study was framed to examine teachers’ pedagogical content knowledge while teaching in their own classrooms. The teacher’s classroom is their ‘real world’ and provided a meaningful context for this study to investigate their knowledge base in a thorough way.

The second feature is to represent ‘the views and perspectives of the people in a study’ (Yin, 2011, p. 7). The research was designed to give teachers many opportunities through interviews and questionnaires to express views about their teaching and various aspects of it, ranging from planning, implementing, assessing and evaluating their performance.

The third feature deals with ‘covering the contextual conditions within which people live’ (Yin, 2011, p. 8). Teachers work in schools that constitute environments with their own culture and which provide sets of relationships with other teachers, parents, systemic personnel and the wider community that the school serves. An investigation of the contextual factors that affected the teachers involved in this study was considered important.

Yin’s fourth feature is that qualitative research should contribute ‘insights into existing or emerging concepts that may help to explain human social behaviour’ (Yin, 2011, p. 8). Essentially, this study aims to examine teachers’ pedagogical content knowledge to build a better understanding of how teachers teach measurement, to explain the complex phenomenon of pedagogical content knowledge of teachers teaching middle primary measurement.

Finally, Yin suggests that qualitative research should ‘use multiple sources of evidence rather than relying on a single source alone’ (Yin, 2011, p. 8). The design for
this study outlined below will demonstrate that multiple sources of evidence were an important consideration.

**The Research Design: The Case Study Approach**

The present research involved investigating teachers’ pedagogical content knowledge as an attempt to make sense of the teaching experience of four different teachers teaching measurement concepts to Years Three and Four students. This required observing each teacher in their classroom to analyse and interpret their pedagogical understandings and classroom practices. The use of qualitative methods to observe each teacher aimed to generate a study providing rich and informative accounts of the phenomenon being investigated. The richness of this approach provides findings that build an understanding of teaching by ‘collecting, examining, and beginning to codify the emerging wisdom of practice among both inexperienced and experienced teachers’ (Shulman, 1987, p. 11). Shulman argues that using the case method is most appropriate: ‘Case knowledge is knowledge of specific, well documented, and richly described events’ (Shulman, 1986, p. 11). The approach aimed to carry out this investigation in a close-grained and thorough way.

Case studies have become a common way of investigating various phenomena in qualitative research. ‘Case study is not a methodological choice but a choice of what is to be studied’, as Stake argues (2005, p. 443). Merriam (1998, 2002) identifies important factors that need to be considered when determining an appropriate research design. The first is the nature of the research questions being asked, arguing that questions appropriate for case studies concern ‘how’ and ‘why’. This present study places particular focus on ‘how’ questions, since in investigating teachers’ pedagogical content knowledge, the central issue is ‘how teachers represent
mathematical content knowledge for their student learners’. As Shulman (1987) argues, the only effective way to answer these questions is by close examination of what teachers do, by observing in detail what teachers are doing when engaged in pedagogical activity and in analysing and interpreting their actions.

Merriam (1998) suggests that research design is also partly determined by the amount of control the researcher has. When investigating teachers’ pedagogical content knowledge in this study, it was essential to observe teachers operating in their regular classrooms, that is, teachers operating in their natural environment. In this sense, a researcher has little, if any, control over the teacher. Within this study, it is not entirely true that there was no control, as negotiation took place with the teachers in regards to the kinds of lessons they would teach. All of the teachers observed in this study were asked if they would teach measurement lessons. However, this was the only form of control that was sought. Each of the teachers then determined what measurement concepts they would teach and how they would structure their classes. Merriam (1998) argues that where there is limited control, the case study is an appropriate research design.

The third factor of ‘the desired end product’ relates closely to the research questions being asked. In investigating pedagogical content knowledge with the focus being on ‘how’ questions, this study is attempting to interpret and describe a contemporary phenomenon. Shulman (1987) provided some explanation of this phenomenon but also acknowledged the need for greater investigation of pedagogical content knowledge. Hence, this study particularly pays attention to specific mathematical topics, such as the teaching of measurement. The desired end product in this study is description, the interpretation and eliciting of meaning from the data, the
portrayal of a complex phenomenon and an attempt to make sense of this phenomenon. This type of result can be obtained most effectively from studying cases and interpreting what teachers do and how they do it, within context. Again, this needs to be performed in a thorough and thoughtful way.

Finally, Merriam (2002) suggested that a case study approach is deemed appropriate if a ‘bounded system’ exists, stating:

The case study is an intensive description and analysis of a phenomenon or social unit such as an individual, group, institution, or community. The case is a bounded, integrated system. By concentrating upon a single phenomenon or entity (the case), this approach seeks to describe the phenomenon in depth. (2002, p. 8)

The phenomenon of teachers’ pedagogical content knowledge is consistent with this notion of a bounded system. According to Creswell (2002), “‘Bounded” means that the case is separated out for research in terms of time, place, or some physical boundaries’ (p. 485), thus placing limits around the focus of the study. Selecting four teachers as cases for investigating pedagogical content knowledge while teaching middle primary measurement provided a ‘bounded system’ and required research procedures appropriate to do justice to the complexity of the phenomenon.

The above four factors identified by Merriam (1998, 2002) all suggest that any investigation of teachers’ pedagogical content knowledge requires the close-grained analysis that a case study offers. This is consistent with the approach taken by Shulman (1987) and others who have previously investigated pedagogical content knowledge (Ball et al., 2001).
**Cross-case comparison.**

This study sought to investigate individual teacher’s pedagogical content knowledge to compare their understandings, as this would reveal something of the different structures found in teachers’ knowledge. Yin (2009) examines the use of multiple case studies to draw a set of ‘cross-case’ conclusions’ (p. 20).

Due to the fact that measurement is a major strand of the curriculum and that no two teachers would necessarily teach the same content in the same way, four cases were considered appropriate for this study, rather than relying upon a single case. This is largely due to the complexity of the phenomenon of pedagogical content knowledge. Complexity calls for cross-case comparison as the research investigation is driven by a conceptual model to be elucidated below.

Soy (1997) advocates the use of multiple cases as a means for examining ‘a complex phenomenon’ in depth, stating: ‘The researcher must determine whether to study cases which are unique in some way or cases which are considered typical’ (p. 2). Therefore, the study included both novice and experienced teachers to explore a range of ‘configurations’ of pedagogical content knowledge. Two beginning teachers and two experienced teachers offered a balance in the selection, though this was not intended to establish a formal comparison between the two types of teachers. As Shulman suggests, all kinds of teachers need to be investigated if a rich account of teachers’ pedagogical content knowledge is to be collected. The two beginning teachers provided literal replications that ‘predict similar results’ (Yin, 2009, p. 54) and the two experienced teachers provided theoretical replications that ‘predict contrasting results but for expected reasons’ (Yin, p. 54). It would be expected that
beginning teachers and experienced teachers would draw on pedagogical content knowledge bases with differing depths and richness.

**Research Procedures**

**The pilot study.**

A pilot study was first conducted for a full ten weeks with one volunteer teacher. The purpose of the pilot study was to trial aspects of the research and ascertain the scope and characteristics of data that could be collected over a series of lessons. The teacher involved in the pilot study taught in a different school to the teachers who were selected for this study. The teacher was interviewed before the teaching commenced and before and after each lesson. These interviews and the entire lessons were digitally recorded and the recordings were transcribed and presented to the teacher to determine their accuracy.

The pilot was used to monitor how students responded to the recording of the teaching and whether recording a teacher’s lessons would affect both the students and the teacher. It needed to be determined whether providing the teacher with a wireless microphone and allowing her to have the control over the microphone as far as being able to turn it off if she wished had any impact on the data collected.

The pilot study allowed the trialling of interview questions, allowed for modifications to these questions prior to the actual study, and enabled the researcher to gauge how many lessons were required to gather rich and useful data. This established that the possible invasive technique of recording the lessons was not problematic, as both students and teacher adjusted to the presence of the researcher very quickly and with no noticeable effect on the teaching and learning environment.
The interview questions were found to provide appropriate stimulus for meaningful responses and conversations with the teacher to elicit information about her planning, teaching practices and her beliefs and the potential of the pilot study to demonstrate rich and insightful experiences was evident. Further, the teacher valued the experience, as she claimed it made her think about her planning and to reflect on her teaching.

A major outcome of this test of the research design was the addition of the reflective questionnaire, since it was apparent that there was no real opportunity during the pilot study for the teacher to provide a final summation and reaction to her participation in the research process.

**Data collection.**

A variety of data collection methods was used. Figure 3.1 illustrates the sequence of data collection and analysis procedures. The data was collected by means of an initial interview with each teacher, observation and recording of the lessons taught by each teacher, interviews associated with each lesson and a reflective questionnaire completed by each teacher, two weeks after the final teaching episode was recorded.
Stage 1 in Figure 3.1 includes the conducting of interviews, observation and recording of the lessons and the completion of the reflective questionnaire. Stage 2 illustrates the process of sorting through the data and writing significant teaching or interview moments as vignettes to identify key themes. The process for writing the vignettes will be described in the data analysis section that follows.

**Observation and recording of lessons.**

Each of the four teachers described in this chapter agreed to be recorded while teaching a series of measurement lessons. The original intention was to record one lesson per week with each teacher for the duration of a ten-week teaching term, although this was reduced to a series of lessons for seven of the ten weeks. This
reduction was due to two reasons. The normal interruptions faced by teachers within school programs, such as camps and special assemblies for important visitors, resulted in some of the sessions being unable to proceed in each case. Secondly, the pilot study had revealed the amount of data gathered over a full ten weeks was quite considerable and that sufficient data could be collected from fewer lessons to provide the basis of a rich and meaningful investigation.

There were two major reasons for observing one lesson per week. Firstly, the practicalities of observing four teachers during the same ten-week period resulted in one visit to each teacher per week. Secondly, measurement is only one of five mathematics strands required to be taught as part of the mathematics curricula in Australian Capital Territory (ACT) schools. Not all lessons in a week focused on measurement concepts. Since measurement was the focus of this study, the teachers concerned were prepared to ensure that at least one measurement lesson was planned for each week and to schedule it for a convenient time to be observed.

While being observed, all the teachers were provided some control over the observation process. The teachers wore a wireless lapel microphone attached to a transmitting device carried in their pocket or strapped to a belt, one that could be turned on or off at the teacher’s discretion, giving the teachers a certain degree of control over what was recorded. If, at any time they felt uncomfortable with a situation, they were able to turn the microphone off and back on again when it suited. Although this did not happen often, all teachers felt more comfortable knowing they had this level of control.

Pedagogical content knowledge is a complex phenomenon that needs to be analysed from the raw material of classroom behaviour, teacher speech and
explanations and the researcher’s observations and interpretations of this material. All
lessons were recorded to provide the primary data for analysis of the teachers’
pedagogical content knowledge. As Stenhouse is clear in pointing out:

> Observation clearly calls for some kind of recording, and the notebook is the
classic form. But it is not easy to keep a good record. Taking notes during
observation is generally intrusive, and field notes are usually written up from
memory as soon as possible after the event. Clear indications are desirable to
distinguish paraphrase from quotation. Sometimes photography can be used
either as a record or as a stimulus for writing. A trained memory is at a
premium. (1988, p. 51)

Stenhouse’s concerns were readily met by recording the teachers’ actions. The
recording was also appropriate from the perspective that pedagogical content
knowledge deals specifically with how teachers represent mathematical content and
much of their representation is via spoken language. The accurate recording of the
language they used was important to the study as this was the basis of much of the
evidence necessary to analyse the levels of pedagogical content knowledge. Each of
the recorded lessons was transcribed for later use when analysing the data.

In addition to the recording of the lessons, there were times when copies of
the teacher’s written explanations provided on whiteboards were recorded by note
taking. Occasionally, some teachers provided copies of worksheets.

**Interviews.**

The purpose of the initial interview was to establish a profile for each teacher,
enquiring into how each teacher planned for teaching mathematics lessons and what
they considered important issues when teaching mathematics. It questioned them
about their beliefs and helped the researcher establish rapport and a level of trust with each teacher, prior to the commencement of teaching their measurement lessons (see Appendix A for the initial interview questions).

Prior to each lesson, each teacher was interviewed about the lesson he or she was going to teach. Interviews were kept informal and focused on what the teacher hoped to achieve during the lesson, how they had planned for the lesson, the kinds of activities they used and why they had chosen these particular activities. The interviews were planned with a few specific questions to provide a basic structure, but they consisted of conversations with each teacher, allowing the teachers and the researcher to expand or discuss relevant issues that arose throughout the interview. This is consistent with Merriam’s position that ‘the largest part of the interview is guided by a list of questions or issues to be explored, and neither the exact wording nor the order of the questions is determined ahead of time’ (Merriam, 2002, p. 13).

All interviews were recorded and later transcribed. The interviews, while not exhaustive, continued long enough until each teacher had nothing more to contribute about their lesson and the topic had been ‘saturated’ (Groenewald, 2004, p. 11). Initially, each teacher was interviewed before any of the teaching episodes were recorded.

Each teacher was interviewed briefly again at the completion of each lesson. Many of the questions were similar to those asked prior to the lesson, except the emphasis shifted to whether they were satisfied with the results of the lesson, any problem areas, and questions regarding if they would plan differently next time. (See Appendix B for the questions for interview prior to each lesson and Appendix C for the questions for interview after each lesson.)


**Reflective questionnaires.**

The reflective questionnaire was administered two weeks after all teaching episodes had been completed to allow sufficient time to gain perspective so that the teachers did not simply reflect on the final lesson or two. It was not intended to be simply an evaluation or reflection on the most recent lessons, nor did it occur so long after the lesson that the teachers had forgotten the details of the experience.

The questionnaire was brief by necessity. It was considered that anything too lengthy and requiring too much detail would not be given the attention required. The questionnaire as set out in Table 3.1 consisted of five questions that required the teachers to respond on a Likert type scale. There was space provided after each of these questions for further elaboration if the teachers chose to do so. Then there were two final, open-ended questions.

Table 3.1

*Questions that Comprise the Reflective Questionnaire*

<table>
<thead>
<tr>
<th>The five questions requiring a five-point Likert scale response:</th>
<th>The two open-ended questions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>How would you rate your own knowledge of mathematics?</td>
<td>When you planned the lessons for this research, what did you consider to be the most important factors?</td>
</tr>
<tr>
<td>How would you rate the level of understanding achieved by the children in your class?</td>
<td>Are there any final comments you would like to make about your teaching of the sequence of measurement lessons?</td>
</tr>
<tr>
<td>What emphasis did your lessons place upon relevance to real life?</td>
<td></td>
</tr>
</tbody>
</table>
While the main emphasis was placed on observation for gathering data for this study, the interviews and reflective questionnaire played a vital role in strengthening the ‘trustworthiness of the data’ (Glesne & Peshkin, 1992, p. 24).

The use of interviews, observation and recording of lessons and the reflective interviews throughout this study provided an important mixture of data that allows corroboration of findings about the teachers’ pedagogical content knowledge when teaching measurement.

**Data Analysis and Interpretation**

This section considers the analysis techniques used throughout this study. The focus on the four teachers’ pedagogical content knowledge required investigating Shulman’s theory. To investigate Shulman’s theory, the transcripts of the individual lessons for each teacher were thoroughly investigated. The data obtained from transcribing each lesson, as well as from the interviews, was categorised. The data categories that were developed focused directly on each of the three knowledge types that contribute to pedagogical content knowledge. These included teacher’s knowledge of mathematics, knowledge of pedagogy and knowledge of learners. Figure 3.2 illustrates the interpretive categories that were used to analyse the transcripts.
Use of vignettes.

As the transcripts were analysed, the categories shown in Figure 3.2 assisted in capturing significant teaching episodes that provided insights into each of the teacher’s knowledge and understanding in these three key areas. These episodes were written up as brief vignettes. While vignettes are used in qualitative research in a variety of ways and different researchers use different definitions (Angelides & Gibbs, 2006), the ‘narrative vignette’, as suggested by Erickson (1986), was used in this study. This kind of vignette is based upon field notes and transcripts and is written up as an account of the teacher’s performance within their classroom context.
The reason for writing the vignettes was to assist in analysing the qualitative data. Erickson suggests:

The vignette is an abstraction; an analytical caricature (of a friendly sort) in which some details are sketched in and others are left out; some features are sharpened and heightened in their portrayal and other features are softened, or left to merge with the background. (1986, p. 150)

Use of vignettes as an evaluation and analytical tool for monitoring teachers is well documented, although use of vignettes generally focuses upon ‘pedagogical issues, not content’ (Veal, 2002, p. 2). ‘In reality, content drives how many teachers instruct and organize activities … Content needs to be included if students and teachers are to advance their understanding about teaching and learning’ (Veal, 2002, p. 2).

Veal’s emphasis on content as well as pedagogical issues is consistent with the focus of this study. The vignettes were written containing records and analysis of specific teaching episodes, reflecting both the classroom interactions that occurred and analysis reflecting the researcher’s extensive experience in mathematics education. The data was collected through interviews and the recording and transcribing of each of the observed lessons was extensive.

These vignettes served three key purposes in assisting with the analysis of such extensive data. Firstly, teaching episodes that the researcher identified produced systematic working accounts of each teacher’s knowledge of content, pedagogy and students, serving an intermediary function in the analysis of the data. This proved to be invaluable in managing the data. Secondly, they provided a key interpretive procedure drawing on the interpretative categories from the model. Finally, the
collection of vignettes then became a secondary source of information, which could be grouped together to identify common themes.

Figures 3.3 to 3.6 provide classification maps of the vignettes for each of the teachers. These maps represent an overview of all the vignettes written and illustrate how the vignettes were classified according to the aspects of teacher knowledge highlighted within the vignette. The colours used for coding the vignettes were based upon the colours from the model shown in Figure 3.2: teacher’s mathematical knowledge (green), understanding of the learner (purple), or pedagogical knowledge (blue). The categorisation of a vignette into one of these areas suggests it to be the prominent feature of the selected teaching episode. A sample collection of four of these vignettes is provided at Appendix D. These vignettes provided a thorough depth by ensuring close-grained analysis of the findings.

Information gained from the reflective questionnaire was also cross-referenced with the analysis of data from lesson observations and interviews. This allowed an examination of each teacher’s self-efficacy upon reflection, two weeks after the teaching had been completed.
Figure 3.3. Map of vignettes developed from Colin’s teaching.

Figure 3.4 Map of vignettes developed from Linda’s teaching.
Figure 3.5. Map of vignettes developed from Lois’ teaching.

Figure 3.6. Map of vignettes developed from Carla’s teaching.
Credibility of the Research

Clearly, the research design needs to be credible and appropriate for the purpose. Patton (2002) suggests there are three key areas that need to be addressed to ensure the credibility of a qualitative research study and to address issues of validity and reliability. These involve ensuring that rigorous methods of data collection are used, the credibility of the researcher and the ‘philosophical belief in the value of qualitative inquiry’ (Patton, 2002, p. 584).

Ensuring integrity, validity and accuracy.

Every effort was made to ensure the accuracy of the data collected and that the interpretation of the data was accurate. Each interview was recorded and each recording transcribed in its totality. Transcripts of these interviews were then supplied to each teacher, who was given the opportunity to read them and agree or disagree with their accuracy, providing a ‘member check’ (Oliver, Serovich & Mason, 2005). Similarly, each lesson was recorded and the recordings were transcribed in their totality. The teachers were given the opportunity to read the transcripts to determine their accuracy. Allowing the teachers to read the transcripts ensured the factual accuracy of their account. Gay et al. (2012) suggest that this factual accuracy ensures ‘descriptive validity’, which adds to the trustworthiness of the research.

The second measure taken to ensure the integrity and accuracy of the research study was the use of multiple methods of data collection. This involved regular interviews, the observation and recording of the lessons and a final reflective questionnaire, two weeks after the teaching was completed. These multiple sources of data resulted in triangulation: ‘Triangulation is qualitative cross-validation. It
assesses the sufficiency of the data according to the convergence of multiple data
sources or multiple data-collection procedures’ (Wiersma, 1991, p. 233).

Triangulation can be used to ensure that what may be omitted from a single
data source is covered by one of the other sources of data. Multiple sources of data
ensure completeness of the study’s findings and one source of data can be used to
confirm another (Gay et al., 2012; Lincoln & Guba, 1985; McMillan & Schumacher,
2006; Seale, 1999; Stenbacka, 2001). Patton (2002) advocates the use of triangulation
by stating, ‘triangulation strengthens a study by combining methods’ (p. 247).

Golafshani (2003) also points out that the term triangulation needs to be redefined,
compared to its more traditional meaning use in quantitative research: ‘For example,
in using triangulation of several data sources in quantitative research, any exception
may lead to a disconfirmation of the hypothesis where exceptions in qualitative
research are dealt to modify the theories and are fruitful’ (Golafshani, 2003, p. 603).

**Researcher knowledge and expertise.**

Within qualitative research, it is well recognised that the researcher is often an
expert in the area within which they are researching. While initially, this could be
observed as a case of the researcher having a bias, it has become accepted that rather
than bias, this simply represents what is regarded as an authoritative voice:
‘Typically, the researcher is ultimately privileged as the authoritative voice’ (Hesse-
Biber & Leavy, 2011, p. 167). In this study, the researcher was an experienced
mathematics educator with many years working in teacher education, conducting
professional development for in-service teachers, as well as ongoing involvement
with teaching children mathematics. Hence, the researcher brought to this study ‘an
authoritative voice’, enabling meaningful and insightful analysis of the data. The
depth and sensitivity of the analysis of pedagogical content knowledge would be inconceivable without the researcher being highly experienced in the field.

Throughout the data analysis and the writing of this research, it is unquestionable that the experience of the researcher in the area of mathematics education provided a particular lens for viewing the data that influenced the kinds of interpretations made. The researcher’s experience ensured the ability to analyse the data and to develop the categories used within the data analysis. Thody (2006) claims that ‘voice’ is important and that ‘the writing or presenting cannot, and should not, be neutral’ (p. 130). The researcher’s expert knowledge of mathematics and well-developed pedagogical content knowledge were essential in validly assessing the pedagogical content knowledge of others.

The researcher chooses the balance among the voices to be reported (Thody, 2006). Within this study, while each of the teachers had a ‘voice’, as evidenced in the way they responded to interview situations and on the reflective questionnaire, the voice of the researcher was dominant during the interpretation of each teacher’s lessons and the final analysis.

**Key underlying assumptions.**

There were three key assumptions underlying this research. Firstly, it was assumed that the responses of the participants to interview questions and questionnaire items would be trustworthy accounts of their knowledge, beliefs and understandings of the issues under discussion. Secondly, it was assumed that each teacher shared a common value of teaching for understanding and would demonstrate inclusive practice. This would be evidenced by a demonstration of a sound knowledge in the areas of the subject, pedagogy and of the students within their class.
Finally, there was the assumption that the research methods used in data gathering, interpretation and analysis would provide credible descriptions of the teachers’ pedagogical content knowledge and a trustworthy description of their teaching practice.

The Participants

Four teachers, named throughout with pseudonyms, volunteered for this study. Two of the teachers, ‘Colin’ and ‘Linda’, were novices, both in their second year of teaching. The other two teachers, ‘Carla’ and ‘Lois’ had both taught for more than 20 years. In agreeing to participate, each of these teachers expressed their willingness to have their lessons recorded and to participate in the research. Therefore, they were neither randomly selected, nor were teachers who were necessarily considered experts. However, they did represent a considerable range of experience for such a small group.

It is necessary to establish some background for each teacher to support the documentation and analysis of their teaching that will be presented in the following two chapters. Each teacher within this study is a context within his or her own right, so it is important to give an account of each. This section also describes the school context for each of the teachers.

Stenhouse (1988) suggests that, ‘a multisite case study approach should seek to cover the range of variables judged to be the most important in relation to the theme of the study’ (p. 50). In selecting four teachers, the opportunity to cover the variable of experience by including both novice and experienced teachers appeared to be appropriate in investigating pedagogical content knowledge. Stenhouse also states, ‘each case studied adds to the collection of cases’ (p. 50). This is consistent with
Shulman’s notion of building a ‘wisdom of practice’ by documenting the work of many teachers, while teaching many different topics in mathematics.

All four teachers in this study taught in schools operated by the Australian Capital Territory Department of Education and Training (ACTDET). This gives a context to the study in which there are several important features. Schools under the control of ACTDET exercise considerable independence in developing and working with their own, school-based curriculum documents. Until recently, there were no centralised curriculum documents other than Every Chance to Learn—the 2007 ACT (ACTDET, 2007) curriculum framework. This framework was provided to schools and teachers with criteria to assist them when selecting content and was intended to influence school-based curriculum development. Prior to this, schools had even more control over their own curriculum development process.

Linda, Carla and Lois all taught at the same primary school on the south side of Canberra and were all involved in middle primary teaching, specifically, Years Three and Four. The school catered for predominantly young families living in its feeder area, with a higher than normal proportion of single parent families. School staff were experiencing considerable change and only a few teachers had been present for more than a few years. Conversely, Colin taught at a school on the north side of Canberra in a more established area. Staff turnover was low and the school's clientele was largely families from a considerably higher socioeconomic status.

The remainder of this section provides a background of each of the four teachers involved in this study, based on information collected by means of an initial interview with each teacher at an early stage of the study. The following discussion of each teacher does not attempt to analyse their teaching in terms of their pedagogical
content knowledge but provides information about each of their educational backgrounds and their experience in the teaching of mathematics. The results of the initial interviews produced responses that enabled the researcher to derive the following structural elements for the profiles of the participating teachers:

- adequacy of initial teacher preparation for mathematics teaching
- teaching experience
- perceived challenges or issues with teaching
- teacher qualities as judged by the researcher
- school expectations and support
- degree of stability or change in their teaching.

**Colin.**

_Adequacy of initial teacher preparation for mathematics teaching._

Colin had taken his teacher education course in South Australia and had been employed by ACTDET upon graduating. Colin believed that his initial teacher education course had done little to prepare him for teaching mathematics. He described the two mathematics subjects that he was required to study during his pre-service course as very theoretical and presented in such a way that most students did not understand the content.

Colin claimed that no time was devoted to outlining the content required for primary school mathematics, as opposed to covering issues like conditions of learning, how to involve parents and mathematics anxiety. This concern over pre-service preparation to teach mathematics was expressed several times and he was quite adamant that his pre-service teacher education course was inadequate in
preparing him for the real classroom. He claimed his course placed a disproportionate emphasis on teaching language. As a result, he felt comfortable teaching language, but the same could not be said for his preparation and his confidence for teaching mathematics.

At one point in the initial interview, he raised questions on the standards of his teacher education course, including how he passed an assignment when he believed he should probably have failed. He explained how his lecturer phoned him about the poor quality of the assignment, but after a brief discussion over the phone, the decision was made to pass the assignment.

**Teaching experience.**

Colin had only completed one full year of teaching at the time of this study. Hence, Colin was a novice teacher settling into a new career and experiencing the realities of classroom teaching on a full-time basis. He was teaching a Year Four class and had the added pressure of studying part-time as he had chosen to add to his qualifications.

**Perceived challenges or issues with teaching.**

Although enthusiastic, his teaching of mathematics was not without its frustrations. He discovered that a ‘school-based curriculum’ meant different things to different people. He reported in the initial interview that when he arrived at the school, he was given the frameworks document and was informed that he would need to develop his program from that document. He soon discovered that other teachers on staff selected textbooks to help them with their planning for mathematics. Hence, these teachers had adopted particular texts and deemed those texts to be the basis of their curriculum.
Teacher qualities as judged by the researcher.

Colin was an enthusiastic teacher who was keen to do well and demonstrated this in the long hours he worked and the interest he showed in his class.

School expectations and support.

Colin felt that his introduction to teaching was very much a ‘sink or swim’ event. His induction to the school was non-existent. As a beginning teacher on probation and without permanency, he stated that the researcher was the first person to come into his classroom, watch him teach and to discuss his teaching in any way. He felt a total lack of support from senior staff within his school.

Degree of stability or change in their teaching.

As a novice teacher in his second year of teaching, Colin was teaching in his first school and had settled into its routines. He was developing an awareness of the school’s routines, the school culture and school practices. Although inexperienced, he felt a sense of stability within his school setting.

Linda.

Adequacy of initial teacher preparation for mathematics teaching.

Linda did not comment a great deal on her pre-service preparation for teaching mathematics. Although probed in this area, she was generally positive in her few responses. She felt that her pre-service teacher education was adequate in preparing her to teach mathematics. She did suggest that her major criticism of the course was that programming for short periods while practice teaching was superficial, compared to full-time teaching in the real world. Part of her concern related more to preparation for classroom management than preparation for teaching the content of mathematics. She raised the issues of catering for individual
differences and managing behavioural problems as significant concerns to her, and felt she had not been adequately prepared in these areas.

**Teaching experience.**

Linda was also in her second year of teaching. She had taken her teacher education course in the ACT and had completed a four-year Bachelor of Education. While a probationary teacher, her appointment by ACTDET was as a permanent teacher.

**Perceived challenges or issues with teaching.**

Linda was teaching a large class of combined Year Three and Four students. She had a large number of behaviour problems to manage and was teaching in a demountable classroom in overcrowded conditions. Survival was a high priority for Linda. With every visit to Linda, it was observed that she had a major control problem that usually required the intervention of a senior teacher or assistant principal.

Linda openly acknowledged that she experienced problems controlling her class. She stated that control was primarily in her mind and it appeared that at times her planning was not always on educational grounds but was based on what would minimise potential problems. This was particularly evident during the initial interview when she was asked what issues received priority when planning mathematics lessons. Teaching a combined Year Three and Four class, she commenced answering this question by stating that she considered the age groups within her class. However, she then explained that if the lesson involved ‘hands-on’ activities, the Year Three and Four students would do the same activity, ‘so we're not in chaos’.
Teacher qualities as judged by the researcher.

Linda often found questions during interviews difficult to answer and did not answer the question that had been asked, or else provided a brief response with little elaboration. However, Linda appeared to be comfortable with being observed and interviewed and maintained a quiet confidence about her teaching and the lessons she prepared.

School expectations and support.

Like Colin, it would appear that as a novice teacher, her induction to the school and to full-time teaching was minimal, evidenced by her uncertainty about the school’s expectations and policies. When asked about the school’s approach to programming mathematics, Linda was quite unsure. She was actually contradictory in her comments about this question. She commenced by stating that, ‘the school has quite a strict system for programming’, but then stated, ‘we haven't agreed with it so we've gone our own way’. When asked if they were allowed to simply go their own way and disregard the formal school approach, Linda replied, ‘I think so’.

Linda was asked whether the school required teachers to use a set text. She was quite vague in her response, finally answering, ‘Well, I think it’s use whatever you want to’. She indicated that from what she could ascertain in her short time in the school, most teachers did whatever they wanted and there was no consistency of texts or approaches being used.

Linda liked to involve parents in her classroom teaching and throughout most lessons that were observed she had parent helpers present, particularly to work with students when they divided for group activities. Her reliance on parental support in
her lessons was clearly related to her need to find ways to keep her class on-task and avoid behaviour problems.

Degree of stability or change in their teaching.

Linda was in her second year of teaching but had changed schools at the beginning of the new school year. Therefore, at the time of this study, Linda was still adjusting to her new school, which was considerably different to her previous school. Her first year had been in a well-established location, drawing from a stable population with a high socioeconomic status. Parental support had been high and discipline in the school had not been a problem. Linda then found herself in a school in a newly developing area. There were considerably more families from a lower socioeconomic level and the school had not established a well-disciplined operation.

Carla.

Adequacy of initial teacher preparation for mathematics teaching.

Carla studied her pre-service teacher education programme in Victoria and explained that her preparation for teaching mathematics was very good. She suggested that she remembered her pre-service days better than she remembered any of her teaching because, ‘it was such a good time of my life’. She studied a three-year course and was required to do mathematics education units in each of those three years. She also studied mathematics as an elective and pointed out that she was required to have passed the subject at the Higher School Certificate level to be allowed to do so. Carla described her preparation for teaching mathematics as quite comprehensive. It covered mathematics as a discipline, methods for teaching mathematics and focused upon the kinds of errors that children make. She
particularly valued this final factor (working with children’s problems) as part of her initial preparation.

**Teaching experience.**

Carla revealed that she considered a career change from classroom teaching after only a few years of teaching and she studied a teacher-librarianship course in Queensland during that time. She stated that she had been desperate to give up regular teaching. She recounted that as she progressed through the teacher-librarianship course, she realised that the classroom was where she really wanted to be. Her desire to leave the classroom changed and she recommitted to the teaching profession.

At the time of this study, Carla was teaching a Year Four class at the same school as Linda. She had taught for approximately 20 years, although she did take some time off during that period when having a family.

**Perceived challenges or issues with teaching.**

Carla maintained that it was the changes that took place in teaching that attracted her back to the classroom. Aspects such as emphasis on group work and less emphasis on just ‘talk and chalk’ were major changes that influenced her in this decision.

**Teacher qualities as judged by the researcher.**

Carla was a positive, confident teacher who enjoyed mathematics and claimed she wanted to help her students develop the same confidence and enjoyment of the subject that she experienced.
**School expectations and support.**

Carla was an experienced teacher and held a well-established position on staff. She was well respected by other teachers. Carla did not rely on support from the school. Rather, she went about her teaching in an independent and competent manner. Carla maintained that because she had always liked mathematics, she had not participated in many mathematics in-service courses. Mathematics was a subject she liked and she claimed that she always endeavoured to provide interesting activities. She held the view that when professional development courses became available, she selected areas where she felt she needed more help. She suggested that she had undertaken more professional development to help improve her teaching of language.

**Degree of stability or change in their teaching.**

Carla was a very experienced teacher. Early in her career, she experienced some instability, with her interest in teaching diminishing to the extent she decided to requalify as a teacher-librarian. However, that was short-lived and she returned to full-time classroom responsibility. At the time of this study, she had been teaching in the same school for several years and was both confident and comfortable in her role in that school. Carla was quite stable and secure in her school setting.

**Lois.**

**Adequacy of initial teacher preparation for mathematics teaching.**

Lois was a senior teacher at the same school as Linda and Carla. She remembered her pre-service preparation well, although she admitted with an embarrassed laugh that it was over 30 years ago. She prepared for teaching in Queensland where she was required to undertake a two-year course of study. It is only relatively recently that she upgraded to a four-year qualification. As part of that
upgrading, she studied a semester mathematics education unit that she suggested had a considerable effect on her teaching of mathematics.

Her initial course had concentrated almost entirely on the teaching of the four operations. She recalled that her lecturer taught them each of the operations, with particular emphasis on long division. They were tested on the four whole-number operations and once they passed the test, there were no other mathematical content topics covered in their mathematics education requirements.

Lois suggested that there was considerable emphasis placed on lesson planning in her initial course, but that planning centred on a very narrow view of mathematics. She maintained that there was no emphasis on understanding mathematics in the way that she now believed to be important, relational understanding.

*Teaching experience and its quality.*

Since finishing her initial teacher education course, Lois had taught continuously for all of the following thirty-plus years. Most of her teaching had been in Canberra and at the time of this study, Lois was at a new school—her appointment being the result of a promotion to senior teacher status. She was responsible for the entire middle school unit—Years Three and Four, and as a result, was team leader over Linda and Carla as well as other teachers in that unit.

*Perceived challenges or issues with teaching.*

Lois indicated that over the years she had tried to do as much as she could in the form of mathematics in-service courses. Particularly in her early years of teaching, she recalled doing several courses, some of which had a major emphasis
related to teaching measurement. One of her main issues as a teacher was to teach
mathematics in a meaningful way so that children would understand the subject.

School expectations and support.

Lois was also an experienced member of staff who was well established
within her school. She had a clear understanding of curriculum and of the
expectations held of her, yet she worked quite independently within her own
classroom setting. She was confident in her teaching and so did not look to others in
her school for support, or expect a great deal of support.

Degree of stability or change in their teaching.

Lois had taught for almost 30 years continuously. She felt quite secure and
stable in her role as teacher. She acknowledged there had been many changes in
education throughout her career but she believed she had kept pace with the changes
and was confident about her teaching.

Teacher qualities as judged by the researcher.

Lois was an enthusiastic teacher who endeavoured to share her infectious
enthusiasm with her students. It was obvious from the outset of this study that her
students held her in very high regard and that the classroom environment was warm,
friendly and contained a level of positive energy. Lois certainly knew how to engage
her class.

Conclusion

This chapter presented key issues relating to the research methodology and
design. Firstly, the research issues dealing with pedagogical content knowledge were
framed, resulting in a focus for the study. The main focal research question was then
broken down to four more specific questions, forming the basis of the investigation
for this study into the depth of each teacher’s knowledge of measurement, students and general pedagogy, as well as other factors which affect each teacher’s knowledge and teaching practice. Secondly, this chapter argued the appropriateness of a qualitative methodology and the adequacy of such for analysis of teachers’ pedagogical content knowledge.

Thirdly, the chapter discussed the research procedures employed to observe and interpret the pedagogical content knowledge of the four teachers. The research procedures adopted were adequate for the complexity of the phenomenon of pedagogical content knowledge and provided a close-grained analysis. They explored teachers in the context of their own classrooms, revealing their personal teaching experiences.

Multiple data sources were employed to ensure rich and useful data, providing credibility and validity of the research, depth of analysis and rigour in applying the conceptual categories. The procedures were strengthened by the systematic and thorough interpretation of the data and by employing vignettes to fully analyse and capture significant aspects of pedagogical content knowledge. Benefits of multiple case studies were outlined, allowing for cross-case comparisons and utilising explicit interpretive procedures linking the model directly to the data analysis. Finally, a detailed profile of each of the four teachers was presented. Each teacher was reported separately, as suggested by Yin’s multiple case study design.
Chapter Four: Teacher Case Studies

Introduction

To gain a clearer insight into how knowledge was demonstrated by each of the four teachers when teaching measurement, they were observed, recorded and interviewed during a series of lessons. This chapter aims to provide a broad context for a further analysis of teacher pedagogical content knowledge by documenting the results of those interviews and observations. The chapter incorporates a discussion of the teaching moments that were considered to reveal insights into the teachers’ pedagogical content knowledge. Some of these teaching episodes reveal strengths and positive aspects of the teacher’s lessons, while other episodes raise aspects of concern and areas of weakness.

It is important to first document what this study revealed about each participant’s teaching, focusing on the results of interviews and teaching that constitute the empirical evidence of this study. As Yin (2011) suggests, ‘the “research” aspect of “qualitative research” means giving careful attention to your empirical evidence’ (p. 263). This will be documented throughout this chapter using a predominantly narrative form and by employing quotations from interviews or teaching transcripts. The use of transcripts helps to present the findings in such a way that the voice of each participant emerges, together with the interpretations of the researcher.

The case of each teacher will be presented separately to gain a meaningful understanding of each teacher’s pedagogical content knowledge. Their teaching will be reported using the four research questions and using the following headings for each teacher:
1. knowledge of mathematics
2. knowledge of students
3. knowledge of teaching
4. other factors that impact on teaching.

Colin

Colin’s pedagogical content knowledge.

Colin was observed as he taught a series of measurement lessons to Year Four covering the attributes of length, area, volume and mass. He was interviewed prior to and after each lesson. During these interviews, he was able to explain his intent for the lesson and his beliefs about aspects of teaching mathematics that were relevant to the lesson. After each lesson, he was given an opportunity to reflect and comment on the effectiveness of the lesson. Table 4.1 provides a summary of the topics Colin taught during the time he was observed.
Table 4.1
Summary of Colin’s Lessons

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Topic</th>
<th>Activity</th>
<th>Recording by children</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Measuring our environment</td>
<td>Practical measurement of objects around the classroom. Estimation required first.</td>
<td>Incidental—some children wrote estimates and measures as memory aids.</td>
</tr>
<tr>
<td>2</td>
<td>Measuring our environment</td>
<td>Continuation of similar activities to Lesson One.</td>
<td>Some wrote estimates, as in Lesson One.</td>
</tr>
<tr>
<td>3</td>
<td>Area understood</td>
<td>Measurement of area of a book cover using grid paper.</td>
<td>None.</td>
</tr>
<tr>
<td>4</td>
<td>Skin-side out (surface area)</td>
<td>First, children drew around their hand and measured the area by using grid paper. Main activity for the lesson involved a group activity to measure the surface area of their body.</td>
<td>None.</td>
</tr>
<tr>
<td>5</td>
<td>Discovering kilometres</td>
<td>Activity on the oval:</td>
<td>Recording individual jogging times for group members.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Measuring a kilometre with trundle wheels.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Estimating how long it would take to jog a kilometre.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Timing the children jogging a kilometre.</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>A tonne of children</td>
<td>Activity measuring the mass of all children to determine the collective class mass.</td>
<td>Worksheet requiring children to estimate the mass of heavy objects based upon pictures.</td>
</tr>
<tr>
<td>7</td>
<td>Building a cubic metre</td>
<td>Activity requiring children to construct the edges of a cubic metre using rolled newspaper. Calculate the volume of the room.</td>
<td>None.</td>
</tr>
</tbody>
</table>

The topics included in the Table above use the titles selected by Colin. Colin believed that by giving his lessons titles and having these written on the board prior to the lesson would help capture the children’s interest, as they would be keen to engage in interesting activities. Most of these activities were practical and only
occasionally were the children required to formally record the results of these activities. Colin only once used a worksheet.

The basis for reporting on Colin’s knowledge is from observation of his teaching and the interviews conducted associated with each lesson. Again, this is not an attempt to identify comprehensively all aspects of his knowledge base, but rather to identify issues in the teaching of measurement, referring to his knowledge of mathematics, students and teaching and finally, other factors that impacted on Colin’s knowledge.

**Colin’s knowledge of mathematics.**

Colin readily admitted that his knowledge of mathematics needed to improve. One way that he dealt with this was to have a mathematics dictionary on his desk to enable him to check the meanings of unfamiliar mathematical terms. He was quite explicit about this during the initial interview.

It became evident that it was not simply unfamiliar terms such as ‘scalene’ or ‘isosceles’ that caused Colin problems. His knowledge of the language of measurement units was limited, although he knew that each unit had a relationship to other units. Colin revealed during interviews that he believed strongly in teaching mathematical relationships to help children develop a more meaningful understanding of linear units. However, it became apparent that while Colin recognised there were important relationships, he did not understand the language used to describe these. The following extract of transcript demonstrates how Colin linked each of the units together without actually recognising that the language used stressed the relationship of each unit to the metre base unit, the metre.
Colin:  *When we're using millimetres, they make up a centimetre, so how many millimetres can we put into a centimetre? How many do you think Victoria?*

Victoria:  *Ten.*

Colin:  *Ten. All right, we've got ten. Ten millimetres in a centimetre. What about centimetres in a metre? If I want to have a metre, how many centimetres would I need to put into that metre?*

Bronwyn:  *A hundred.*

Colin:  *A hundred. All right. And then our final one. We've got metres in a kilometre, how many?*

Class:  *A thousand.*

Colin:  *A thousand. Alright.*

Colin has used the sequence of units by their size to express the relationships. Taking the units from smallest to largest, he discusses with the class the relationship between consecutive units. Hence, he commences with, ‘how many millimetres can we put into a centimetre?’ The answer of ‘ten’ is clearly correct. However, this relationship is not the important relationship if the students are to make sense of the language, milli-metre.

As the discussion progressed, Colin looked at the relationship between centimetres and metres and metres and kilometres. Colin's problem is not blatantly obvious, as most of the relationships he deals with are valid. Having dealt with millimetres, there were only three units remaining to examine in terms of relationships. The unit ‘metre’ occupying a central position in the sequence of
centimetre, metre and kilometre, assured that Colin's approach connected both ‘centimetre’ and ‘kilometre’ to ‘metre’.

Colin's lack of understanding of the language used for the units may not have been evident had it not been for the following interaction:

*Robert:*  *Does ‘milli’ mean ten and ‘centi’ mean a hundred?*

Looking at the researcher, the teacher called out, ‘Can you answer that? Does ‘milli’ mean ten and ‘centi’ mean a hundred?’

Robert's question is clearly linked to the relationship that Colin had emphasised as important. Robert asked the question based on Colin's teaching point that there are ten millimetres in one centimetre. Whether Robert had previous notions of ‘milli’ meaning something other than ten is unclear. It is possible he had previously associated ‘milli’ with ‘thousand’ and was now attempting to sort out the confusion that confronted him.

Colin's response of, ‘I'm not sure’ and deflection of the question to the researcher present in the room provided the undisputable evidence that Colin himself did not recognise all of the important relationships between the units. More importantly, he was not aware of which relationships governed the use of language that he was attempting to introduce. It would appear that he had not considered what terms like ‘milli’, ‘centi’ and ‘kilo’ actually meant and that their meaning was in direct relationship to the unit ‘metre’, as shown in the following Figure.
Colin acted as though the terms are arbitrary and need to be memorised in the sequential order in which he dealt with them, as shown in the next Figure.

A similar situation occurred during his lesson teaching mass when he said, ‘Alright, kilo means thousand, so a tonne could be a thousand kilograms’. What he failed to recognise was that the prefix ‘kilo’ is linked to the unit ‘gram’, thus kilo—gram is the same as 1000 grams. It is unclear how he was able to draw the conclusion that because kilo means 1000, as the word ‘tonne’ also implies 1000.
Another use of language that is worthy of mention is Colin’s use of ‘mass’ and ‘weight’. This distinction is technically a real and easily defined distinction, yet Colin used the language of mass and weight as if the two were the same. The reason for raising this issue is based upon Colin’s belief in the importance of modelling correct language, which he believed he did ‘for the kids’ sake, so they get the right thing’. His usage of the language was mixed and he modelled these terms as if they were corresponding. Given his belief of modelling appropriate language at all times, it appeared evident that he had used ‘mass’ and ‘weight’ without recognising the difference between the two terms.

One further teaching episode illustrated Colin’s lack of understanding of key mathematical terms. When questioning the class about how they might measure the volume of the room, the children’s responses were varied, with some referring more to capacity rather than volume. Clearly, ‘volume’ and ‘capacity’ are terms that both refer to volume, yet ‘capacity’ uses litres as the base unit, whereas ‘volume’ uses cubic metres. Colin did not attempt to explore the difference between the two terms. He did not refer to the different kinds of units used to distinguish between them, when one is concerned with how much you can put into something (capacity) and the other with how much space an object is occupying (volume). However, a more formal relationship between the two and their units could certainly have been left to another lesson (for example, how one millilitre is equivalent to one cubic centimetre).

Colin demonstrated throughout his lessons that he was keen for his students to measure with accuracy. Yet, from the commencement and throughout his teaching of all lessons, he did not focus on this important aspect when discussing with his class what determines an appropriate unit for measurement. At all times he stressed that,
’when you measure something large, a large unit is needed and when you measure something small, a small unit is needed’. He did not connect the need for small units to measure accurately, irrespective of the size of the object being measured. There are times when Colin’s position may be in direct opposition to measuring accurately.

Another important insight into Colin’s knowledge of mathematics occurred during a lesson on the oval when the children were measuring a kilometre. He stated it was important for them to see a kilometre so that they could develop a mental model and be able to visualise it as a unit more easily. This is clearly a large linear unit of measurement and it may have been the large size of the unit that led Colin to this strategy. He did concede during the lesson that it is harder to visualise such big units. However, it raises the question as to what Colin saw as the purpose of estimation. If he saw estimation as part of the process of constructing mental models, then the activity did not seem to be appropriate. Colin had previously used estimation in a way that suggested that he did see it as important in the process of constructing mental models. That is, a mental picture of the unit is formed, applied to the external situation (estimate) and then checked to see the accuracy of the mental models by measuring. Based on the estimate, one then modifies the mental model of the unit accordingly.

However, in this situation, the estimate required from the children was: ‘How long do you think it will take you to jog a kilometre?’ The children were not only required to visualise the kilometre, they also had to take into account their running speed, whether they could keep up a constant speed for a kilometre, to use their mental models of units of time (minutes) and then estimate how many minutes it will take them to run the kilometre. Without acknowledging the evident change in focus,
Colin proceeded through the activity with the belief that estimating time (and speed) was an appropriate estimate when helping children construct a mental model of a kilometre. Colin confused attributes or, at the very least, mixed attributes as his focus moved from the linear unit (kilometre) to dealing with units of time (minutes).

One final aspect of Colin’s mathematical knowledge was related to his knowledge of measuring equipment. In two of the lessons, Colin required children to use tape measures, rulers and trundle wheels, as well as other measuring equipment. The children in the class used the trundle wheel without any instruction as to how to use it accurately. Starting from zero may seem obvious, but the children appeared oblivious to the numbers around the wheel. They counted clicks and believed that was all that was required. Clearly, not using the numbers caused differences in measurements. Colin did not realise that the children were not using the trundle wheels correctly until it was discussed in the post-lesson interview. He appeared to dismiss this as if it were unimportant for his objectives.

However, when Colin completed the reflective questionnaire, two weeks after the observation of his teaching, he did acknowledge that he needed to improve his mathematical knowledge. He wrote:

*I know I need to keep improving my mathematics knowledge. Sometimes I need to check on meanings of mathematical terms and how to do some things. I sometimes forget formulas but it is not anything that gets in my way of teaching as I check them before I teach.*

Colin’s final comment supports the view that even though he believed he needed to improve his mathematical knowledge, his knowledge was adequate because he checked on unknown content before he presented it to his class.
In summary, while Colin strongly emphasised his desire to teach for understanding, the evidence suggests that his own mathematical knowledge was predominantly instrumental. There appeared to be many gaps and often his understanding of concepts was incomplete.

**Colin’s knowledge of students.**

Colin discussed the students in his class in terms of their abilities and their needs on several occasions. He appeared to know the more capable students and their strengths as well as those who were not so capable. Further, he often commented on their learning in terms of how they had been taught previously by other teachers. Colin clearly believed that his approach to teaching was different to other teachers his class had encountered previously. He repeatedly insisted that his class had just been taught formulas and the mechanics of mathematics without any real understanding. While he knew his students had predominantly an instrumental knowledge of mathematics and that he had a belief that relational understanding was more desirable, he often did not demonstrate that he had sufficient knowledge of how to convince students that his representational approach was better. There were instances where it was clear the children did not fully understand why they were being asked to do activities in the manner Colin required.

During an activity in which the class was required to measure the area of a book cover, Colin engaged one child in a brief discussion:

*Colin:*  Okay, so I wanted you to count up all the squares. Would multiplying the width and the length as you said have given you the same answer, if you did it carefully and accurately?

*Angus:*  Well, it didn't work for me. I got about a hundred, two hundred off.
This lack of connection continued to be demonstrated by Angus:

*Colin:* Why do you think I wanted you to use the squared paper and not just measure and multiply?

*Angus:* I think you just wanted us to use it as a first-up experience.

*Colin:* First-up experience to do what?

*Angus:* Well, because you are probably going to teach us how to measure it properly but you were just giving us that so that we understood it better.

From the comments by Angus, it is clear that he did not see counting squares as a 'proper' technique for measuring. He had difficulty expressing the reason why he thought Colin had asked them to count all the squares. However, he appeared to exercise good faith in Colin as a teacher and assume that it was to help them understand it better, and that Colin would subsequently teach them how to measure area properly.

During this lesson, Colin insisted that children use grid paper to measure the area of the book cover. However, some children still demonstrated a reluctance to count squares. Colin became somewhat frustrated and appeared not to understand why the class was reluctant to count. There came a stage where he approached the researcher present in the classroom and stated: ‘I’m fighting a losing battle here. They're stuck with this formula, see, it's thrown them’.

The only plausible explanation Colin could offer was that having been taught the formula, there was a reluctance to attempt any other way. By suggesting, ‘it's thrown them’, Colin was proposing that because it was a technique that worked, even though it was one that they did not understand, they were unable to measure area by
any other means. The counting of squares on grid paper bore no relationship to the formula in their minds and, therefore, the tedious job of counting many small squares seemed to be a waste of time, compared to applying the formula. Colin reinforced this interpretation in the interview after the lesson, when he made the following comments:

_They'd been taught the formula and they felt they didn't want to go back and have to count all the squares. They knew how to do it really quickly. But they didn't understand the formula and they didn't know how to use it. Well, they sort of knew how to use it, but they had no idea of checking it. They got an answer, they had no idea of visualising what that answer looked like as in centimetre squared and that's where the problem is._

It became clear from discussing this issue with Colin that while he had the knowledge that representational theory was desirable, he did not have the teaching knowledge to know how to use representations with children who already had established an instrumental understanding of mathematics.

Another notable incident during Colin’s lesson on mass demonstrated a lack of knowledge about children and their sensitivities. While only comprising a single incident in the lesson, it is significant, as Colin did not realise that he had caused a problem. During the lesson, each child was required to come to the front of the room, measure their mass and then write the measurement on the board. One girl who clearly regarded herself as being overweight was reluctant to stand on the scales and record her actual mass. One can only assume that she was going to record a measurement quite different from those who had been before her.

_A brief description of the events follows:_
The girl initially said that she knew her mass.

*Colin:* *You know yours, try it anyway, jump on.*

She stepped on to the scales but just as quickly stepped off. Colin noticed that she had repositioned herself off the scales before the dial had stopped moving.

*Colin:* *Did you weigh yourself? Step on.*

The girl stepped back on to the scales, but this time leant onto the whiteboard ledge. This was a clear attempt to ensure a lower reading of her mass. However, the teacher once again insisted:

*Colin:* *You can’t lean on anything, just step on. That’s right.*

By this time, the girl was quite red faced from embarrassment and looked decidedly uncomfortable as she recorded her mass of 47 kg on the board. Up until this point, most of the other children had recorded measurements in the thirties, with a couple recording 26 kg and 28 kg. As the girl returned to her seat, facing the observing classmates, she looked even more embarrassed.

Colin demonstrated in this episode that when the needs of the students clashed with the needs of the content of the lesson, he did not have the knowledge to anticipate student sensitivities. He did not have knowledge of strategies of how he may have presented this activity differently and yet still achieve his intended outcomes.

**Colin’s knowledge of teaching.**

As a recent graduate, Colin was enthusiastic and remembered much of what he had learnt in his pre-service mathematics education units. He was aware of many
of the planning issues a teacher needs to address and many of the beliefs discussed in
the previous section were based on his knowledge of how to teach mathematics.

Colin had a sound knowledge of representational theory and the importance of
using various modes of representation to enable children to develop relational
understanding. However, while he knew that it was important to represent
mathematical content, his repertoire of representations for specific content was often
less extensive. One example of this was the lesson on the oval when his class had
been required to measure a kilometre. In the post-lesson interview, he stated clearly
that he had planned the activity with the intention of helping the children construct a
mental picture of a kilometre:

*If we look at the measurement strand in the syllabus, there are activities that
while you know that you need to cover them within the syllabus, it just tends
to talk about a kilometre. The syllabus stresses the relationship that a
kilometre is a thousand metres but it doesn't have anything to visualise it at
all, whereas that's what I wanted to build on today.*

Having completed this lesson, Colin gave no indication that this was only one
of a set of experiences to enable the children to construct their mental models of a
kilometre and, in fact, indicated the contrary. In Colin’s mind, it was clear that the
children had been provided with this experience and that it was sufficient. When
asked by the researcher what further experiences could be provided to the children,
Colin struggled:

*Researcher: How else could you measure it?*
Colin: *I don’t know (laughs), how else could you measure it? What, using a car are you saying?*

Researcher: *I’m just asking you for your ideas … how can we measure a kilometre?*

Colin: *I would say to them to measure a certain amount of area but it's still really hard. I see what you're getting at but I find it difficult to answer that one.*

This question appeared to take Colin by surprise and provides additional evidence that he had not considered the cognitive processes involved in constructing mental models, particularly regarding ideas as complex as a kilometre. He operated on the belief that even if children have experienced an idea only once, then they will be able to construct an appropriate mental model of the idea. His knowledge of a variety of representations appears to be lacking. He operated from a limited number of representations, yet had the belief that these are adequate.

Nevertheless, Colin knew the advantages of prior testing before commencing teaching a new topic, knew how to group children for cooperative activities and how to encourage each to actively participate. Throughout the interviews, he demonstrated a sound knowledge of the curriculum and the requirements placed upon him as a teacher.

**Other Factors Impacting on Colin’s Knowledge**

The conceptual framework of Chapter Two identified three main factors that have an impact on teachers and their teaching of mathematics: the teacher’s beliefs, teacher self-efficacy and the cultural context of the school. These are now discussed in Colin’s case.
Colin’s beliefs about teaching mathematics.

During the interviews, Colin expressed quite clearly and with considerable conviction many of his beliefs about teaching mathematics. Colin stressed the importance of children’s prior knowledge and claimed that he often assessed this to assist with his planning. He believed it was essential to cater for all students, including both high and low achievers:

After testing the kids, I know what areas we're going to go through. I decide on the topics for the term to begin with and then we look at that particular topic. And also, when I'm planning, I'm thinking, because of the class, you know, they're quite good at mathematics. But, there's still some strugglers too, so you have to balance them in a way that you can extend the higher achievers and keep them interested, because they are the ones that really, really enjoy maths, so I want to keep that interest going. Whereas with the lower achievers, you want to work with them, let them achieve some degree of success in a way. (Initial Interview)

Colin also expressed his belief that there is always a need to provide extension and consolidation work for children to help each child reach their potential and ensure that he catered for all the children in his class.

When teaching mathematics, Colin felt there is a real danger of being too abstract and believed that measurement needed to be practical and that formulas should be derived, not just memorised. This would ensure all children understood and give all students the opportunity to make sense of mathematics. He also recognised the need to teach mathematics in a way that was fun for his students, something that was central to much of his planning:
So, if I'm planning a lesson working through my programme, I'll think about, how can I present this activity to get the class interested to start with? And also to not confuse, to be able to get the information across. And then, you also want to make it fun. (Initial Interview)

In the first lesson, as Colin discussed with the class linear units of measurement, he encouraged the children to think of people who would use these units as part of their everyday employment. The students identified carpenters, architects, carpet layers and picture framers as some of the occupations needing to measure using the various linear units. During an interview, Colin later explained that mathematics needs to be made relevant to children, and he encouraged them to think about occupations that used linear measurements for this reason. There is a clear understanding, on Colin's part, that children need to see mathematics as useful and applicable in daily life.

Colin believed that representations of mathematical ideas were paramount to children developing meaningful understanding. He believed that the mental models children created were heavily influenced by the external representations engineered by the teacher. This did not mean the teacher needed to provide all external representations. He believed that with appropriate activities, children could represent mathematical ideas in ways of their own. Throughout the observed lessons and associated interviews, he consistently spoke of his belief that children need to make sense of mathematics and how children can construct their own mental models of mathematical ideas through appropriate representations. This was an important aspect of his planning; a belief that Colin stated repeatedly.
Colin demonstrated this explicitly during his lesson dealing with measuring a kilometre. At the commencement of the lesson, he asked the class:

*Who can picture a kilometre in their head? What is a kilometre? We’ve talked about centimetres; we’ve done some work with metres. What, what is a kilometre?*

Once outside on the oval, where the children were to measure a kilometre, Colin asked the following question:

*First of all, we want to make an estimate. We want to make a guess at how long it will take to jog a kilometre. What do you think now before you even look at a kilometre, at how long it is? Remember, when we make our estimates we’ll be able to see how far we have to run but just in your heads now, how long do you think it will take you to jog a kilometre?*

Colin once again emphasised the importance he placed upon mental models. His words, ‘we'll be able to see how far we have to run **but just in your heads now**’ provides evidence that he was encouraging the children to practise visualising the unit and that he wanted his class to construct mental models of a kilometre.

In another of Colin’s lessons, he had the children construct a cubic metre using rolled newspaper rods for edges. With their models constructed, the children were able to see a cubic metre. This was an example of Colin’s belief that external models enabled children to construct internal models of a concept. Colin had revealed during interviews that he believed strongly in teaching mathematical relationships to help children develop more meaningful understandings and that relationships were
easier to develop if concepts were understood by using meaningful representations or models of the mathematical concepts.

Colin introduced an activity called ‘Skin-side out’, intended to involve children in measuring the surface area of their bodies. The activity was posed as a problem-solving investigation for the children to work through, the goal being to calculate how many square centimetres of skin covered their body. The situation was described to the class in the following way:

*This is a question I want to ask you. If we could unwrap our skin off our body, doesn't sound too good does it, but if we could unwrap our skin off our body, how much area would it cover?*

As Colin expressed it, the major objective was to provide a problem-solving lesson for the children, as he believed problem solving was an important mathematical process. Working in groups, the children were to find a strategy that would enable them to measure the surface area of the body of one of the students, using sheets of newspaper.

This emphasis on problem solving appeared throughout his teaching and in other activities, he encouraged his class to look for strategies to solve the problems he posed, such as calculating the area of a book cover or the area of their hands.

Discussing this belief, he again made the connection to relevance and enjoyment, stating that if children are engaged in investigations like calculating the area of one’s skin, they are more likely to enjoy the mathematical activity.

Colin also expressed and demonstrated clear beliefs about several aspects specifically related to the teaching of measurement. Firstly, he stressed the importance of accuracy when measuring. During Colin's lesson on measuring length,
children were instructed to use their metre rulers to measure objects of their choice around the classroom. Having drawn up a table with the headings ‘objects’, ‘estimation’ and ‘measurement’, they were to select an object, make an estimate of its length and then measure accurately to determine the closeness of their estimate. The following comments from the teacher when two children were measuring him demonstrate his concern for accuracy and the process he wanted them to follow:

*Teacher:* Now you’re going to have to do this accurately.

*Child:* (Measuring the teacher.) A metre and seventy centimetres.

*Teacher:* Did you do an estimate first?

*Child:* Yes.

*Teacher:* Right.

Clearly, it can be observed the emphasis placed on measuring accurately and ensuring the children estimated first. Here, Colin’s beliefs regarding estimation in the measurement process are evident. He believed that children should be encouraged to estimate prior to performing the measurement and emphasised consistently that he taught in such a way that he wanted his students to construct meaningful or relational understandings of mathematics. He believed that by watching children estimate he was able to obtain an insight into their understandings.

After the lesson, students made their own metre-long strips and estimated lengths of various objects before measuring them. Colin was asked:

*Researcher:* What sort of things were you looking for as you moved around the room?
Colin: I was looking for their understanding, looking at how they estimate. If they can understand, if they can visually look at something and see. If they're looking at how wide the door is. One girl had three hundred centimetres wide, then I want to pick up her thinking, you know, why she thought that, and then after she could relate back. I didn't have to say to her no, that's totally wrong. She could relate back, well, that can't be right because that is three metres. I just wanted them to be able to identify an object, be able to look at it, make estimation and then use a ruler that they made up and measure that.

Here, the belief that estimation reveals understanding is clearly demonstrated. Colin believed that children’s estimation practices reveal the extent to which their internal representations of units of length have developed. With this emphasis on estimating and developing mental models of units, Colin believed that children should have more than just knowledge of the various units for measuring each of the measurement attributes. He believed that children needed to use the units. Measurement to Colin was a practical aspect of mathematics and as a result, he planned activities where children were often required to construct units and then use them.

Colin expressed quite strong beliefs in the need to model appropriate mathematical language, that this was admittedly an area that he found problematic and that he often relied on a mathematics dictionary. It was important ‘for the kids’ sake so they get the right thing’:

Colin: I find as a second year out teacher there are some terms that I don't understand. I'm just becoming familiar with the curriculum and I'm
always going to a maths dictionary just to look up what everything
means.

Researcher: What would be an example?

first read it I say, what's the definition of a scalene triangle and, it's
natural, if you're not dealing with it all the time, you're not going to be
able to do it.

Researcher: I noticed the maths dictionary on your desk.

Colin: And I use it all the time. For the kids’ sake so they get the right thing.

Finally, Colin articulated a number of times that he believed quite strongly
about the need for children to learn to use equipment in an appropriate way. He
indicated, ‘You have to keep it really structured’, saying he did not believe in just
letting children play with the equipment. Colin believed that the play approach
adopted by some teachers was having a negative effect on children’s attitudes to
mathematics. He gave the impression that he was using the equipment for serious
learning activities and there was no room in his class for anything but ‘real’
mathematics.

Self-efficacy.

Two weeks after the teaching was completed, Colin rated himself against the
dfive questions on the reflective questionnaire (Table 4.2).
Table 4.2

Colin’s Responses to the Final Reflective Questionnaire

<table>
<thead>
<tr>
<th>Question</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How would you rate your own knowledge of mathematics?</td>
<td>3</td>
</tr>
<tr>
<td>2. How would you rate the level of understanding achieved by the children in your class?</td>
<td>4</td>
</tr>
<tr>
<td>3. What emphasis did your lessons place upon relevance to real life?</td>
<td>4</td>
</tr>
<tr>
<td>4. What emphasis did your lessons place upon problem solving?</td>
<td>4</td>
</tr>
<tr>
<td>5. What emphasis did your lessons place upon estimation?</td>
<td>4</td>
</tr>
</tbody>
</table>

1=Poor 2=Below Average 3=Average 4=Above Average 5=Excellent

From Table 4.2, it is evident that Colin’s self-perceptions were quite high. He rated himself as average concerning his knowledge of mathematics and above average for all other questions, reflecting his level of confidence displayed throughout the study. From the outset, he was eager to be a participant and throughout maintained an enthusiasm for teaching and a sense of self-belief. He saw himself as thorough, well-organised and providing interesting relational activities for his children—activities designed to be engaging. As a result, Colin believed that he taught effectively and influenced student learning outcomes positively. His frequently stated belief in teaching for understanding in a relational manner demonstrated his confidence that he was achieving desirable outcomes for the students in his class.

Cultural context of the school.

Colin was teaching in a school that would be considered quite traditional in that its architectural design required each teacher to be responsible for their class in a self-contained room. Apart from the library, there were no open area spaces for team teaching or more flexible approaches to teaching. Hence, Colin found himself
teaching in a relatively isolated situation and once in his classroom, had virtually no other contact with colleagues. He raised this issue of isolation during interviews, commenting that the researcher was the first professional to enter his room and show an interest in what he was doing. He also reflected upon the fact that his pre-service teacher preparation emphasised team relationships and that in his school, there were none, as teachers went about their teaching quite independently with little opportunity for sharing or collective planning.

The school appeared to provide little assistance to Colin as a novice teacher in his second year and there was little parental involvement in the school and his classroom. The school caters for a higher than average socioeconomic area within the ACT and, as a result, Colin reported that most children came from families where both parents worked and often in a professional area. Consequently, he rarely had parents volunteer to assist with aspects of classroom activity during the day, a quite common occurrence in many primary schools.

Linda

Linda’s pedagogical content knowledge.

Throughout the time Linda was observed, she taught a series of measurement lessons to Years Three and Four, with emphasis on the attributes of capacity and volume. During the accompanying interviews, she was asked to explain her intent for the lesson and her beliefs about aspects of teaching mathematics relevant to the lesson. After each lesson she was asked to reflect and comment on its effectiveness. Table 4.3 provides a summary of the topics Linda taught during the time she was observed.
Table 4.3
Summary of Linda’s Lessons

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Topic</th>
<th>Activity</th>
<th>Recording by children</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Place value</td>
<td>Activity from workbook linking MAB_{10} pictures to number values up to 100. Set of mental arithmetic for early finishers.</td>
<td>Recording in workbook.</td>
</tr>
<tr>
<td>2</td>
<td>Making a milkshake</td>
<td>Teacher made milkshakes for children while they observed the process.</td>
<td>Copying milkshake recipe from board into their books.</td>
</tr>
<tr>
<td>3</td>
<td>Measuring the capacity of a milk bottle</td>
<td>Estimating and measuring the capacity of a milk bottle using teaspoons.</td>
<td>Recording estimate and actual measurement.</td>
</tr>
<tr>
<td>4</td>
<td>Things we measure</td>
<td>Activity measuring a variety of objects, determining the attribute to be measured and selecting appropriate units.</td>
<td>Recording estimates and actual measurements.</td>
</tr>
<tr>
<td>5</td>
<td>Things we measure</td>
<td>Continuation of similar activities to Lesson Four.</td>
<td>Recording estimates and actual measurements.</td>
</tr>
<tr>
<td>6</td>
<td>A witch’s brew (capacity)</td>
<td>Activity involving following a recipe to make a ‘witch’s brew’.</td>
<td>Writing in their workbooks about the problems they encountered throughout the activity.</td>
</tr>
<tr>
<td>7</td>
<td>Area of our hand</td>
<td>Activity requiring children to use grid paper to measure the area of their hand.</td>
<td>Drawing of the hand and recording estimate and measurement.</td>
</tr>
</tbody>
</table>

Linda had initially agreed to allow the observation and recording of a series of lessons related to the measurement strand of the curriculum, and, in particular, she stated she would like to develop a sequence of lessons dealing with capacity. As can be observed from the Table above, Lessons One and Seven did not fit within this plan.
When the researcher arrived for the first lesson, Linda had the children work through a place value activity from their workbook, with no explanation as to why she had changed, and in Lesson Seven, Linda again changed from the agreed plan. She had been at an in-service course the day before and thought the children would enjoy one of the activities she had seen at the course. She stated this lesson was in place of the capacity lesson she had intended to do and that she would not be doing the planned capacity lesson until later in the year.

Throughout the sequence of lessons and interviews, it became quite evident that the knowledge required for Linda to teach measurement was weak for each of the three identified areas.

**Linda’s mathematical knowledge.**

During the initial interview, Linda stated:

*I never liked maths at school. I did advanced maths to Year Ten. I went to Queensland high school, and then when I did Years 11 and 12, I dropped back a level and I really found that I didn't grasp a lot of the concepts.*

It became evident during the follow-up interviews when Linda was asked what she had hoped to achieve throughout the lesson, that her responses often highlighted outcomes that were not mathematical. On one occasion, Linda responded, ‘to see if they could handle pouring water and not spilling too much’. After the next lesson, she stated her intended outcome was to work with the Year Four children, ‘to give them a go at what the Year Threes did last week’.

Linda consistently demonstrated her limited knowledge of mathematics. From understanding relationships between concepts to knowing appropriate mathematical language, it was apparent that many gaps existed. This was particularly evident
during a lesson when the children were asked to list ‘things we measure’, ‘tools for measurement’ and ‘what they measure’. It is worthwhile examining the attributes that the children identified, as several misconceptions became evident. Further, these misconceptions largely went undetected by Linda and certainly uncorrected.

When the ruler and the tape measure were listed as tools, the class decided that they were used for measuring length and width. While this is quite correct, the children saw these two ‘dimensions’ as quite different ‘attributes’. Linda did not attempt to draw their attention to the idea that ‘length’ as an ‘attribute’ takes on various forms: length (the dimension), width, thickness, height and depth. Therefore, no time was spent actually exploring this attribute and helping the children to develop a meaningful understanding with Linda assisting the children to understand that rulers or tape measures can be used to measure these properties as various forms of length.

Another measuring tool that was listed was ‘cups of water’. While ‘cups’ may not usually be considered as a formal metric unit of capacity such as the litre or millilitre, when one considers the experiences of the children, particularly when cooking, the cup must be accepted as a tool. They experienced difficulty identifying the attribute that was being measured with the cup and could not go beyond saying, ‘how much’ and ‘cupful’. Linda introduced the term ‘capacity’ and it was interesting to note that she informed the class that it was a ‘tricky word’. This particular attribute had been the focus of Linda’s two previous lessons, so it was surprising that she did not consider it important for them to understand the attribute or to use the appropriate language as taught previously.

Despite her focus on capacity, at no stage during the entire sequence of lessons did Linda refer to the unit ‘millilitres’ to the class, only to ‘mils’, and as a
result, the children only used the term ‘mils’. While language can often prompt understanding, Linda failed to recognise the importance of language in this situation, and did not enable her students to develop understanding through use of appropriate language.

Linda did not appear to think mathematically while she was teaching. In one particular lesson when children had been filling a 600 ml milk bottle with teaspoons of water, they were instructed to count how many teaspoons and to ensure the teaspoon was always full. The groups reported counts of 204 and 215. The plastic teaspoons were 5 ml teaspoons (the researcher verified this empirically after the lesson). Given this and the knowledge that the milk bottle contained 600 ml, then a reasonable measurement would have been close to 120 teaspoons (i.e. 600 ÷ 5 = 120). When one considers that there are only 120 5-millilitre measures in a 600 ml milk bottle, then the children’s measurements have to be considered as quite inaccurate.

Linda did not appear to have anticipated the above calculation and did not detect the children’s inaccurate measures, suggesting that she had not thought about how many millilitres of water a teaspoon holds. Again, it needs to be emphasised that Linda’s main objective for this lesson was for the children to work with millilitres and to improve their understanding of this unit, yet it would seem that the children only consolidated measuring with an arbitrary unit—the teaspoon—and did not linked this to millilitres at all. If the focus was in fact millilitres, then it would have been essential that Linda link the measures ‘teaspoon’ and ‘millilitres’. An initial investigation as to how many millilitres make up a teaspoon would have been appropriate.
Linda’s knowledge of mathematics was limited to the extent that she did not accurately differentiate between standard units and arbitrary units, such as ‘teaspoons’ and ‘bottles’. These were never explained or linked to standard units. During a lesson where students were given many boxes of different sizes, Linda instructed the children to measure as many attributes as they could. She provided a variety of measuring equipment for the children to use, including rulers, tape measures, blocks and bottle tops. The mixture of equipment catered for measurements using both standard and arbitrary units. Linda demonstrated that she did not understand what constituted different attributes and their relationships. Figure 4.3 shows common attributes children should learn during their measurement experiences throughout primary school and illustrates that the attribute ‘length’ includes all properties of a shape or object that is measured with linear units.

![Diagram of Measurement Attributes]

Figure 4.3. Common measurement attributes.

Linda does not appear to understand the ‘double’ use of the term ‘length’ to both name the class of linear characteristics and the longest characteristic of an object, for example, the length of the table compared to its width or height. The
activity focused entirely on one attribute—length, and the different applications of that attribute. Linda saw ‘length’, ‘width’, ‘depth’, ‘height’ and ‘thickness’ as different attributes, rather than ‘length’, ‘area’, ‘volume’ and ‘mass’ being the different attributes.

This confusion over attributes was demonstrated in another lesson when Linda led a discussion about what various instruments measured. The interaction that took place about what a beaker measures is particularly interesting:

Linda: *What do we use beakers for, Ben?*

Ben: *Measuring.*

Linda: *Measuring what?*

Ben: *The depth of stuff.*

Linda: *Right. Beakers can measure how deep something is if you've got some water there.*

Ben’s initial response was quite vague—‘measuring’. When he was encouraged to provide more detail, he responded with ‘depth’ as an attribute. Linda’s response to this is quite surprising. She actually confirmed that the answer was correct and restated, ‘Beakers can measure how deep something is’. However, she clarified this with the condition, ‘if you’ve got some water there’. It is unclear what Linda was thinking at this time. In receiving a ‘correct’ response and agreeing with it, Linda then moved on to the next instrument. All discussion of beakers and the attribute they are used for ceased and this was clearly considered satisfactory.

It is a concern that the teacher did not appear to realise that beakers are used to measure capacity—the main focus of her lessons, providing evidence of her poor
understanding of the concept ‘capacity’ and how it may be estimated and measured. While many beakers are calibrated, the calibrations are not in millimetres or any other linear unit. The calibrations are in units measuring capacity, such as millilitres.

The point of clarification Linda made, ‘if you’ve got some water there’, would suggest she confused the use of a beaker and water to find the volume of irregular objects by the displacement method with measuring depth. Not only has she not recognised a measuring instrument used for capacity but she also failed to recognise that depth is merely another instance of the attribute ‘length’.

This discussion highlighted weaknesses in Linda’s mathematical knowledge and understandings. When Linda responded to the first question on the reflective questionnaire regarding her own knowledge of mathematics, she acknowledged that her mathematical knowledge was not strong. She wrote:

*I know I need to improve my mathematics knowledge. I got very nervous, as it got closer to teach these lessons, which is why I changed the first lesson to place value, because I had been working with that topic. The children knew what we were doing and I felt more comfortable. The researcher made me feel a bit better, so I gave the other lessons a go.*

This comment was made once Linda had been given two weeks to think about the experience of her lessons and this was the first time that she had explicitly admitted that she was aware that her knowledge needed to improve.

**Linda’s knowledge of students.**

Linda did not appear to know where her children were located in their mathematical development. She often mentioned that her activities were to determine what they knew. On more than one occasion, she stated that she did not know
whether the class had done any previous work on capacity. Further, she made many
assumptions about the mathematics children would have experienced at home, both
when she was talking to the group and in specific situations with individual children.

Based upon the fact that every home would have drink bottles, milk cartons
and the like, Linda assumed her students would have a familiarity regarding capacity
and the units for measuring capacity. Her intention was to build on this knowledge
and this assumed knowledge was referred to throughout one of the interviews:

*Researcher:* What experience have they had with measuring using millilitres
before? Any at all?

*Linda:* No. Not in our classroom this term.

*Researcher:* With the unit millilitres, have they been introduced to that before in a
previous year level, or have you done work with it before, or are you
slowly leading up to that?

*Linda:* I haven't done work with it. I'm not sure if each class has been through
that in previous years.

*Researcher:* What would be your assessment of the group?

*Linda:* I think they probably only know what mils are from their experience with
containers at home.

*Researcher:* What do you think their understanding of millilitres would be?

*Linda:* Oh well, they seem to know that it was a measure because they said that
the little bottle contains fifty ‘somethings’ and they probably equate that
to, you know, how we use centimetres to measure length.
Her responses were based upon some very tenuous assumptions. Linda believed that because objects with capacities that were measured in millilitres were readily available within the children’s home environment, they would understand the unit. She assumed that availability equated to knowledge.

While Linda generally related well with her children at a social level of interaction, she did not know their mathematical capabilities nor did she try to extend children to reach their potential. She had a combined Year Three and Four class and admitted that they mostly did the same work. Her reason for this was to avoid disruptive behaviour, explaining it was easier to keep the class under control in this way.

Prior to Lesson Four, Linda was questioned about her planning:

*Researcher:* What influenced you in the way you planned today’s lesson?

*Linda:* I wanted something a little more structured than normal.

*Researcher:* When you say more structured, what do you mean?

*Linda:* Well, basically, everyone is doing the same thing.

With her major planning strategy being to have all the children ‘doing the same thing’, it was clear that classroom management was a higher priority than encouraging all children to reach their potential.

It was not evident that she knew what understandings her Year Four students had developed that her Year Three students still needed to develop. Further, it was not evident that she had any awareness of what each group needed to build on their existing knowledge. During the interviews, Linda provided evidence that her
activities did not provide specific, clearly articulated mathematical objectives and that she was not always aware of the children’s previous experience:

Researcher: I’d like you to tell me what it was you set out to do and how well you think you achieved it.

Linda: Right, the water activity I had was mainly just working with Year Fours, so it was just to give them a go of what the Year Threes did last week, so it's basically the same objectives. And with the measuring of boxes, it was just to see if the children could think of as many different ways to measure them as they could.

Researcher: Right.

Linda: And, focusing on the capacity side of it.

Researcher: So what experience have they had previously with measuring volume or the capacity of the boxes?

Linda: Well, it's only measuring of the water into the bottles last week. Whether or not they've put objects into a box before and counted what can fit, I'm not sure.

Another example of Linda’s lack of awareness of the children’s prior knowledge was demonstrated during the final lesson of the sequence. She explained later that her final lesson dealt with area because she ‘realised there were gaps’ in this content:

Researcher: What were the gaps that you picked up? When you said you were working with volume and you said there were gaps, can you identify anything in particular?
Linda: They didn't really understand when I said you have to find the area first; they didn't know what area was.

Researcher: Did that surprise you?

Linda: For the Year Fours it did because I think according to what they should be up to. They should know what ‘area’ means. And even today, when I was questioning them first of all, they weren't sure exactly what it was.

Researcher: Is it something that you've done earlier in the year and they've forgotten or is it something you're assuming from documentation that they've done in previous years?

Linda: You know, we've talked about it informally, but I'm assuming that they, the Year Fours should have done something.

Linda could not say with certainty that her class had been taught about area in any formal way. Her reply that she was ‘assuming’ this described Linda’s approach to planning for her class. She appeared to have some knowledge of what students in Year Three and Year Four should be expected to have learnt and then assumed that her class would have covered this content. She justified these assumptions by reminding the researcher that this was her first year at this school and, therefore, she did not really know what had been taught to her class in previous years.

**Linda’s knowledge of teaching.**

Linda prepared each of the lessons for this study with accompanying notes for the researcher that gave brief overviews of the lessons, stating what the lesson was about and what she hoped to achieve. Following is an example:
Maths Lesson—Make a shake

This is the last activity in a series of work cards I have been using.

All the other activities have been run with the children working individually or in pairs. I left this activity until last for a whole class activity because it could be messy.

We haven't done any measuring of volume this term, so I'm using this lesson as an assessment of where the children are.

Other concepts covered: problem solving, multiplication and time.

Her planning revealed that she understood how to plan the organisational aspects of each lesson. She made decisions as to whether group work would be included and if so, she organised for parent helpers to be available. This was essential to Linda as she had poor behaviour-management strategies and needed parents to help supervise.

Her lack of mathematical knowledge noticeably affected her planning. The entire sequence of lessons was to be focused on measurement. Yet, the first lesson dealt with place value. The reason for the change was due to Linda lacking confidence in teaching activity-based measurement lessons. She admitted that she taught the place value lesson because the class contained quite a few children with behavioural problems and she believed they would be less trouble if they all did the same place value worksheet. This revealed another problem with her ability to plan for teaching; she found it difficult to judge the amount of work needed to engage the children for a full lesson. When asked:

Researcher:  How well do you think you achieved your planned outcomes?
Linda: *I think it went fairly well. Most of the children understood the tens and ones. However, as a lesson overall, a lot of them finished quicker than I anticipated.*

It was this inability to select appropriate activities that could fully engage the class that caused many of the behaviour problems. Linda generally selected activities that did not occupy all the time set aside for the lesson and since the content often lacked any real mathematical substance, the children finished early and became distracted. On most occasions when she had activities planned, she taught the same activity to Years Three and Four. Fear of ‘chaos’ was the basis of this decision. She explained this from the outset during the initial interview:

*I'll do the activity altogether, sometimes I know it'll be too easy for the Fours or too hard for the Threes, so I have separate activities, but generally, when I'm using hands-on material we all do the same thing so we're not in chaos.*

She demonstrated consistently that she did not know how to deal with problem children and, usually, once she reached a point of frustration, she would tell them to sit out on the veranda and not to come back until they were prepared to participate in a well-behaved manner. Another behaviour-management strategy Linda employed was to threaten and, in some instances, impose worksheets for them to do rather than participate in the hands-on activity.

She did not have a good knowledge of how to sequence and plan appropriate amounts of content for each lesson. All of her lessons contained minimal content and usually did not engage the children in legitimate mathematical activity.
Other Factors Impacting on Linda’s Knowledge

Linda’s beliefs about teaching mathematics.

Throughout the interviews, Linda expressed several of her beliefs about teaching mathematics. Linda emphasised her belief in the need to assess the children’s prior knowledge. This was considered important for student needs to be assessed and for appropriate teaching to take place, to avoid her class getting to upper primary with major deficits in their mathematical understandings. The previous year, she had taught a very weak Year Five and Six class. During the initial interview, Linda made the following comment: ‘I've had a Five/Six last year and they were very weak in some of the concepts, so I know that Three/Four is the place to get them’.

Linda believed that children gained familiarity with mathematical concepts from their home experiences in real life. Several times during the interview when asked whether she had taught aspects of capacity to this class before, or whether previous teachers had taught them, Linda gave responses like: ‘They probably know what mils are from their experience with containers at home’. As a result, Linda frequently assumed prior knowledge based upon this notion.

Linda believed that mathematics needed to be relevant and often her intention when teaching was to link the mathematical content to real-life situations. She selected activities that she believed children would enjoy. Examples of this are the lessons including making milkshakes and hands-on measuring activities using water, blocks and a range of other equipment. When asked by the researcher what she considered important about mathematics, Linda replied:

*Because it's a real-life skill and we use it all the time, I always try and integrate real-life aspects to it, not in every lesson but throughout the day.*
Even if we're ruling a margin or measuring something, I'll just point out that we are measuring, we are doing maths.

Linda believed that it was not only important that children see mathematics as relevant but that they also find it enjoyable. This relates back to her school experience. She claimed that she did not understand mathematics, particularly in her later high school years and she wanted children to enjoy mathematics and to see it as a useful subject. Her own experience at school led to a very strong belief that she did not want children to experience mathematics in a negative manner. She saw tedious pages of operational mathematics as unpleasant and consequently had a belief and a strong commitment to activity-based mathematics, so that her planning frequently focused on what she termed ‘hands-on’ activities. She was adamant in her belief that these needed to be fun and yet she still believed that the type of mathematics she had experienced in school had its place. She believed it would be a deterrent for bad behaviour and consequently used pages of operations as a punishment for children who disrupted her lessons.

Linda expressed her belief that hands-on activities were especially important, particularly if children were to fully understand. Several times during the interviews that followed her lessons, Linda expressed the view that because they had been involved in a hands-on lesson, she was quite confident that the children understood the mathematical content covered in the lesson.

Based upon the evidence of her lessons, Linda believed estimation was an important process. However, she believed it was quite acceptable for estimations to be ‘wild guesses’:
Well, pretend that you had fizzy drink in that carton. How many do you think you'd get out of it? Just have the wildest guess and write it down for me. And we might have a competition to see who gets closest.

A similar comment was made on another occasion:

Linda:  How did you work that out?

James:  It's just a guess.

Linda:  Okay, it's just a guess.

James:  This is embarrassing. I can't get it right. This is embarrassing.

Linda:  Just have a guess. Have the wildest guess, okay?

James:  I don't know how I got it. I just wrote the answer down.

Linda:  That's fine. It's just a guess.

At no stage did Linda model estimation in a more mathematical way or discuss with her class why estimation was important. She did not encourage the students to refine their estimations. Estimations were guesses with no apparent justification for why she was actually including them within her teaching.

Self-efficacy.

Two weeks after the teaching was completed, Linda rated herself against the five questions on the reflective questionnaire. Table 4.4 provides her ratings for the first five questions. From this, it is evident that Linda’s self-perceptions were not high. She rated herself as below average concerning her knowledge of mathematics and the emphasis she placed upon problem solving and average for all other questions.
Table 4.4
Linda’s Responses to the Final Reflective Questionnaire

<table>
<thead>
<tr>
<th>Question</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How would you rate your own knowledge of mathematics?</td>
<td>2</td>
</tr>
<tr>
<td>2. How would you rate the level of understanding achieved by the children in your class?</td>
<td>3</td>
</tr>
<tr>
<td>3. What emphasis did your lessons place upon relevance to real life?</td>
<td>3</td>
</tr>
<tr>
<td>4. What emphasis did your lessons place upon problem solving?</td>
<td>2</td>
</tr>
<tr>
<td>5. What emphasis did your lessons place upon estimation?</td>
<td>3</td>
</tr>
</tbody>
</table>

1=Poor 2=Below Average 3=Average 4=Above Average 5=Excellent

Linda provided an honest and insightful response to the question, ‘Are there any final comments you would like to make about your teaching of the sequence of measurement lessons?’:

Mathematics is the subject I find hardest to teach. If I get suggestions from other teachers, I usually try them. I changed Lesson Five because of an in-service course I went to the day before that lesson. They suggested an activity that I used and it worked well. Mathematics is the area I wish I had other teachers to plan with. As a new teacher, I find it difficult to plan sequences of lessons. I agreed to be involved in this research because I thought I might get ideas to help with my teaching. The discussions with the researcher were great. They did help me understand mathematics and teaching better.

This response was perhaps the most open that Linda had been throughout the observations of her teaching. Throughout the observed lessons, it was evident that she did not believe she was affecting student learning significantly. She stated on a
number of occasions that she was more focused upon avoiding ‘chaos’ and that maintaining control of the class was a higher priority than engaging the students in meaningful activities. She admitted that her Year Four students often did the same activity as her Year Three students so she could control the class more easily.

Overall, Linda was not confident in her ability to teach mathematics and this was evident from the outset of this study when she changed her agreed plan to teach a measurement lesson. Additionally, her final lesson was changed when she was shown an activity at an in-service course that she decided would be easier to implement and had been told by the person running the course that it was a good activity.

**Cultural context of the school.**

Linda taught in a school that was only a few years old, one situated in a newly developed area in the ACT, housing young families. It was not recognised as a high socioeconomic area and many children came from single parent families. The school had already established a reputation for having a significant number of children with behavioural problems.

Architecturally, it consisted of open area ‘pods’ that encouraged teachers to work together and share planning throughout much of the school. However, Linda was teaching her combined Year Three and Four class in a demountable classroom that was more traditional in that it was self-contained, placing Linda in an isolated teaching situation. She planned individually and did not have close contact with other staff members, other than during recess and lunch breaks. The isolation she experienced, despite the open plan of most of the school, enabled Linda to avoid pressure to adopt better professional practices, which was also exacerbated by the fact
this was her first year at this school. The culture of this school was significantly different to her previous experience.

Parent involvement was encouraged throughout the school and this was a successful part of the school’s operation. Most days, parents were present and assisted Linda in supervising students and helping to keep the children on-task. On occasions, more parents were present than Linda actually needed and it is questionable as to whether Linda dealt with this situation well. She often found it difficult to manage the children and having large numbers of parents present at times made her management issues more difficult.

Carla

Carla’s pedagogical content knowledge.

In the time Carla was observed, she taught a series of measurement lessons to Year Four with the emphases being on the attributes of time, length and capacity. After each lesson, she was given an opportunity to reflect and comment on the effectiveness of the lesson. During these interviews, she was asked to explain her intent for the lesson and her beliefs about aspects of teaching mathematics relevant to the lesson. Carla was also provided the opportunity to comment on these issues in the reflective questionnaire. Table 4.5 provides a summary of the topics Carla taught during the time she was observed.
Table 4.5
Summary of Carla’s Lessons

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Topic</th>
<th>Activity</th>
<th>Recording by children</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Quadrilaterals</td>
<td>Activity classifying different quadrilaterals. Emphasis on right angles.</td>
<td>Drawing shapes and recording the number of right angles.</td>
</tr>
<tr>
<td>2</td>
<td>Telling time using both digital and analogue means</td>
<td>Estimating time to perform certain activities and then measuring the time.</td>
<td>Recording results of each of the activities.</td>
</tr>
<tr>
<td>3</td>
<td>Timelines</td>
<td>Mapping their day on to a timeline.</td>
<td>Recording on the timeline.</td>
</tr>
<tr>
<td>4</td>
<td>Measuring the length of objects</td>
<td>Estimating and measuring the length of objects using both arbitrary and standard units.</td>
<td>Recording the estimate and actual measurement.</td>
</tr>
<tr>
<td>5</td>
<td>Measuring heights</td>
<td>Measuring their heights using streamer tape.</td>
<td>Making a class graph with strips of streamer tape.</td>
</tr>
<tr>
<td>6</td>
<td>Measuring with millimetres and centimetres</td>
<td>Activity estimating and measuring a variety of objects using centimetres and millimetres.</td>
<td>Recording estimates and actual measurements.</td>
</tr>
<tr>
<td>7</td>
<td>Measurement: length and capacity</td>
<td>Two activities involving estimating and measuring. One emphasised length while the other emphasised capacity.</td>
<td>Recording their results on a worksheet.</td>
</tr>
</tbody>
</table>

Throughout the interviews and her teaching, Carla demonstrated a sound knowledge in each of the three areas that follow. From the outset, Carla discussed what important teacher knowledge was when she first became a teacher. Reflecting on teaching when she first commenced, she stated:

*It was all to do with procedures and teaching and a bit of it was for yourself, your own personal development. Most of it was to do with classrooms, so that was quite good and looking at where children had problems. And I think that was really good, you know, to be able to pick up why children had problems and how to help.*
Carla, as an experienced teacher, considered she had a sound knowledge base but even though she had developed throughout her career, she was still heavily influenced by her pre-service preparation.

**Carla’s knowledge of mathematics.**

Carla was both confident and competent in her ability to understand mathematics. She had always loved the subject and had studied it to a higher level than most primary school teachers had. When asked about her pre-service preparation, particularly in respect to teaching mathematics, she responded:

*I can remember that probably better than I can remember any of my teaching. I think because it was such a good time of my life. We did maths every year. I did a three-year course. We had to do maths for three years and I majored in maths as well because I always liked maths and I thought it'd be an easy one to do. You had to have had HSC maths to do a major in it, so they assumed you could add and subtract, anyway.*

It was evident that Carla’s well-developed understanding and love of mathematics enabled her to design many of her own activities for teaching. Even when she modified existing activities, the modifications were based upon her understanding of the mathematics involved and her ability to identify weaknesses in the activity that would make it unsuitable for her class.

Carla recognised the importance of estimation as part of measurement. She stated that she liked to include estimation at least twice a week, as she believed estimation was a mathematical skill that children would take into the rest of their lives. Carla emphasised to her class that estimation involved ‘intelligent guesses’ that required using strategies that she taught the class. When she required her class to
estimate, she also required them to compare their estimates to the actual measurement and discussed how the students could refine their estimates, so that they were close to the measurement.

Throughout this study, Carla demonstrated a comprehensive understanding of many other topics in addition to measurement. During interviews, she often illustrated the point she was making by referring to other examples. She discussed place value, fractions, decimals, shapes, angles and number patterns when being interviewed. She also demonstrated an understanding of relationships between many of these topics, providing evidence of a sound, relational understanding.

Carla’s choice of activities demonstrated her knowledge of the mathematics that the children needed to learn. She used clapping and clicking fingers to help the children understand the notion of repeating a unit and counting to determine measurement. She used clocks and timelines to help her class develop an understanding of the many aspects of time.

Further, Carla’s knowledge of mathematics became evident when she discussed her planning. After Lesson Four, Carla discussed what she would teach next in the sequence of lessons. She emphasised the importance of several aspects of measurement:

*I think after that we’ll look at comparisons and the ordering of measurements.*

*Then, I think we will do some more work on reading scales, because even though they can measure accurately, they have to know how to read the scale.*

*Taking things back to zero all the time. And, after that, I think I’d like to go to longer measurements, after seeing them today. I think we might go out to the*
oval and measure things out there with the trundle wheel. We might do that next week.

These key ideas of comparison, ordering, reading scales with emphasis on where to commence the measurement reading and moving to measuring longer distances evidenced Carla’s thorough understanding of the essential components of the measurement process.

**Carla’s knowledge of students.**

Carla knew her class well. During interviews, she often commented on their ability and provided evidence of their prior knowledge and any gaps in their knowledge. Often, her comments went beyond measurement to the focus of the lessons being observed. During the initial interview, she discussed her students’ lack of understanding of basic facts and place value concepts:

*Carla:* They have trouble with tables and their addition and subtraction facts. They have a lot of trouble, even though they're Year Four, with regrouping and we've just had a focus on regrouping and not even when we're doing subtraction, but it's just regrouping numbers, and there's still a little cluster in there who have no idea of what they're doing. They don't really understand big numbers at all.

*Researcher:* What do you mean by big numbers?

*Carla:* Well, anything above a hundred is foreign to a lot of them. They don't understand, they just write them. You might as well write a million as write two hundred and something to those ones. So we spend a lot of time going back over those things. They do forget them very quickly. We've
spent time on place value and I know they still don't understand place value. So, that's something I keep going back to, doing those things all the time.

During teaching, Carla moved around the room and demonstrated an awareness of students who were likely to need help with misconceptions and errors. When children were measuring, she was observant of how they used equipment and whether they were reading the measurement units correctly. On one occasion when children were measuring lengths of streamer paper, which they had cut to be the same as their height, the following interaction took place:

**Carla:** How long is the tape?

**Child:** Twenty-six.

**Carla:** Twenty-six what though? Have a look again. Is it twenty-six centimetres?

Your ruler's thirty centimetres, so is his height smaller than your ruler?

What does this say? What does that stand for? One?

**Child:** One metre.

**Carla:** One metre, so it's one metre what?

**Child:** Twenty-six.

**Carla:** So how many centimetres altogether? How many centimetres in a metre?

**Child:** There's a hundred. 1.26 metres.

**Carla:** Good boy. That's very good.

On another occasion, Carla provided the class with lengths of string to measure. She wanted the measurements to be accurate and indicated during the
interview before the lesson that she anticipated some students might not be careful to keep the string straight. During the lesson, this did occur. Some children bent their tape measures around the shape of the string rather than hold the string straight. Carla was quick to notice this and through discussion with each of these children, helped them to understand how their measurement could be more accurate.

One further example demonstrated Carla’s awareness of how her students carried out their measurement tasks. This occurred during the lesson when they were measuring their heights with streamer tape. As well as measuring the length of tape, they were then required to glue their strips of tape onto a graph Carla had prepared and had pinned to a display board. Some children were not careful when gluing their strips of tape and did not align the bottom of their tape with the horizontal axis. Hence, the graph would have showed them as taller than their actual height if Carla had not anticipated this error. She was quick to detect these errors and have the children place their strip of tape correctly before the glue set. This created an important discussion, not only about the need for measuring accurately but also for representing information accurately when displaying it in graphical form.

At times, Carla used children who were more capable or who worked faster to assist children who were having problems. She was always able to identify a child who needed help and to address their needs either personally or by asking another child to help them:

*Carla:* It's seventy-seven. Go and write in seventy-seven centimetres. You finished them all?

*Child:* Yes.
Carla: You might be able to help someone that hasn't finished. Now where's Guy? He might need a bit of help.

Child: There he is.

Carla: Okay, you go and help Guy. Now, get Guy to measure these things, okay? So, you could take him and help. Perhaps if you go and measure him over here as well.

After each lesson when Carla was interviewed, she was often asked what problems the children had encountered during the lesson and what problems she had anticipated during her planning of the lesson. She was always able to identify the errors children had made. On one occasion, after children had been measuring with string, she identified the children who had not kept their string tight and straight. On another occasion, when the class was making a graph of their heights by gluing streamer tape onto a chart, she identified the inaccuracies caused by not sticking their tape on to the bottom axis. Overall, Carla demonstrated that she knew her class and their needs very well, both collectively and as individuals.

**Carla’s knowledge of teaching.**

Carla demonstrated a particular style of teaching that was teacher directed. She knew the curriculum thoroughly for the levels she was required to teach. Throughout the interviews, she gave a clear understanding of many planning issues, with particular emphasis on catering for all the students in her class. Carla was strongly committed to her directed approach. She expressed the view that children needed formal teaching and that it was the role of the teacher to plan carefully and provide structure to lessons so that children would learn. She made the following
observation during the initial interview about the approach used by other teachers in the school:

They have had four years of discovery and they haven’t discovered anything. They’ve had a wonderful time playing with equipment but nothing has ever happened. Their concepts and understandings haven’t developed at all. It is really directed now and this is how you do it.

When asked to clarify what she meant by ‘directed’, it was revealed that what she meant was that her role was to model concepts for the children using a variety of representations. She explained that with other teachers, the students had ‘been let go and no one has shown them how to use the materials. We still have the materials in front of us, but it is directed in how to use the materials’.

Carla was confident in her knowledge of teaching and her knowledge of how students learn, to the extent that most of her activities were self-designed rather than being dependent upon any set textbook. She spent considerable time designing activities for her class. During the initial interview, Carla revealed that she often worked late at night designing activities:

I often make games and activities for my class. I spent to midnight last night staying up making new games and I've got to get them laminated, because I want to get the kids working over there on problems that they have to solve.

On another occasion, Carla indicated that she mostly made her own worksheets to match the content she was teaching:
I do most of my own worksheets in anything I do. It's very seldom, unless it's really good, that I photocopy something out of a book and hand it to the kids. I usually change it around and get what I want that way.

In doing this, she felt she was able to match the activity more closely to the direct needs of the children in her class.

Carla demonstrated that her sound knowledge of mathematics influenced the teaching decisions she made on a daily basis. Her knowledge of the measurement process and all of the small steps that make up this process enabled her to sequence activities appropriately and to build on existing knowledge and experiences in a meaningful manner. She recognised the need to go beyond estimation and measuring. Comparing, ordering and appropriate reading of scales were all aspects that Carla included in her sequence to assist her class to develop their individual understanding and sense of measurement.

Other Factors Impacting on Carla’s Knowledge

Carla’s beliefs about teaching mathematics.

Throughout the interviews, Carla quite clearly expressed many of her beliefs about teaching mathematics. Carla made mention of children’s prior knowledge, stressing that it was important to build upon this knowledge and to plan according to their needs. She believed this made planning a complex task. To ensure all children were given the opportunity to develop mathematically, Carla stated that she often needed to teach topics she would not have seen as necessary, as she believed the children should already have been taught the relevant content in earlier years. Often, she spent much longer teaching a topic than she may have initially planned:
You can plan all you like, but really, the class determines where it goes to, and it might take you the rest of the year to get through what you planned because they come up with all these problems in the meantime that you don't see as a problem. But it is for the children, so you teach that.

Carla expressed her belief that mathematics should be enjoyable. She explained that she had children in her class who were not always successful, particularly when doing operations in mathematics. She wanted these children to find mathematics enjoyable, even if at times they experienced difficulty:

I try and keep the enjoyment level up in between when they don't have a lot of success in the everyday pluses and minuses, but in things like that it doesn't matter if they're right, they still enjoy it. I love maths and I keep saying to kids I think maths should be the best part of the day and I always try to get kids onto loving maths.

Carla believed that enjoyment was more important than children seeing mathematics as relevant to their lives. She believed relevance was not always easy to achieve, or to even determine if she had been successful. Her reason for this was evident during the interview after Lesson Four, when she said cryptically, ‘Our “real life” is not a child's “real life”’. She felt that relevance would mostly be realised once they became an adult. She questioned how much mathematics a child consciously used in their ‘real life’ outside school.

Carla believed that students needed exposure to a variety of modelling situations to develop an understanding of mathematical concepts, rather than being left to explore and play with equipment without being shown how it can represent
mathematical ideas. During the initial interview, she expressed her belief in directing the children when using equipment and demonstrating how to use it:

Researcher: Do you use materials still in your directed approach?

Carla: Oh yes, we do. Yes, we've got all the cubes and the blocks up there. We often, even when we do very directed things, we still have the stuff in front of us, but it's directed in how to use the equipment. They've been let go and no one's shown them how to use it before.

Carla’s belief in modelling was stressed quite emphatically a little later in the same interview:

*I think modelling, not just in maths but in language and in all areas is something that if children don’t see it modelled, they really don't know what to expect. I think that's one of the most important things a teacher does is model; I mean you model behaviour, you model your dress, you model everything.*

This need to model appropriate use of materials and language for the children was an issue Carla continued to stress during other interviews. After the first lesson while being interviewed, she became quite passionate about the issue:

*I think it's fairly sad when you get to be ten and you don't know a triangle from a rectangle. You can play with them for the rest of your life but I think it's time I told them the truth.*

Carla believed that problem solving was important from a mathematical perspective and that using carefully selected problems for children to solve was an effective way of providing enjoyment for the children as they experienced mathematics:
Problem solving is one of my areas that I like getting kids into and we've got a maths centre area set up there. I give them all sorts of different problems, because that's how I get them to enjoy maths. Otherwise, they can't see that maths can be fun.

Carla stated that she incorporated problem solving into her teaching in two key ways. She reported during the initial interview:

Sometimes problem solving can be for early finishers. We have cards that early finishers go to, and they are problem-solving cards. And the ones over there, I expect to rotate everyone around there so that everyone problem solves.

Carla believed problem-solving tasks served as an effective classroom management strategy. Early finishers were directed to engage in problem solving from a collection of problem-solving cards she provided. She believed that early finishers needed to engage in meaningful, mathematical enrichment and problem solving was the best way to ensure this occurred. However, she recognised that if this was the only way problem solving was experienced, some children would never engage in a problem-solving experience. Hence, she believed that at times, all students should participate in problem solving. To accommodate this, she had a second source of problem-solving cards.

Due to Carla’s belief that students were often allowed inconsequential ‘play’ with mathematics equipment, she took a controlled and teacher-directed approach to most of her lessons. She demonstrated processes and gave clear instructions. Throughout the lessons observed, the children were mostly given activities to
complete individually, after Carla had provided instruction. During some practical measuring activities, Carla did encourage the students to work with a partner.

While group work was not a prominent feature of Carla’s teaching, at no stage did she articulate a belief that there was no value in the role of group work. Neither did she articulate its virtues. Rather, she believed her input was the key to her students learning and that this was necessary to provide ‘structure’. During the interview after Lesson Four, she said:

When teaching measurement it needs to be structured, that's why I gave them that practice thing first. The aim was to run through that, and I'll probably do a bit more on that. It's to structure it, so they have definite things to measure and then compare them, but estimate them first. I think I'll do some more on that with millimetres and centimetres. They each need to have a go at it, following my directions.

As a result of this approach to teaching, Carla did not place any emphasis on children working in groups and even if she considered group work valuable, it was not as valued as the direct instruction she provided.

Estimation as part of the measurement process was important to Carla. She believed it was more than simply a pre-measurement activity that children needed to carry out. Her belief was that estimation was a critical skill that children needed to develop, which extended considerably further than their school activities:

I do a lot of estimation with these kids. I usually like to start off at least two lessons a week with estimation, because I think estimation is one thing you use for the rest of your life. Even if you can't add and subtract, you have to be able to estimate and come out with something.
Carla believed that teachers need to discuss children’s estimations with them and to determine how close the estimations were to the real measurement. It was important for children to think about what they needed to do to ensure that their estimates were becoming more accurate. She also thought that children needed to be taught to measure accurately. This was consistent with her belief that children need to have the use of equipment demonstrated to them. If they knew how to use the equipment, then she believed that they were more likely to measure accurately.

During the interview after Lesson Three, Carla reiterated this conviction:

*I think that's what I try to teach them, to get it through, that when you're measuring something we have to be accurate. We spoke about this yesterday as well, how we had to try to be accurate in our measurement.*

Overriding all other aspects of teaching measurement was Carla’s belief that she, as the teacher, needed to model, demonstrate and provide accurate and meaningful information. Only then would practice and personal involvement by the children result in meaningful learning. This belief influenced her use of concrete materials, as far as she would allow children to use them:

*Materials are an aid but they are not the only aid. I think they need a little bit of understanding before they're actually thrown in to use them. They don't even know what they're looking for half the time when they’ve got the materials. The modelling by the teacher first is so important.*

In essence, Carla believed that the role of materials reinforced understanding, but only once she had taught the class the concepts, using a directed approach.
Self-efficacy.

Two weeks after the teaching was completed, Carla rated herself against the five questions on the reflective questionnaire. Table 4.6 provides her ratings for the first five questions. From this, it is evident that Carla’s self-perceptions were quite high. She rated herself as average concerning her emphasis on real life and problem solving but above average for the other questions. Throughout the study, Carla consistently demonstrated a high level of confidence. She managed her class well and was aware that while other teachers in the same school found classroom management challenging, she did not share this problem. This ability to engage children and her considerable experience resulted in her self-belief as a teacher and belief that she was making a positive impact on her students’ learning.

Table 4.6
Carla’s Responses to the Final Reflective Questionnaire

<table>
<thead>
<tr>
<th>Question</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How would you rate your own knowledge of mathematics?</td>
<td>5</td>
</tr>
<tr>
<td>2. How would you rate the level of understanding achieved by the children in your class?</td>
<td>4</td>
</tr>
<tr>
<td>3. What emphasis did your lessons place upon relevance to real life?</td>
<td>3</td>
</tr>
<tr>
<td>4. What emphasis did your lessons place upon problem solving?</td>
<td>3</td>
</tr>
<tr>
<td>5. What emphasis did your lessons place upon estimation?</td>
<td>5</td>
</tr>
</tbody>
</table>

1=Poor 2=Below Average 3=Average 4=Above Average 5=Excellent

Cultural context of the school.

Carla taught in the same school as Linda. Like Linda, she was one of the teachers who were required to teach in one of the demountable classrooms. However, the demountable classroom she was in was considerably larger and was able to
accommodate two classes. Therefore, Carla was able to plan with the other teacher occupying the same demountable. She explained that normally, they would team teach and share the planning. During the lessons taught for this research, Carla and her colleague agreed to teach separately. This was not Carla’s preferred way of teaching. She stated from the outset, during the initial interview:

*I think maths is better when it is planned with more people. I’m not used to planning by myself because I plan everything else with others. I do not like planning by myself. It is too restrictive. I like being with other people and talking to them when planning.*

Carla had parent helpers on a regular basis, but managed the numbers of parents by encouraging a roster system that ensured she did not encounter the same problems as Linda, in sometimes having too many parents in the room. This would work against her whole approach of carefully structured teaching to maximise student understanding.

**Lois**

**Lois’ pedagogical content knowledge.**

The fourth teacher was Lois, who taught a series of measurement lessons to Year Four with the emphases being on the attributes of length, area and mass. During the interviews prior to teaching, she was asked to explain her intent for the lessons and her beliefs about aspects of teaching mathematics relevant to the lessons. After each lesson, she was given an opportunity to reflect and comment on the effectiveness of the lesson. Table 4.7 provides a summary of the topics Lois taught during the time she was observed.
Table 4.7  
*Summary of Lois’ Lessons*

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Topic</th>
<th>Activity</th>
<th>Recording by children</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><em>Making a metre ruler</em></td>
<td>Children made their own metre ruler from folded newspaper.</td>
<td>None, other than marking the scale on their ruler.</td>
</tr>
<tr>
<td>2</td>
<td><em>Using metre rulers</em></td>
<td>They used their ruler to measure objects within the classroom.</td>
<td>Recording their estimates and measurements.</td>
</tr>
<tr>
<td>3</td>
<td><em>Perimeter</em></td>
<td>Measuring the perimeter of various rectangles and squares.</td>
<td>Measuring the perimeter of shapes on a worksheet.</td>
</tr>
<tr>
<td>4</td>
<td>‘<em>Thing</em>: measuring length using both arbitrary and standard units’</td>
<td>Measuring a variety of dimensions of the picture of the stegosaurus called ‘<em>Thing</em>’.</td>
<td>Recording their measurements.</td>
</tr>
<tr>
<td>5</td>
<td><em>Measuring the area of ‘<em>Thing</em>’</em></td>
<td>Children measured the area of ‘<em>Thing</em>’ drawn on the carpet.</td>
<td>Children partitioned the drawing and counted MAB pieces to calculate area. Recorded total for each section and added to gain final area.</td>
</tr>
<tr>
<td>6</td>
<td><em>Measuring the mass of ‘<em>Thing</em>’</em></td>
<td>Children made their own ‘<em>Thing</em>’ by making play dough animals.</td>
<td>No recording.</td>
</tr>
</tbody>
</table>

Lois was an experienced teacher who demonstrated a sound knowledge of teaching. Her classroom management was such that she rarely experienced any serious behaviour problems. She had a good rapport with her class and knew each of her children well enough to be aware of their individual needs.

**Lois’ knowledge of mathematics.**

Lois saw mathematics as a means of communication. She emphasised the importance of mathematical language for children so that they could discuss the
subject using appropriate terms. After Lesson Two, Lois made the following comments:

Language is so important for concept knowledge and development. It comes right back to introducing the terminology and linking it to models and other representations of the concepts. That was further highlighted this week when we did work on perimeters. Once again, the terminology I used was linked to the shapes I have drawn and we were talking about boundaries and perimeters. Today wasn't just that I'd gone from last week's onto this week's. I've actually been following through all week.

This last point made by Lois emphasised that the importance she placed upon language extended beyond the lessons observed. She had been working all week emphasising language and encouraging the children to learn and use it appropriately.

Lois also saw mathematics as connections and relationships. She emphasised to her class quite frequently the importance of showing relationships and making connections, whenever possible. During the lesson when the children were measuring the area of ‘Thing’, Lois explored the relationship between a square metre and multi-base arithmetic blocks (MAB) flats. This enabled the children to cover ‘Thing’ with flats and make the appropriate conversion to square metres. The establishment of this relationship with the children demonstrated that Lois herself had the knowledge required to plan this kind of activity initially.

Lois’ belief in establishing connections and relationships is based upon a mathematical knowledge base of her own that initially appeared quite sound. However, there were instances during the observed lessons where Lois demonstrated gaps in her knowledge of mathematics. Two such incidents were associated with
teaching the class about the importance of selecting appropriate units for measurement. Lois only emphasised a single criterion for selecting appropriate units. She taught her class that small units were used for small objects and large units for large objects. During Lesson One, the following interaction took place:

Lois:  *When would I use millimetres?*

Child:  *When you’re measuring something short.*

Lois:  *When I’m measuring something short or something ... ?*

Child:  *Small.*


Child:  *Tiny.*

Lois:  *Tiny, okay. When might I measure in kilometres?*

Child:  *When you’re measuring the road.*

Child:  *Something real big.*

Lois:  *Something really, really big.*

At no stage in this discussion did Lois suggest that the reason for selecting a particular unit could also be determined by the level of accuracy that is required when measuring. This lack of emphasis on the need for accuracy, being an important consideration when selecting appropriate units, was further evident in Lesson Five. During the introduction of this lesson, Lois quickly revised the concept of mass. She went through the relationship between grams and kilograms with the class. She asked the children what kinds of things they might measure in grams and what objects they could see around the room that could be measured in kilograms. The discussion that
took place was similar in drawing the conclusion that, ‘light objects are measured in grams and heavy objects are measured in kilograms’.

There were other instances during her teaching where Lois demonstrated that although her knowledge was certain, it was habituated. She easily slipped into incorrect usage of terms. When teaching ‘mass’, Lois was emphatic to the class that ‘mass’ was the correct term and not ‘weight’. Analysis of the transcripts revealed that she used the terms ‘weight’ and ‘weigh’ considerably more than she used the term ‘mass’.

Lois was largely reliant on her own instrumental learning. Although from a pedagogical perspective she understood that there was a difference between instrumental and relational understanding, her own mathematical understanding was lacking deep relationships and connections. In fact, sometimes confusion resulted because of her own misunderstandings. There was a teaching incident when Lois wrote ‘200 mm = 10 cm’ on the board. Lois had been assisting the class to determine how many millimetres in a metre. She used a metre ruler and had the class count in tens along the scale of the ruler. The problem occurred when she counted ten for every half-centimetre. One would expect that Lois would have realised the error and self-corrected. When the class reached 20 cm on the ruler, Lois wrote ‘200 mm = 20 cm’ on the board, but due to counting ten for every half-centimetre, a child corrected her with, ‘That’s supposed to be four hundred’. She then erased the ‘200’ and replaced it with ‘400’.

The lesson dealing with mass contained another example of Lois’ weaknesses in mathematical knowledge. She had focused on mass for the first 20 minutes of the lesson. The activity associated with this lesson was for the children to make their own
'Thing’ using a recipe for play dough. At no stage did Lois’ language indicate that the lesson focus had shifted from mass, yet the activity required the children to use a dessertspoon as their unit for measuring the flour. Without any explanation or apparent acknowledgement that she was aware that she had done so, Lois changed the focus suddenly to the attribute of capacity/volume. One minute the children had been hefting and feeling, the next, Lois asked them to estimate how many level dessertspoons of flour would be required to fill a cup, underscoring that they must estimate first. It is unclear how Lois associated this estimation with the first 20 minutes of the lesson.

It was unsurprising that a girl from one group called out: ‘Mrs __, Mrs __, we don’t understand’. The children had returned to their desks for the group activity following the introductory 20 minutes where the focus had been on using units appropriate for measuring mass. It is likely that their previous school experiences had taught them that when a concept is introduced in a lesson, the activity usually consolidates the concept. Yet, they found they were asked to do an activity that did not appear to be related to the concept ‘mass’ but involved volume/capacity.

While Lois was clearly an experienced teacher, the gaps in her own mathematical knowledge base and the dependence on predominantly instrumental-type knowledge affected her teaching. There were times when her class was confused and times when Lois herself appeared confused. Therefore, while her knowledge of teaching may have been sound and her knowledge of her students was good, the actual teaching of these measurement lessons was negatively affected by poor mathematical knowledge.
**Lois’ knowledge of students.**

Lois demonstrated a good knowledge of her students. She set down clear expectations that she expected to see them adhere to when in her class. These expectations related to behavioural standards as well as engagement in learning. During the initial interview, she made it clear that she expected them to try, even if they made mistakes:

*Researcher:* You want them to learn from your attitude to them, that when they make mistakes that's part of the learning experience?

*Lois:* Oh very much so, and I tell them that quite often. They will say to you, ‘She hates work that's not neat, but making a mistake, providing we've tried, is nothing to be fearful of’.

Lois knew her students well enough that she was aware when to intervene and when to leave them to explore a task further by themselves. She anticipated the kinds of errors students were likely to make and often pre-empted these situations by asking questions or making changes to her planned lesson.

During one teaching episode when the children were measuring perimeters using millimetres and metres, Lois had prepared two worksheets for the children to complete. She turned to the researcher and stated: ‘I'm going to change that now; I think I'll just stick to this for now. Otherwise, I'll confuse them’. Then she turned back to the class and said: ‘Let me see how quietly you can go back to your seats. Only work through sheet one. You may work with a partner quietly’. Lois read the reactions of her class to the tasks and realised that it would be more beneficial to complete one worksheet thoroughly than to attempt both sheets and risk some children not fully understanding what was required of them.
During another interaction with a group of children, Lois sensed they were experiencing difficulty measuring with their thirty-centimetre rulers. The confusion was being caused by the half-centimetre markings along their ruler. Instead of centimetres and millimetres marked on their ruler, they only had centimetre and half-centimetre markings. The following interaction took place:

Lois:  Oh, here, how long's this one?

Child:  That's eight.

Lois:  No, it's?

Child:  Eight and a half.

Lois:  No, it's?

Child:  Seven.

Lois:  Seven and a half. Okay. Be careful of those half strokes, okay? Be very careful of the half strokes.

Child:  Seven and a half?

Lois:  Yes, because it has gone past the seven but has not yet reached the eight. It's seven and a half. You're doing an excellent job, but don't go too fast so that you make mistakes.

Lois’ repeated reply of ‘No, it's?’ was intended to make the children think and to make them look at the ruler and try to interpret the scale more meaningfully. Throughout her teaching, Lois made frequent such attempts to encourage the children to think and to avoid making errors. However, better verbal cues might have been,
‘Look more carefully at the units’. The pedagogical precision of her language mediating instructions rests on her prior mathematical understanding.

**Lois’ knowledge of teaching.**

Lois planned many of her own activities rather than relying upon textbooks and planned lessons to maximise the opportunity for the class to develop a relational understanding of mathematics. She endeavoured to design activities that covered more than one specific concept. During the initial interview, she stated, ‘One of the things that I place priority on, is I’ve always been really keen on trying to get a lot of “mileage” out of the one activity that I’m doing’. The series of lessons Lois planned around ‘Thing’ aimed at doing just that. She planned for the children to further their understanding and consolidate their skills at measuring length, area and mass using a variety of different resources.

Lois had knowledge of structuring learning to challenge children, not just by appropriate activities, but also by using questioning effectively. When appropriate, she would intervene and ask questions that would redirect their thinking or challenge what they were doing. When a group of children were covering the picture of ‘Thing’ on the floor with MAB, Lois ensured they understood what they were doing and that they understood the relationships between the different pieces. The following interaction took place:

*Lois:*  
*You would count them. Which ones do you think you would count first?*

*Girl:*  
*The big ones.*

*Lois:*  
*The big ones. Which ones do you think you would count next?*

*Girl:*  
*These ones.*
Lois: And how many of these would you need to make up one of the big ones?

Girl: Ten.

Lois: Ten. Then what would you count next?

Girl: The ones.

Lois: And how many of those would you need to make...?

Girl: A hundred.

Lois: You would need a hundred.

Once Lois was satisfied that the group were both covering the shape totally and were aware of how to count the blocks in a meaningful way, she moved on to others in the room.

Lois’ approach to teaching was based upon her knowledge of how children learn. She recognised that you cannot teach a concept once and hope that it has been understood and planned her teaching on the principle that children need to revisit concepts for them to be understood and retained. After Lesson Five, she summed up her teaching by stating:

_I always come back and make sure that the relationship is learnt. That it's not a hit and miss thing, like I've kept coming back. I can’t just stop and say I've done my measurement, and I've done my mass, and I've done length and that's it for the year. Now I need to keep coming back and tying in new things for models, to keep that visual image going, because I think that is really important. Not just the visual image, but also the felt image, when you're talking about mass as well and when you're talking about liquid measure._
Lois linked previously learnt concepts to new ones and demonstrated throughout her lessons that her beliefs about teaching were matched with a sound knowledge of teaching principles and strategies.

**Other Factors Impacting on Lois’ Knowledge**

**Lois’ beliefs about teaching mathematics.**

Throughout the interviews, Lois expressed quite clearly many of her beliefs about teaching mathematics. She expressed her concern from the first interview that the children in her class were not very confident in their approach to mathematics. Lois stated that often the children’s own perceptions were that they were quite good at mathematics, though this was only in a narrow computational sense:

> And I don't think it matters how experienced you are, because there's a lot of work that needs to be done. In fact, if you were to look at the way that they tackle each bit of apparatus, and that happened with the place value work that I've been doing, and number patterning work, each of the aspects in fact, has shown me that there are grave problems.

She clearly believed that the children’s prior knowledge in measurement was poor and that this was a problem across most areas of mathematics. However, Lois also admitted that at times she did not always take this lack of prior knowledge into account when actually planning a lesson and found herself making assumptions that were inaccurate. After the second lesson, she made the following comments:

> When it comes to their concept knowledge and development, I think about what I have previously done before with fourth grade, and even the making of that ruler, folding along a diagonal. I had just assumed. It comes right back to the thing of just assuming and forgetting that you're with a group. I just
thought that I need to go back. Even from the language last week, I knew that I had lost them, because I had assumed that they would know some of the key terms, and they didn’t. I’ve actually been following through all week trying to teach areas that initially I had just assumed they should know.

Lois believed that teachers should determine the children’s prior knowledge and that new content needed to be linked to existing knowledge. Yet, she admitted that in her planning she often misjudged their level of prior knowledge and as a result, had to spend considerable time teaching content she had assumed they knew.

Lois expressed mixed beliefs about the need for mathematics to be relevant and enjoyable. She certainly acknowledged that mathematics was a subject that was necessary in preparing students for life. During her initial interview, she stated:

*It's a very necessary part of life. I mean, it's necessary not in just everyday life but it's a necessary part of school they need for the workplace and throughout education, all vocational or career options. It is one of the things that can very much limit children, if they do not experience success in mathematics.*

Though Lois believed that to help children see the relevance of mathematics they needed to enjoy the subject, she appeared to have mixed feelings about this:

*Well, one of the important things is, it will appear more relevant to them if you really try and get them to enjoy maths. At the same time, I'm not a great believer in life is one ‘turned on’ event. Sometimes you just have to get on and do things, even if you are not enjoying it.*

Lois took the view that mathematics was necessary and fundamentally, its importance should be reason enough for them to work hard and learn the content of
the subject. In another interview (Lesson Two), Lois reiterated this point. She stressed that it is the importance that the teacher places on the subject and the teacher’s function as a role model that helps the children also view it as important:

*I know that for kids, it's really hard for them to focus on mathematics, particularly if they are finding it difficult. I think I try and give meaning and emphasise the importance of the subject. I think that children very much can pick up an empathy with the importance that you place on a subject and the way you act as a model, in the way that you react, or the way that you think about things. I think that, as a teacher, that's really where my role can help. If children see that I think it is important, then they are more likely to also believe it is important.*

Clearly, she was realistic about the number of children who experienced difficulty with mathematics, but held the belief that a teacher who acted as a positive role model could influence them to success and enjoyment.

Lois was well aware of the distinction between instrumental understanding and relational understanding and used this language on several occasions:

*There is a concern, from my perceptions that I’ve had from the kids coming through, that gets back to instrumental teaching. Although people would purport to do a lot of concrete work, what is happening, I think, is that the links are not being made between the related concepts. They are not learning in a relational way. Concepts are still isolated.*
Lois believed that relational understanding was dependent upon helping children establish an understanding of relationships and it was important for them to be able to visualise. After Lesson Four, she made the following comment:

That lesson had a lot to it. It was interesting. The main problem was that in the problem context, they didn't automatically switch on to the tens rule. It comes back to instrumental teaching doesn't it? Not visualising.

To help children visualise, Lois believed that modelling and having them construct models was important. Modelling is an important form of representation. Lois also made the following comments after Lesson Five:

When I selected the activities, I tried, as far as possible, to ensure that they made appropriate links to other related concepts. I believe that there comes a point where you can model, and you can have children creating models, yet there comes a point that there is a discipline entailed in there. Some may say that is almost pre-empting back to an instrumental approach. But I think there is a point with some children within teaching, where you can select activities that in fact accomplish nothing unless there is this focusing and a real discipline of learning. This discipline needs a considerable teacher-directed approach, rather than just letting children investigate on their own.

Here Lois, once again, acknowledged the distinction between instrumental and relational learning. While she uses the language of Skemp (1976), she argues that teachers need to take control of the teaching situation in a direct way. This teacher-directed approach needs to include providing appropriate models as representations of key concepts, as well as having children create models. While in her mind this leads
to relational understanding for the children, she thinks others may see this as just another form of instrumental teaching, due to the teacher-directed nature of the interactions.

Lois believed that problem-solving skills were an important part of mathematics. She did not often discuss them explicitly, although on a few occasions she was quite emphatic about their importance. During the initial interview, she stated:

*Problem solving is often tied up in much of what I do with the children. We have looked at a lot of strategies and often they need to use some of these, like draw a picture or create a list. It’s really important that they can use these strategies.*

Following Lesson Three, Lois stated that the nature of the activity that required the children to select dimensions of ‘Thing’ and to measure them with appropriate units involved ‘inherent problem solving’. She indicated that having the children select appropriate units from either arbitrary units or standard units involved ‘a lot of inherent problem solving’. This was made explicit one final time with the following comment, made in a response to the reflective questionnaire:

*Problem solving is one way that children can demonstrate relational understanding because they need to know which information to draw on to help them solve the problem.*

This knowledge of what information to draw on was consistent with Lois’ belief that relational understanding was dependent upon seeing the connections between concepts. She believed that group work was an important part of her
teaching and subsequently planned activities that required the children to work together. Lois explained that this was particularly important because there were several students with specific learning needs in her class. These children were, at times, withdrawn from the main classroom for individual assistance. However, during the sequence of lessons taught for this research, these children remained with the class. Lois explained that how she grouped the class was important. She stated:

I need to ensure they are in a group where the learning of the others in that group will not be affected. At times they can distract other children, but there are quite a few children who accept them and help them. They benefit from working with the others but I have to be careful how I structure the groups.

Lois did not offer any other justification or explanation for why she believed groups to be important. However, she did use group activities extensively in her teaching. A significant comment during the interview after Lesson Four provided an insight into how Lois linked group activities to whole class instruction. She felt it was more efficient to deal with problems collectively rather than moving around to each group and dealing with the same problems repeatedly:

Often, when I’m moving around and working with a particular group, I realise there is a problem. Like today, I had been working with one group, and they were having a problem using the equipment. It’s better for the whole class to have the experience if you can collect them all together because then you’re coming from the same point all the way through. Then they can go back to their group work.
Lois saw measurement as a central component of mathematics. She believed that through measurement there were many connections to other topics in mathematics. Lois made this point from the outset. After the first lesson, she stated:

*I like to teach measurement, particularly formal measurements, so that it connects to decimals. Then, recording can often relate to money. I wanted to also use it for fractions work. Once again, it gets back to this relational type teaching. And we've also been doing work on place value involving hundreds, so this relates to a hundred centimetres—all of that lesson fitted in with these relationships.*

As well as the connections to other mathematical ideas, Lois believed strongly that her class needed to understand the relationships between the various units of measurement. In one lesson, she spent considerable time establishing the relationship between millimetres and metres. In another, she focused on grams and kilograms. As Lois stated in her final interview:

*It is really important, in all of the lessons on measurement, that the children can see all the different units and for them to know how many of one is needed to make the other. They need to be able to make fairly simple conversions from one unit to another.*

Lois always took time to introduce to the children the measuring equipment that was to be used for each lesson, believing that if children were to learn to measure accurately, they needed to know how to use the equipment. This was particularly important if equipment was being used in an unusual way. This was evident during the lesson in which the children were measuring ‘Thing’. When measuring the area of
the chalk picture of ‘Thing’ that Lois had drawn on the carpet, Lois supplied boxes of MAB for the children to use. As MAB were normally used for place value and operation type activities, Lois spent time with the class ‘mapping’ MAB flats onto a square metre so that the children would understand how many flats they would need to make a square metre.

**Self-efficacy.**

Two weeks after the teaching was completed, Lois rated herself against the five questions on the reflective questionnaire. Table 4.8 provides her ratings for the first five questions. From this, it is evident that Lois’ self-efficacy was quite high. She rated herself as average concerning her knowledge of mathematics but above average for all other questions. This high self-efficacy was evident in the confident manner in which she conducted her lessons. She continually demonstrated a real enthusiasm for teaching and a belief in herself as an effective teacher. Her high level of self-efficacy enabled her to be creative with her planning. She did not need to be reliant on textbooks or worksheets. Creation of lessons around ‘Thing’ resulted from her belief in herself and her ability to design effective and engaging activities.
Table 4.8
*Lois’ Responses to the Final Reflective Questionnaire*

<table>
<thead>
<tr>
<th>Question</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How would you rate your own knowledge of mathematics?</td>
<td>3</td>
</tr>
<tr>
<td>2. How would you rate the level of understanding achieved by the children in your class?</td>
<td>4</td>
</tr>
<tr>
<td>3. What emphasis did your lessons place upon relevance to real life?</td>
<td>4</td>
</tr>
<tr>
<td>4. What emphasis did your lessons place upon problem solving?</td>
<td>4</td>
</tr>
<tr>
<td>5. What emphasis did your lessons place upon estimation?</td>
<td>4</td>
</tr>
</tbody>
</table>

1=Poor 2=Below Average 3=Average 4=Above Average 5=Excellent

**Cultural context of the school.**

Lois was a senior teacher in her school and had responsibility as team leader for supervising other teachers responsible for Years Three and Four. She was responsible for both Carla and Linda. Interestingly, Lois believed she provided leadership, although the other two teachers did not support this view.

Lois planned separately from other teachers in her unit. This was of particular interest as Lois was teaching in one of the open area ‘pods’ within the school. This school claimed that they utilised the open area architecture for team teaching and for sharing planning responsibilities. She had a designated area of her ‘pod’, which was separated from two other teachers in the same ‘pod’ by portable partitions. These partitions provided areas to display children’s work, yet, at the same time, they served in providing Lois with a more private and separate teaching area. In the interview after Lesson Five, Lois stated:

*Well, I tend to very much rely on having taught for so long. I've always been a little bit inclined to look at resources and then tend to formulate what I want*
to do. I find planning my own work means that I can do exactly what I think is important. I've got my objectives and rather than trying to modify from somebody else’s, I find it easier to just do the thing myself.

Lois preferred a more traditional approach to teaching where teachers took responsibility for their own class. While this was her preferred style of teaching, there certainly were teachers in other ‘pods’ who shared planning and engaged in team teaching. Even though the school advocated a more open philosophy of teaching, it allowed for teachers who preferred to teach on their own to do so. Lois was one of these teachers.

**Ranking of Each Teacher’s Knowledge Type**

A summary of the results reported in this chapter in terms of whether each teacher’s knowledge is weak or strong for each type of knowledge is presented in Table 4.9.

Clearly, no two teachers will have the same knowledge and there will always be degrees of knowledge. Skemp (1978), in distinguishing between relational understanding and instrumental understanding, helped to clarify further the differences from one individual’s knowledge base to that of another. The classification of teachers as weak or strong is not an attempt to generalise or describe their knowledge beyond the scope of this study.
Table 4.9
Classification of Depth of Each Teacher’s Knowledge Type

<table>
<thead>
<tr>
<th>Knowledge type</th>
<th>Knowledge of teaching</th>
<th>Knowledge of mathematics</th>
<th>Knowledge of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colin</td>
<td>Developing</td>
<td>Weak</td>
<td>Developing</td>
</tr>
<tr>
<td>Linda</td>
<td>Weak</td>
<td>Weak</td>
<td>Weak</td>
</tr>
<tr>
<td>Carla</td>
<td>Strong</td>
<td>Strong</td>
<td>Strong</td>
</tr>
<tr>
<td>Lois</td>
<td>Strong</td>
<td>Weak</td>
<td>Strong</td>
</tr>
</tbody>
</table>

Weak knowledge does not mean no knowledge. In the case of the two novice teachers, it is quite likely their knowledge in each area was developing and as they gain classroom experience, their knowledge could be expected to improve. For this reason, Colin has been described as having ‘developing’ knowledge of teaching and of students. The evidence within this chapter suggests that he has some knowledge in each of these areas but as a beginning teacher, he clearly does not have strong knowledge, as in the case of the more experienced teachers.

Throughout this study, there was sufficient evidence as reported within this chapter to indicate each teacher’s weaknesses and strengths in each of the three knowledge types identified as essential to pedagogical content knowledge.

Conclusion

This chapter presented the results of each of the four case studies. The results presented have been based upon observations of a series of each of the teacher’s lessons, interviews associated with each lesson and a reflective questionnaire completed two weeks after their teaching concluded. In presenting the results in this chapter, the model was applied vigorously and each of the three knowledge
categories, knowledge of mathematics, knowledge of students and knowledge of teaching, was analysed. The ability to provide detailed and rich knowledge descriptions of each participant’s knowledge in these areas resulted from the identification of key incidents and first-writing vignettes that captured the essence of the critical incident. These vignettes then acted as a secondary data source to assist in reporting the results, so that the research questions were carefully examined.

Documenting these results for each teacher has been essential to enable meaningful interpretation and analysis to be presented in the next chapter. The following chapter will examine these findings and provide a cross-case analysis of the results. The documented findings within this chapter will be analysed further in an attempt to provide answers to the research questions. In particular, the implications on pedagogical content knowledge will be analysed when deficits in one or more type of knowledge exist.
Chapter Five: Analysis of the Model

The previous chapter applied the researcher’s model for pedagogical content knowledge to each of the teachers by examining their mathematical knowledge, knowledge of students and knowledge of teaching. Discussion of each teacher in terms of their beliefs, their self-efficacy and the cultural context of the school provided additional information about factors that impacted on their pedagogical content knowledge. This chapter will provide further cross-case analysis (Yin, 2009) through an extended discussion of pedagogical content knowledge as it was revealed in the four teachers’ varying experience and competence.

Chapter Two discussed Shulman’s notion that pedagogical content knowledge needs to be documented and through extensive case studies, an understanding of the nature of pedagogical content knowledge needs to be built. The following researcher-designed model is presented to demonstrate what is meant by pedagogical content knowledge.
Figure 5.1. Researcher’s model of the three essential components of a teacher's knowledge base when teaching mathematics.

Within this model, the representation of each body of knowledge by congruent circles is significant. The use of congruent circles implies that, ideally, each body of knowledge is equally important and that extensive knowledge in each of the three areas is essential to maximise the area of the three overlapping circles, representing pedagogical content knowledge. Clearly, when each of these three types of knowledge is strong, the pedagogical content knowledge any teacher may possess is also strengthened. This is consistent with what Shulman (1987) suggests is required for effective teaching.

However, as the evidence from Chapter Four demonstrates, not all teachers possess rich pedagogical content knowledge. Of the four teachers studied, only one, Carla, demonstrated a depth of understanding in all three areas resulting in strong pedagogical content knowledge. Each of the other three teachers demonstrated
limitations in one or more of the three areas, limitations that, in principle, require action to improve.

This chapter now examines the variations in pedagogical content knowledge among the four teachers. These are variations within the structural elements of the model caused by limitations in one or more areas of knowledge. It will be shown how the model provides explanatory value as it enables the examination of variations in each knowledge type, and hence, variations in pedagogical content knowledge. Changes in the richness of any of the three knowledge types possessed by teachers will be viewed as changes in the structural elements of the model. The following three sections of this chapter will deal in turn with variations in the form of limitations for each of the three types of knowledge required by teachers. The final question examines other factors impacting on how these teachers use their knowledge.

**Impact of Variations in Teachers’ Knowledge of Mathematics**

Each of the four teachers revealed insights into their mathematical knowledge through their teaching and interview responses. There was considerable variation among the four teachers and of the four, only Carla demonstrated strong mathematical knowledge, while the other three had major weaknesses.

To analyse the results reported in Chapter Four, an interpretive framework was designed to examine the effect of limited mathematical knowledge. Figure 5.2 summarises the major kinds of evidence of weaknesses in mathematical knowledge and the associated implications on the teachers’ pedagogical content knowledge.
There were three main ways these teachers demonstrated weak mathematical knowledge:

1. making errors that were not self-corrected
2. showing a lack of understanding of mathematical language
3. avoiding teaching concepts due to lack of understanding.

These three problem areas resulted in less effective teaching and could be considered as having impacted upon each teacher’s pedagogical content knowledge.

This impact was notable in four distinct areas, highlighted in Figure 5.2:

1. children’s learning
2. teachers’ planning
3. teachers’ use of representations and how they incorporated those representations into their lessons

4. teachers’ ability to make accurate judgments.

This interpretive framework will form the basis of the discussion that follows. The impact on teachers’ pedagogical content knowledge is supported throughout this discussion from the evidence reported in Chapter Five and summarised in Figure 5.2.

**Errors that were not self-corrected.**

Firstly, teachers made errors when teaching that went beyond accidental errors that were self-corrected. These errors resulted in confusion among their students. The one area in which Lois showed a weakness was her own mathematical knowledge. The previous chapter documented a teaching incident in which Lois taught the class that there were 200 millimetres in ten centimetres. After some time, Lois did realise an error had been made and corrected the error due to questions by a student. It could be argued that this was nothing more than a minor error made by Lois that was quickly corrected. This would not be an unreasonable interpretation, in that Lois did correct herself. In the course of a day, teachers are bound to make small mistakes like this, and as long as they correct them, no harm is done. Conversely, one can ask how a teacher of mathematics can reach the stage of writing ‘200 mm = 10 cm’ without realising the error. Therefore, it raises questions about Lois’ internal model of a centimetre. The ruler used was marked at half-centimetre intervals and the markings made this quite clear.

While Lois’ lessons were interesting and learning did take place, opportunities were not always maximised due to her own lack of understanding. It was clear that Lois’ knowledge of linear measurement was predominantly instrumental and not
relational. She knew the facts but did not have a good understanding of the size of a centimetre. While she initially trusted her visual judgment and her belief about her understanding of the size of centimetres, she came to a situation where this conflicted with her memorised facts. Once this conflict was evident, she was forced to reassess her initial judgment and was then able to resolve the situation successfully, not without causing confusion among her students.

Another teaching moment for Lois that illustrated her instrumental understanding was in discussing with her class when a unit is appropriate. She focused only on the idea that if the object being measured is small, then a small unit is required. Conversely, when an object is large, then a large unit is needed. While this may sometimes be true, another important consideration that must be made when determining the appropriateness of a unit is the precision or accuracy required. It is almost certain that Lois realised the need for accuracy in many life situations, yet when teaching measurement to her class, she did not emphasise this aspect.

A lack of understanding of mathematical language.

Secondly, a lack of understanding of mathematical language and concepts emerged as a concern regarding the teachers’ knowledge of mathematics. Colin demonstrated confusion over the use of the terms ‘mass’ and ‘weight’ despite his belief in modelling correct language. His usage of the language was mixed and he modelled the language as if they were synonymous.

This distinction is technically real and easily defined. However, it must be acknowledged that the distinction becomes more clouded when one looks at it from the perspective of societal use of language. It is common to hear people use the terms
‘weighing’, ‘weight’ and ‘weigh’. Organisations such as Weight Watchers exist based on this commonly accepted misuse of mathematical language.

In the lesson dealing with mass, Lois also insisted that the children needed to use the word ‘mass’, though she herself had problems using this language consistently, using the term ‘mass’ 11 times and the term ‘weight’ 28 times. This also supports the assertion that Lois resorted to her established instrumental knowledge. After the lesson, Lois commented: ‘It’s so hard to get out of old habits. I know it should be “mass” but I am so used to using “weight”’.

During this same lesson dealing with mass, Lois introduced the activity where the measuring focused upon capacity and volume. This abrupt change of focus cannot be considered an accidental or a temporary diversion, as the children were reading from the sheets Lois had provided, it was clear that Lois had deliberately included the activity. During the interview after the lesson, Lois in no way indicated that she had been teaching anything other than mass. It must be considered that Lois was herself confused about the important attributes of mass. While she discussed with the class that mass was about how heavy and how light things were, Lois appeared to be of the misconception that volume/capacity is an equally important attribute of mass or, more problematically, that they are the same.

Lois may have intended to teach both mass and capacity/volume in this lesson. There is no teaching principle that suggests a teacher should teach no more than one idea in a single lesson. However, the evidence does not support this viewpoint. Lois at no stage acknowledged that she had dealt with volume/capacity and during the interview stated that the lesson was an introduction to work on mass. She stated that there was still a considerable amount of work to be done to help the
children construct meaningful understandings of mass. Therefore, it seems incongruous that Lois would intentionally provide an activity focusing on quite a different mathematical idea without providing a reason or explanation at some stage for doing so. The most plausible explanation appears to be that Lois believed the activity of estimating the number of level dessertspoons of flour needed to fill a cup was contributing to the children’s understanding of mass. Students exposed to these misconceptions from a teacher are inevitably more likely to learn the same misconceptions.

**Avoidance of teaching concepts due to lack of understanding.**

The third way in which these teachers demonstrated weak mathematical knowledge was to avoid teaching concepts due to their own lack of understanding. It was particularly evident in the case of Linda, as she avoided teaching to any depth and avoided a topic rather than attempt to teach something she did not understand. The very first lesson she taught dealt with place value rather than measurement, a topic she felt more confident with because the class had been learning it the week before this study commenced.

From analysis of each of Linda’s lessons, it is evident that many were content-free or at best, minimal mathematics was involved. During the capacity lesson where the children were required to measure the capacity of a milk bottle using teaspoons, Linda had not thought about how many millilitres of water a teaspoon holds. Had she known this, she could not have helped but realise that the measurements made by the children in her class were quite inaccurate.

While Linda’s main objective for this lesson was for the children to work with millilitres and improve their understanding of this unit, it would seem that the
children only consolidated measuring with an arbitrary unit—the teaspoon—and did not link this to millilitres. If the focus was in fact millilitres, then it is inexcusable that Linda had not linked the teaspoon and millilitres. An initial investigation as to how many millilitres a teaspoon holds would have been appropriate. Linda did not seek to pursue the discussion beyond the point of mentioning that the differences were due to filling the teaspoons with varying amounts and the spillage that occurred.

The impact that a lack of mathematical knowledge has on the overall teaching practice of any individual teacher is illustrated in Figure 5.3. Using the model in this way to represent a limited mathematical knowledge powerfully depicts the consequences for a teacher’s pedagogical content knowledge. Although a teacher may well have a good knowledge of teaching skills and a thorough knowledge of the students in his or her class, weak mathematical knowledge impacts upon the pedagogical content knowledge for that particular subject or topic.

![Figure 5.3. The impact of weak mathematical knowledge: diminished PCK.](image-url)
Colin and Lois, with their predominantly instrumental knowledge and Linda with quite weak mathematical knowledge, all demonstrated a diminished pedagogical content knowledge. As Figure 5.3 demonstrates, a deficit in just one type of knowledge impacts considerably on pedagogical content knowledge. The varying degrees of weakness of mathematical knowledge demonstrated by Colin, Linda and Lois, resulting in diminished pedagogical content knowledge, produced four major negative effects.

**Impact on children’s learning.**

For each of the three teachers who provided evidence of weak mathematical knowledge, it was evident that the children in each of their classes did not gain meaningful learning experiences because of the sequence of lessons. This was evidenced by their inability to use language correctly, teaching errors without recognising they had done so and, in the worst case, simply not teaching any significant mathematics. The lessons Linda taught about milkshakes and filling containers were lacking in any real substantive mathematics. The language that was modelled by Linda was poor. Children only ever referred to ‘mils’, since that is all they heard Linda use.

Lois and Colin’s mathematical knowledge was stronger than Linda’s was and with these two teachers, learning did take place, but the tendency was for it to be more instrumental than relational (Skemp, 1978). Colin wanted to teach for relational understanding but much of his teaching was limited by his own gaps in mathematical knowledge. Lois and Colin both made errors and even when they corrected these errors, some children still appeared confused. Colin’s attempt to teach area in a relational manner caused more confusion than he had anticipated. Having been taught
the formula for area of a rectangle as \( A = L \times W \) by a previous teacher, Colin

eventually conceded to the researcher: ‘I’m fighting a losing battle!’

Children in Linda’s class were not presented with very much mathematical content in any of her lessons, whether it was a lesson where Linda made milkshakes while the class watched, or the lesson where they measured all the attributes of objects provided—so long as each attribute was length related. Linda barely acknowledged other attributes that the children could have measured.

The titles Linda gave her lessons at times lacked mathematical focus. She taught a lesson entitled ‘The Child Sweetener’, in which the children needed to measure 20 millilitres of blue cordial, 20 millilitres of red cordial, one teaspoon of sugar and mix these together in a cup. This was the basis of an entire lesson. When the children had completed their mixture, they were allowed to drink it if they wished. The discussion that concluded this lesson focused more on how to avoid spilling and making sure that too much sugar had not been used. No mathematical discussion took place. Although the children measured using a flask graduated with five millilitre increases, only the word ‘mils’ was used and the activity was not used to help the children gain any understanding of the millilitre as a measure. The measuring throughout this lesson was inaccurate and a significant amount of spillage took place.

In contrast to the activities used by Carla and the learning that was evident in her classroom, it is clear when teachers themselves have weak mathematical knowledge, there is a noticeable, negative impact on the depth and richness of learning for the students in that class.
Impact on teachers’ planning.

The impact of poor mathematical knowledge affects teachers’ ability to plan effective mathematics lessons, an impact that varies depending on the degree of weakness. Linda was the most extreme in this area, as most of her lessons lacked all but minimal mathematical content. Although Linda’s lessons provided evidence of some planning, the focus of that planning was not mathematical content. A lack of methodical coverage of topics was also evident.

One particular example of how Linda was quick to change from her intended plan was her final lesson. She had been to an in-service course during the week and because the presenter had suggested an activity dealing with measuring the area of children’s hands, Linda incorporated the activity into the sequence of lessons for this study. She did so because someone else said it was a good activity and she was willing to change her plan at the word of another. It could be argued that flexible planning is a positive aspect. If this was the case, it could provide evidence of a more positive aspect of Linda’s planning. However, it is more consistent with her lack of confidence displayed at other times. She abandoned Lesson One for a place value lesson because she felt more secure continuing a topic they had covered the previous week. She was quite honest in her reflective questionnaire in providing the reason why she agreed to teach the lessons for this study. She hoped to learn from the researcher as the study progressed.

Colin also displayed problems with planning, directly related to his weaker mathematical knowledge. In particular, he could not determine appropriate incremental increases in difficulty of his content. He moved from measuring the area of a book cover to the activity he called ‘Skin-side out’. The children were not
equipped for this kind of progression. The task of measuring the surface of the body by translating it first to sheets of newspaper was far too difficult. Rather than a problem-solving lesson that Colin had planned, it became a lesson where Colin finally had to provide a ‘recipe’ approach to the activity. The lesson outcomes were not achieved as Colin had intended. Had Colin a stronger mathematical knowledge, he would have realised the difficulty level of this activity and structured it differently.

**Impact on teachers’ use of representations.**

While it is possible to think of measurement as skill-focused, the way measurement concepts are presented to children requires carefully planned representations. Colin’s representation of a kilometre is one that demonstrates the impact of weak mathematical knowledge. This was an interesting experience for the children as it was discovered that a kilometre was approximately two and a half laps of the oval. One cannot help but speculate that many of the children (if not all) would have had problems constructing a mental picture of how long a kilometre is based on a representation that was curved and repeated itself. An analogy would be to estimate the length of a coiled rope compared to a straight rope, as illustrated in Figure 5.4 below.

![Figure 5.4](image)

*Figure 5.4. Linear v. ‘coiled’ representations of distance.*
It is easier to visualise and estimate the length of a straight object than it is a coiled object. A kilometre that is made up of two and a half laps of the oval is indeed a ‘coiled’ representation. In that the children were using trundle wheels where each click measured a metre, the mental picture of a kilometre could only be constructed as they saw each of these metres as a partition of the final distance. Therefore, in their minds, they had to not only ‘see’ each metre but they also had to ‘see’ the final distance with its thousand partitions to construct a meaningful mental model of a kilometre. ‘Seeing’ two and a half laps would not have been an easy way to experience a kilometre, as the children would have mentally had to ‘uncoil’ these laps and visualise the distance in a straight line.

Colin further confused the children in his class by representing a kilometre in terms of time. Each child was working with their own individual mental representation of a kilometre, each of which were not only different, but many of which were inaccurate. Therefore, one could ask, what did the children learn about a kilometre and about estimating a kilometre? There was no discussion about factors such as speed of jogging or why estimates of a linear distance were being made using units of time. At the end of the lesson, the children learnt that a kilometre is two and a half laps of their school oval and to jog it took varying amounts of time—a different time for every child.

In terms of using appropriate modes for representing mathematical concepts (Lesh et al., 1987), it is vitally important that teachers understand the salient features of the mathematics they intend to represent.
Impact on teachers’ ability to make accurate judgments.

When pedagogical content knowledge is diminished due to the teacher’s mathematical knowledge being weak, the ability of that teacher to make accurate assessments and judgments is also weakened. This was evidenced several times in the way that teachers accepted student responses that were either incorrect or inappropriate, something particularly evident with the two novice teachers, Colin and Linda. Further, on occasions, incorrect information or applications of the measurement process were reinforced, resulting in students developing misconceptions or poor skills for measuring.

Colin accepted different responses to children measuring his classroom with a trundle wheel because he did not model how to use it correctly by ensuring the zero was appropriately place at the commencement of measuring. A teacher needs to be able to observe his or her students and make judgments about their performance. Detecting errors and managing them is an important feature of pedagogical content knowledge. Colin did not detect the incorrect use of the trundle wheels and, even when different measurements were shared during the discussion, he did not raise this as an issue and explore possible reasons. His own lack of knowledge prevented him from making important judgments throughout the lesson and in dealing with the resulting errors.

Linda consistently ignored errors made by the children, such as the degree of errors made when measuring milk bottles with teaspoons. At no stage did Linda show concern and engage with the errors being made. Her lack of response suggests that her own lack of knowledge prevented her from realising errors when they were made.
Hence, she could not observe her students and make appropriate judgments about their performance.

As discussed in Chapter Four, a final example illustrating how weak mathematical knowledge impacts on teachers’ judgments was when Colin was being measured by some of the students in his class. Within the space of just a few minutes, Colin had been measured three times. He expressed concern about the students’ estimating and the need for accuracy. Surprisingly, three different measurements were obtained and yet, each of the interactions was concluded with ‘Right’; in each case, interpreted as an appraisal of the students’ efforts.

If accuracy was as important as Colin stressed to the class, then it would appear that Colin had just relinquished what many would call a real ‘teaching moment’. He lacked the ability to make appropriate judgments about the children’s responses and how they were measuring. It would not have taken long to have raised the dilemma with the class and discussed the situation. ‘How can it be that three people have just measured me and all three have got different results?’ is a question that could easily have been posed. The discussion that could have prevailed should have emphasised three important measurement principles:

1. Knowing where to commence measuring and aligning the measurement instrument carefully at the start position should have been emphasised.

2. Knowing that measuring length (or height in this case) is the measurement between two points in a straight line could have been a focus of the discussion. Not contaminating the accuracy of the
measurement by letting the paper metre strip ‘bend’ around clothing or body parts such as shoulders and chin, needed discussion.

3. Recognising the importance of when to measure accurately by using indirect comparison could also have been discussed. The teacher could have modelled how a referent can be used for measuring. He could have stood against the door and had a child mark above the top of his head, then using the metre strip to measure the height of the mark, thereby accommodating previous principles.

It became clear throughout this study that with the teachers who demonstrated weak mathematical knowledge, their ability to observe, assess the situation and make appropriate judgments was also weakened.

**Impact of Variations in Teachers’ Knowledge of Students**

Each of the four teachers revealed insights into their knowledge of students through their teaching and interview responses. There was considerable variation among the four teachers. Table 4.9 in the previous chapter revealed that the two experienced teachers demonstrated strong knowledge of their students, while the two novice teachers demonstrated weaker knowledge of their students.

Figure 5.5 provides an interpretive framework for analysing the impact of a teacher’s weak knowledge of students. There were five main ways that teachers demonstrated weak knowledge of their students. They were:

1. Not sequencing content appropriately.

2. Using teaching approaches that did not match the student’s prior experience or learning styles.
3. Having unrealistic assumptions about their students’ prior knowledge.

4. Having a lack of sensitivity to their students.

5. Not realising the impact of their inappropriate reinforcements on student learning.

It has already been demonstrated that when one type of knowledge is weak, the overall effect is to impact on the teacher’s effectiveness, as a result, having diminished pedagogical content knowledge. Weak knowledge of students impacted on the same four distinct areas of teaching as discussed in the previous section, although in different ways to that when mathematical knowledge was weak.

This section of the chapter will discuss these five weaknesses in teachers’ knowledge of their students. The evidence will be drawn exclusively from the cases of Colin and Linda, as the two teachers who demonstrated weak knowledge of their students. These five weaknesses will be dealt with separately, although there are close relationships between each of these. They are:

1. not sequencing content appropriately

2. using teaching approaches that did not match the students’ prior experience or learning styles

3. having unrealistic assumptions about their students’ prior knowledge

4. having a lack of sensitivity to their students

5. not realising the impact of their inappropriate reinforcements on student learning.
Not sequencing content appropriately.

Colin planned two activities in sequence that both required the children to measure the area of irregular shapes, the first being their hand and the second their entire body. Colin did not anticipate that the difference in difficulty between these two activities would be as great as it proved to be. Firstly, there is the issue of the children’s previous experience. Colin had already established that their measurement experience was limited and that, as far as previous work with other teachers, it had been predominantly instrumental. Their ability to visualise (use internal representations) square centimetres was limited.

The children had experienced one previous lesson from Colin where they had measured the area of various book covers. It was during that lesson that Colin had established that many of the class could not articulate what the term ‘area’ meant. He
underestimated what was required for the students in his class to make the leap required to move from one activity to the next. Ultimately, the second activity could have been appropriate for the culmination of a unit of work on ‘area’, with a class who had developed a sound awareness of the measurement process and demonstrated a good understanding of the concept ‘area’ and the units of measurement used for it.

Colin’s lack of experience may have led him to underestimate the complexity of the task. He could have provided additional experience measuring the areas of other plane shapes (triangles, circles), as well as extra irregular shapes and could have involved objects from the children's real world to maintain interest and relevance. A more gradual development would have allowed for the introduction of skills needed by the children not previously encountered, for example, partitioning a shape into sub-shapes.

**Using teaching approaches that did not match the student’s prior experience or learning styles.**

Colin asserted several times that his class had previously been taught instrumentally and that he wanted to teach relationally, yet he did not know how to change the way the children wanted to do things, based upon their previous experience. When Colin set the children the task of measuring the area of a book cover, he wanted them to use squared paper and derive the formula as a result. However, as he moved around observing children, Colin noticed that quite a few of the children were measuring dimensions and multiplying them. After several incidents where he instructed children to count squares rather than use the formula, Colin called the class to attention to insist that everyone count the squares and not use the formula.
Even after insisting that children use the grid paper, some children still demonstrated a reluctance to count squares. Colin became somewhat frustrated and appeared not to understand why the class was reluctant to count. There came a stage where he approached the researcher present in the classroom and stated: ‘I’m fighting a losing battle here. They’re stuck with this formula, see, it's thrown them’.

The only plausible explanation he could offer was that having been previously taught the formula, there was a reluctance to attempt any other way. By suggesting, ‘it's thrown them’, Colin was proposing that because it was a technique that worked, but one that they did not understand, they were unable to measure area by any other means. The counting of squares on grid paper bore no relationship to the formula in their minds and the tedious job of counting many small squares seemed to be a waste of time compared to applying the formula. Colin reinforced this interpretation in the interview after the lesson, when he made the following comments:

_They'd been taught the formula and they felt they didn't want to count. We know how to do it really quickly. But they didn't understand the formula and they didn't know how to use it. They got an answer, they had no idea of visualising what that answer looked like, as in centimetre squared and that's where the problem is._

Colin's assessment of the situation appeared to be a reasonable one. As he moved around the class reminding children to count all the squares, he met responses that indicated the children could not see the value in counting so many squares. During the following interaction, the child concerned was literally amazed to think he was being required to count all the squares:
Colin: I'm a bit worried about this. You've got your book cover here. You've done your estimate, ninety-five. Have you used this?

Child: Yes.

Colin: And what did you find? (Pause.) You put it there and you put it there, right?

Child: Yes.

Colin: Now, have you added? You just count these, you count these squares here. In this area here. And that will...

Child: All of these? (With amazement.)

Colin: All of them, yes.

Child: Oohhh.

Colin: That's the only way, because you're getting some weird figures there aren't you?

Child: Yes.

Colin: Alright.

Colin did not realise that what was evident to him was being missed by the class. He hoped that by counting, the children would derive the formula and thereby come to a meaningful understanding of it: if they were asked to count the following number of squares, that they would see a pattern and not just count in a random manner. Although it appeared that this conceptual approach by Colin was a desirable approach to be taken by any teacher, there are at least two explanations as to why his class did not make his intended connections.
Firstly, by trying to use objects from the children's daily lives, Colin possibly created situations more complex than the children were ready to deal with. The children became too preoccupied counting part squares and keeping tallies; the task appeared to be quite time consuming and of little relevance to them. Secondly, while the measurement used grid paper and allowed the children to be actively involved, it did not allow the children to manipulate the basic unit easily. The use of square tiles for measuring objects with exact measurements may have enabled Colin to accomplish his objectives more easily. Then, when the children had discovered the relationship between counting squares and the formula, perhaps they could have used grid paper as a transitional approach before using the formula exclusively.

This example illustrates the difficulty teachers can face when their approach is different to what children have previously encountered. Changing students to a relational approach when they are accustomed to instrumental teaching is not easy to achieve. Children who are used to instrumental strategies are generally looking for shortcuts and rules to follow and often have not developed the desire to understand the content they are learning. This style of learning often involves achieving answers and not understanding.

This is the type of knowledge of his students that Colin lacked. He did not have the knowledge of how students respond to varied teaching strategies. His belief that his strategies were appropriate and would lead to relational understanding was what Colin considered important.

**Having unrealistic assumptions about their students’ prior knowledge.**

Linda’s experience of measuring a litre capacity using cups showed that her students found the task more complex than she anticipated. Linda stated during the
interview, ‘It's always hard to tell from a carton of milk how many cups you're going to get out of it because of the shape of the container’. The milk carton was a regular shape (a rectangular prism), although the top of the carton certainly provided some variation in the shape that could have made the task of estimating more difficult. The shape of the cup was possibly more distracting since it was not a rectangular prism or even closely related to one, and its internal diameter varied due to being a truncated cone. This would have made the task of visualising the relationship between the two quite difficult. For children to make meaningful estimates, there may have been a need to relate the situation to real-life experiences of pouring drinks from a litre container. Even then, there is no guarantee that the kinds of glasses the children drank from at home had the same capacity as the disposable coffee cups used.

Linda did try to make this connection with one child as she moved around the room but the outcome was unsuccessful and the child was possibly left more confused than before. Firstly, he did not have milk in one-litre cartons at home. His family purchased plastic, two-litre containers, the shape of which is quite different. Secondly, they only drank ‘fizzy’ drink. Therefore, his only experience pouring drinks was from soft drink bottles, which are normally two litres, or 1.25 litres. Once again, they are a different shape, and it is unlikely that he used drinking glasses the size of a disposable coffee cup. Although Linda did try to provide an analogous situation to help the child make a meaningful estimate, it was one that he was unable to relate to and use as a reference.

A final factor making estimation difficult is the effect that changes in dimensions have on volume with three-dimensional objects. If each of the dimensions of the cup were half the dimensions of the milk container, it would have one-eighth
of the volume. This is not something that is immediately evident visually, particularly to children with little experience in this area.

Linda tried to connect the task to the students’ home experiences; a connection that may initially seem an appropriate one for her to make. However, on another occasion when Linda was relying on their home experiences, it was quite evident that these experiences were not providing the prior knowledge that Linda assumed. Linda stated during interview that her class had not had any previous experience with estimating with her and she doubted whether they had any in previous years.

Colin also demonstrated unrealistic assumptions about the prior experience of his class in the ‘Skin-side out’ activity, a task that proved to be quite complex and too difficult for all in Colin’s class. Intended as a problem-solving exercise, it became a lesson in which Colin described each step in a procedural way and the class did their best to follow as if it was a recipe. This created a far more complex activity than Colin’s students were prepared for at this stage of their measurement learning.

Colin also misjudged his students’ prior knowledge when he asked them to estimate the area of their hands. He asked the class to estimate how many square centimetres would be needed to cover the palm of their hand in the belief that the class had used square centimetre paper before for measuring and that they would have a reasonable mental representation of a square centimetre to operate with for the purposes of their estimations. The task of mentally superimposing square centimetres onto one's hand was quite different to actually drawing around the hand on square centimetre paper and then making the estimate.
This distinction is one that Colin did not acknowledge. However, it supports the notion of the distinction between the role of internal and external representations in making mathematics meaningful for children. It reveals Colin’s inexperience in making judgments about students’ prior knowledge.

**Lack of sensitivity to students.**

Teachers have a responsibility for the development of the students in their classes. In doing so, they have a responsibility to provide a safe and secure environment for their students, not only from a physical perspective, but also emotionally. Both Colin and Linda unintentionally embarrassed students at times in the teaching observed in this study, suggesting that neither had the sensitivity to know when this was occurring or the knowledge of the kinds of situations to avoid, ensuring children were not embarrassed.

When Linda asked the question, ‘How many cups of milk would I need for ten people?’, it would appear that one student, Samantha, had thought carefully about the question but based her answer upon how many times one would repeat the recipe, rather than the total number of cups of milk. When considering the question, it is likely she acknowledged that the recipe provided for two. Therefore, when asked how many cups of milk for ten people, she calculated that she would need to multiply all of the ingredients in the recipe by five, and she provided an answer of ‘five’. While her thinking may be correct, she did not answer the question asked, ‘How many cups of milk?’, since each recipe has two cups of milk, then five repeats of the recipe means she should have responded with an answer of ten cups.

The thinking behind Samantha's answer to the second question, ‘How many teaspoons of flavouring would I need for ten people?’ is less obvious. It appeared
there was some reluctance to answer in that she was afraid she might be wrong again. The teacher had possibly inadvertently added to her confusion by saying, ‘This is where Samantha might be on the right track’. Previously, Samantha had answered ‘five’ but the correct answer was ‘ten’. This time, Samantha offered the answer ‘ten’ hoping that it would be right. Instead, Linda responded with, ‘Ah dear! Let me ask somebody else first’. Samantha immediately appeared embarrassed and did not volunteer any further answers to Linda’s questions in that particular lesson.

Chapter Four referred to the incident of measuring the mass of a larger girl. This incident raises a serious issue all teachers at some time will be confronted with, the need to be sensitive to children’s feelings. The girl appeared to consider herself as considerably larger than her classmates and hence was very self-conscious about measuring her mass. Colin failed to pick up three overt signals of self-consciousness, none of which he read correctly. In doing so, he demonstrated a lack of knowledge of the kinds of issues to which children may be sensitive and easily embarrassed.

**Not realising the impact of their inappropriate reinforcements on student learning.**

Both Colin and Linda frequently reinforced their students’ responses positively even when the responses were incorrect. Colin would reply to an incorrect response with the reply, ‘Right’, and rarely took time to correct the student or show why there was an error. As a result, the children in his class moved on to the next task unaware that they had made an error, as Colin had appraised their result. Over time, this approach must result in students developing misconceptions based upon their teacher’s inappropriate reinforcement.
Linda also reinforced inappropriately. This was particularly evident when she required students to make estimates. She put forward to her class that estimates were ‘wild guesses’ and as such, when students made ‘wild guesses’, they received positive reinforcement.

The impact that a lack of knowledge about their students has on the overall teaching practice of any individual teacher is illustrated in Figure 5.6. A teacher may well have a good knowledge of teaching skills and a strong mathematical knowledge, but weak knowledge of their students impacts upon the pedagogical content knowledge for that particular subject or topic.

Colin and Lois both displayed a lack of knowledge of their students and the needs of their students. As a result, they both demonstrated a diminished pedagogical content knowledge. As Figure 5.6 illustrates, a deficit in the knowledge a teacher requires about students impacts considerably on pedagogical content knowledge. The varying degrees of weakness of the knowledge of their students demonstrated by Colin and Linda resulting in diminished pedagogical content knowledge produced four major negative impacts.
Figure 5.6. The impact of having a weak knowledge of students: diminished PCK.

These four areas of impact are the same four areas that were identified in the previous section when discussing deficits in mathematical knowledge. Given that pedagogical content knowledge is the central intersection of the model, then it stands that whichever body of knowledge is weak, there is a diminished pedagogical content knowledge. While the four areas of impact are the same, the factors contributing differ and these impacts are manifest in different ways to when the deficit was mathematical knowledge.

**Impact on children’s learning.**

Having a weak knowledge of the students in their class affected both Linda’s and Colin’s teaching and hence affected their students’ learning opportunities and potential. Not having a strong mathematical knowledge can clearly affect students’ learning. However, when a teacher does not fully understand the nature of their
students as learners, there must be an impact upon those students and their learning. Colin’s teaching provided evidence of this when he demonstrated a lack of understanding of the appropriate steps to take within his lessons to ensure students progressed within a topic. As a result, he assumed far too much. Moving through a sequence of activities as he did on ‘area’: measuring the area of a book cover, measuring the area of a hand to finally measuring the surface area of their body, proved to increase in difficulty level far too quickly for his class. This inappropriate sequencing is partly due to Colin’s weak mathematical knowledge but it also is due to his lack of knowledge of how quickly students can progress through these types of tasks.

Linda also stated throughout the series of interviews that she did not have a good sense of her students’ prior knowledge and maintained that many of her lessons aimed to discover what her students knew. Her lack of knowledge of her students resulted in different experiences for her students to those Colin provided. Whereas Colin progressed too quickly for his class and the activities proved to be too difficult, Linda provided activities that were generally too easy and that often required copying, such as the recipe for the milkshake. She had some questions requiring simple mental calculations that the class found easy to do, and yet Linda classified this as problem solving.

The learning of the students in these two teachers’ classes was affected for different reasons. Colin assumed too much and his students found his activities too difficult. This resulted in confusion and often the problem-solving focus he had hoped for disappeared, as Colin had to provide a systematic procedure for them to follow. Conversely, Linda had minimal expectations of her students and provided
only small amounts of work for them to achieve in each lesson. The one exception to this is that she assumed that children learn a considerable amount from home and that they would know about capacity and millilitres, based upon their experience of drinking milk and soft drinks at home.

Whether one assumes too much of their students or too little, the result will have similar consequences in errors made and confusion when the needs of the students are not being addressed and when knowledge of how they learn is weak. Linda and Colin both lacked knowledge of their students, and in both cases, their students’ learning was affected.

**Impact on teachers’ planning.**

Planning lessons in any subject clearly has one major purpose, to provide meaningful and engaging experiences so that students learn. This study has provided evidence that when a teacher’s knowledge of their students is weak, there is a significant impact on their planning. The types of activities, the types and levels of expectation will all affect the way teachers plan. In this study, Colin demonstrated that his understanding of his students was sufficiently weak that he was unable to plan activities with appropriate rates of progression. He assumed too much, misjudged his children’s rate of learning and could not determine when activities were too complex and involved multi-step strategies beyond the capabilities of his class.

Linda often planned the same activities for her Year Three children as she did for her Year Four children. This lack of knowledge of her students and the ability to plan for varying ability levels within the one class resulted in inappropriate activities that were not challenging and not involving enough to last the duration of a full
Linda’s lack of knowledge also contributed to her classroom management problems because she did not plan interesting activities at appropriate levels to keep her students engaged, nor did she take into account the role students should have within a lesson. While she did plan some lessons where they were engaged in measuring objects, other lessons required them to sit and observe the process in a passive way, resulting in misbehaviour and limited learning.

**Impact on teachers’ use of representations.**

Shulman (1986) claimed that what characterises pedagogical content knowledge of specific ideas is the quality of the representation of those ideas, ‘the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others’ (p. 9).

Chapter Two established the importance of multiple representations as well as the relationship between external representations chosen by teachers and internal representations or mental models created by the student. When a teacher lacks knowledge about how students learn, the rate at which they learn and their ability to translate external representations into internal representations, then their selection of representations, examples and explanations will be inappropriate. The analysis of both Colin and Linda’s teaching behaviour support this claim.

There were two instances of the negative impact on Colin’s use of representations as a result of weak knowledge of his students. The task of estimating a kilometre was complicated by the children being asked to estimate the distance using a unit of time, and the activity requiring children to estimate the area of their hand, when the children in his class did not have such sophisticated mental models of
square centimetres. Colin demonstrated that he did not know the abilities of his students well and, as a result, required them to use representations that were too complex and assumed too much prior experience.

Linda also demonstrated that her explanations and their implicit representations were limited and misjudged her children’s prior experience. Making an estimation is a skill that teachers need to help their students develop. Linda’s view that estimations need only be ‘wild guesses’ meant that her students were never required to modify their internal representations of units. The activities did not place any real importance on the role of estimation as working with representations and developing internal representations. Linda’s knowledge of students and how they construct their knowledge was limited to the extent that she did not offer adequate representations for her class to assist them in making meaning from her explanations and teaching activities.

During the milkshake lesson, the way Linda modelled the measurement process failed to take into account the importance of this representational aspect of the lesson for her students’ learning. If her knowledge of how students learn had been stronger, it is doubtful she would have modelled measuring capacity in such a way that every cup varied in quantity. The children commented throughout the process that her cups were only half-full, which was denied by her.

Clearly, knowing about students, their prior experiences, how they learn and the rate they learn is important to effective teaching. When a teacher’s knowledge of students is weak, there is a negative impact on the effectiveness of their teaching due to diminished pedagogical content knowledge.
Impact on teachers’ ability to make accurate judgments.

Teachers are constantly in a position where they need to make judgments about students and their needs. Is the rate of progression appropriate? Are the students developing appropriate understandings? Are their skills developing appropriately? Linda and Colin both lacked the depth of knowledge of their students’ prior experiences with measurement and, as a result, they both misjudged the sequence for their instructional activities. Colin progressed far too quickly and presented the children with tasks significantly too complex and difficult for their year level.

Conversely, Linda prepared activities that were simple and lacked challenge and, at times, required a passive, observational role. She misjudged the extent that children learnt from their home environment and assumed that children understood capacity and millilitres simply from being exposed to capacities being printed on drink bottle labels. She planned her activities based upon this judgment. As a result, the children did not develop a good understanding of capacity and the units used for measuring capacity.

Both teachers lacked the ability to make judgments about what would embarrass a child and, as a result, both teachers inadvertently were responsible for embarrassing children in their classes. As Colin and Linda misjudged situations, they consequently made inappropriate decisions. These decisions affected their teaching and the learning of children within their class. With a weak knowledge of their students, the result was diminished pedagogical content knowledge.
Impact of Variation in a Teachers’ Knowledge of Teaching

The earlier parts of this chapter discussed the impact of poor mathematical knowledge and poor knowledge of students. This section will examine the impact of weak knowledge of teaching. Table 4.9 in the previous chapter revealed that of the four teachers, Carla and Lois demonstrated a strong knowledge of teaching, whereas Colin and Linda had weaknesses. Linda showed weakness in all four of these areas though Colin showed fewer, and for this reason in Table 4.9 Colin was classified as ‘developing’ for knowledge of teaching.

General teaching knowledge covers a broad range of issues and those aspects of teaching discussed in this section are restricted to those for which this study provided evidence. These are shown in Figure 5.7:

1. knowledge of clear outcomes for the lessons
2. knowledge of effective discipline and classroom management strategies
3. knowledge of the curriculum and school policies
4. knowledge of assessment and evaluation.
**Figure 5.7.** The impact of weak knowledge of teaching on pedagogical content knowledge.

### Knowledge of clear outcomes for the lessons.

Teachers need to know what it is they hope to achieve as a result of the lessons they plan and teach. In mathematics lessons, these outcomes link very closely to the teacher’s knowledge of mathematics, as one would expect that many of their outcomes would be mathematical in nature. However, it is not uncommon for a mathematics lesson to have other intended outcomes, such as having children develop group skills or alternatively, the ability to work independently. A good teacher anticipates these outcomes and determines these at a suitable level, so they present a challenge to the students and not set too high or too low expectations. Hence, it is clear that in determining outcomes successfully, there are close links to the knowledge a teacher has of their students.

Both Lois and Carla were able to clearly express their intended outcomes for each of their lessons and had a good knowledge of the expectations they held for their lessons.
students. This is one area Colin also demonstrated good knowledge. He planned quite thoroughly and planned adequate amounts of work for each lesson. His major problem was that his outcomes were unrealistic at times and not always appropriate for his class. This was due more to his weak mathematical knowledge and his inadequate knowledge of his students than his knowledge of teaching.

Linda clearly did not demonstrate knowledge of her outcomes. When she did state her outcomes, they were often lacking mathematical focus. Prior to her lesson measuring the capacity of a milk bottle with teaspoons, she made the following comments during the interview:

*Researcher:* At the outset of the lesson what are your intended outcomes?

*Linda:* Right. Well following on from the milkshake lesson when we were just talking about arbitrary measuring, I want to know how much they know about mils.

*Researcher:* Can you elaborate further?

*Linda:* To see if they can handle pouring water and not spilling too much.

When asked, Linda could not elaborate further on what it was she wanted them to know about millilitres. She appeared uncomfortable and changed focus to a non-mathematical outcome related to how well they could pour water.

In the following lesson, a week later, Linda repeated this activity with the Year Four students and asked the Year Three children to measure as many things as they could, given a quantity of different boxes. Prior to the lesson, Linda was once again asked about her outcomes:
Researcher:  Tell me what it is you are setting out to do. What are the intended outcomes for this lesson?

Linda:  Right, the water activity is mainly just working with, most of them are Year Fours, so it is just to give them a go of what the Year Threes did last week. And with the measuring of boxes, it is just to see if the children can think of as many different ways to measure them as they can.

Linda’s reply provides evidence that she had not thought through the outcomes for this lesson. Firstly, it would appear that in giving the Year Four children the same activity as the Year Three children from the previous week, she did not want them to miss out on what they perceived as a ‘fun’ activity. Whether Year Four and Year Three should be doing the same activity is questionable if the Year Four class have progressed beyond the level of Year Three. Secondly, the outcome of the lesson seems to be based more on treating both groups fairly. Since Year Three had fun the previous week, it was now appropriate to let Year Four participate in the same ‘fun’ activity. Linda could not articulate clear mathematical outcomes for these lessons.

Linda’s inability to clearly articulate her intended outcomes was evidenced again when asked prior to the lesson measuring with 20 ml bottles:

Researcher:  What do you intend to achieve in this lesson?

Linda:  Once again, it is following on with the mils and trying to get them to use a smaller unit; we will be using the twenty mils. I think it is important for the kids to realise that they need to have their measurements exact today and, so they will need exactly twenty mils.
Interestingly, Linda stated that her outcome was to work further with millilitres and have the children use smaller ‘units’, ‘the twenty mils’. From a mathematical perspective, this is quite confusing. Is a 20 ml bottle a smaller unit? Was Linda still, in fact, using arbitrary units—small bottles—that had ‘20 ml’ printed on the label? Linda did not, at any stage, plan activities or state outcomes that described how she was going to assist the children develop an understanding of a millilitre. Linda consistently approached and delivered her lessons without being able to articulate a clear outcome for the lesson.

**Knowledge of effective discipline and classroom management strategies.**

When Colin, Lois and Carla taught, children in their classes were well controlled and generally behaved themselves. This was largely due to the well-planned lessons and activities that engaged the children. Occasionally, each had reason to talk to individual students to ensure they were on-task and not distracting others. This was not a major problem and constituted nothing more than the regular classroom management in which teachers engage. The teacher that demonstrated a real lack of knowledge of effective discipline and classroom management strategies was Linda. This was a major problem area for her and in every lesson, there were times when she struggled to manage her children’s behaviour. During interviews, she acknowledged this and used the expression that she tried to avoid ‘chaos’. When discussing her planning, Linda stated:

*I'll do the activity altogether, sometimes I know it'll be too easy for the Fours or too hard for the Threes, so I have separate activities, but generally, when I'm using hands-on material, we all do the same thing so we're not in chaos.*
It became evident that a contributing factor to her discipline problems was her poor planning. She often did not have sufficient work planned and had difficulty predicting the rate at which they would work. She acknowledged after one lesson:

*Linda:*  
As a lesson overall, a lot of them finished quicker than I anticipated.

*Researcher:*  
If you were to teach that lesson again would there be anything you'd do that would be different?

*Linda:*  
I'd probably do it the same way but be a bit more organised with the hands-on things, so it just ran a bit more smoothly.

Linda’s lessons did not run smoothly and this was largely due to her inability to organise hands-on lessons well. As a result, she spent considerable time trying to maintain control. Even the behaviour-management strategies others had suggested to her were ineffective. As she explained:

*Linda:*  
The way the desks are arranged, they're in groups and if they're all working together, they get a point.

*Researcher:*  
So that's part of your classroom management strategy that you use? What if they misbehave?

*Linda:*  
Well, the first stage it’s their name on the board, and then two crosses besides their name and then it's time out.

*Researcher:*  
But what does ‘time out’ actually mean? Are they physically removed from the classroom?

*Linda:*  
They're supposed to go to a desk at the back of the room and sit there for five minutes and then come back. But with some kids, if they're just being
noisy, I'll send them on to the veranda and they're supposed to come back
in when they're ready to.

This final comment from Linda indicated the problem. Her class had
discovered that they could have free time on the veranda with no consequence and it
was up to them when they returned. At times, more than one child could be on the
veranda, a situation that they seemed to enjoy. Conversely, Linda removed them so
that she could ‘teach’ without their constant interruptions. Her strategy was
ineffective but she did not know of strategies that were more effective. The constant
disruptions Linda experienced meant that her lessons became disjointed and,
combined with other factors that have been discussed in previous sections, the result
was that her students did not engage in any substantive mathematics learning.

Knowledge of the curriculum and school policies.

The two experienced teachers, Lois and Carla, had a good knowledge of
school policies regarding programming and assessment. Their knowledge of
curriculum documents was sound and they referred to curricular requirements
throughout their interviews. Neither of the two beginning teachers had a good
knowledge of either curriculum requirements or school policies.

During Colin’s initial interview, he described in considerable detail how the
school-based curriculum that he was required to use simply was not followed by him
or many of the other teachers:

Colin: Our school has tried to provide various checklists. There's not a teaching
sequence or anything set down.

Researcher: So when you talk about the curriculum, you're talking about this
school’s curriculum?
Colin: Yes. School-based curriculum. Sometimes I find, and I brought this up last night with the staff, I feel I'm in a big vacuum. We really do not have clear guidelines. So, this year with mathematics, I just wrote down what I planned to do. I try to do quite a lot of hands-on activities for measurement and things like that. I think that with my class I want to work right through what I have planned for the year. No one else has evaluated it at all.

Researcher: I know this is only your second year of teaching, so how different is this to what you taught last year?

Colin: I have found it very difficult because we didn't have a scheme of work. When I arrived last year, I was given a textbook and was just told to use that.

Researcher: And so you just worked to that textbook?

Colin: Well, I couldn't. I just did not like it.

Researcher: But that was the idea as far as the school staff was concerned?

Colin: That was it. That was all, that was my teacher's resource that was given to me. That was all I could really use as my curriculum. Then I started thinking that I have to get other stuff, so I just tried to grab whatever.

Researcher: What about other teachers on the staff? Were they dissatisfied as well?

Colin: They were, but they tended to stick with the set text. Day one when I first started, I was overwhelmed with stuff. Someone dropped these books on my desk, 'Oh, these are yours' and you know, you don't think anything of the curriculum and it was never mentioned what I was supposed to do
with them. And even about second term or half way through third term, it was just, ‘Oh, how are you going?’

The interview revealed not only Colin’s lack of knowledge of the curriculum but also that as a beginning teacher, he received no support in this area from senior staff or more experienced teachers within his school.

Linda’s situation was similar. Although her school did have a set programme they were required to implement, Linda made it quite clear that she did not follow it:

Researcher: How is mathematics programmed? What does the school base its requirements on?

Linda: Right. The school’s got quite a strict system for programming, although we haven’t agreed with it so we've gone our own way and justified that.

Researcher: And you're allowed to do that as long as you justify it?

Linda: I think so.

The ‘we’ that Linda referred to were the other Year Three and Four teachers. They had decided not to follow the required school programme and to plan their own. This left Linda to determine her own programme and the evidence from observing her teaching and interviewing her suggested that she did not have sufficient experience with curricula to design and implement her own.

The final lesson that Linda taught for this research was not one she had initially planned. She had attended an in-service course only a few days earlier and had been shown an activity for finding the area of a hand. She liked the activity and decided to use it for her final lesson. Linda’s willingness to change lessons so easily was largely due to not having a clear direction or guidance with her teaching. As soon
as she felt there was something better available to her, she was quick to use it, whether it was appropriate for her students and their needs or not. This resulted in a ‘cobbling together’ of activities rather than a carefully sequenced development of understandings.

Further evidence that Linda lacked knowledge of the curriculum is based upon the lack of differentiation in content that she taught her Year Three students and her Year Four students. This was illustrated in the following comment made by Linda:

*Researcher:* How do you cater for the differences between Year Three and Year Four if you're doing the same activity?

*Linda:* (Silence.)

*Researcher:* Do you sometimes give slightly harder examples or problems for the Year Fours to do?

*Linda:* Yes.

Linda was quite unsure about how to answer these questions. She did not have an explanation for how she catered for the difference between the two year levels. The researcher’s comment about ‘giving slightly harder examples’ was agreed to quite readily as a way of providing an answer. No further elaboration was offered.

The lack of knowledge of teaching and the curriculum often resulted in lessons lacking focus and consequently, children appearing to be confused rather than engaged and showing understanding. This was particularly true in the case of Linda. The children in her class were easily distracted and developed minimal understanding. Her own focus was on classroom management rather than teaching.
Her teaching lacked any significant use of representations of the content, as often the content was neither mathematical nor curriculum based.

**Knowledge of assessment and evaluation.**

Of the four teachers involved in this study, Lois and Carla monitored their classes throughout each lesson. Both teachers were quick to detect children who appeared to be confused. They carried out constant observation of their class and moved around, interacting with the children. They questioned and asked students to elaborate on their answers. To Carla and Lois, assessment and evaluation were part of their developed teaching repertoire. They had good knowledge of how to carry out assessment, both informally and formally and demonstrated the ability to make accurate judgments in a variety of contexts.

Colin, although considerably less experienced, also monitored his students and at times provided evidence of making accurate judgments. However, his poor mathematical knowledge sometimes prevented him from making appropriate judgments. He required his students to record in written form in approximately half of the observed lessons. This was a weakness in his planning for assessment, as it restricted Colin’s opportunities to make judgments about his students’ written accounts of mathematics. He often limited himself to assessment situations based upon discussion or observation of their engagement in activities. When the children worked in groups, it was not at all clear that Colin was able to make accurate assessments of each child’s understanding on an individual basis.

Linda did not appear to plan assessment and evaluation at all. Her lessons were often so lacking in substantive mathematical content that assessment would have proved difficult. Linda did have her students record in written form in every
lesson. However, as was evidenced in Table 4.3 in the previous chapter, most of the written work required of her class was recording estimates and actual measurements. When one considers that estimates to Linda were ‘wild guesses’, the children’s responses did not bring about any assessment where judgments of student learning took place. When Linda checked her students’ work, it was from a position of compliance rather than making academic judgments. Accuracy of the students’ estimates and measurements was not judged. Another recording requirement involved Linda asking her class to copy the milkshake recipe from the board. Once again, there was no need for any assessment other than for compliance reasons—only noting whether the children had copied the recipe as instructed.

The impact that a lack of knowledge about teaching has on the overall teaching practice of any individual teacher is illustrated in Figure 5.8. A teacher may well have a good knowledge of their students and a strong mathematical knowledge, but weak knowledge of teaching impacts upon the pedagogical content knowledge for that particular subject or topic within a subject. When teachers are lacking knowledge of their outcomes, classroom management strategies, curriculum, assessment and evaluation strategies, the impact upon teaching effectiveness is considerable.

Linda’s teaching demonstrates that when knowledge of teaching is weak, there is an immediate impact on children’s learning. Linda had limited knowledge of curriculum and did not have clear intended mathematical outcomes with high expectations of her students; her students did not engage. This lack of engagement resulted in minimal learning. Also, the lack of clarity and focus on specific mathematical content resulted in confusion, errors and inaccuracies when measuring.
Figure 5.8. The impact of having a weak knowledge of teaching: diminished PCK.

**Impact on children’s learning.**

In the case of Linda, this lack of knowledge of teaching, in combination with weak knowledge in other areas, brought about a cycle that proved virtually impossible for her to break. This cycle is illustrated in Figure 5.9. Lack of knowledge resulted in lack of clarity in outcomes contributing to an increase in the number of students who became distracted, resulting in the need for Linda to discipline the students for inappropriate behaviour. As a result, the focus of the teacher narrowed, based upon her need to control the class. However, as in the case of Linda, her lack of knowledge of appropriate classroom management strategies commenced the cycle again. The more disruptive her students became, the less her focus on mathematical outcomes and the more her focus shifted to control. This cycle at times appeared to spiral for Linda, with behaviour management becoming increasingly more important.
and the mathematical outcomes of far lesser importance. The result was minimal learning for the children in her class.

Figure 5.9. The cyclic effect of weak knowledge on children’s learning.

**Impact on teachers’ planning.**

Planning effective lessons requires strong pedagogical content knowledge as the mathematics to be taught, the students to whom it will be taught and the way in which it will be taught all need to be considered. When a teacher has weak knowledge of teaching, this impacts specifically on the way the mathematics will be taught. Colin demonstrated weak knowledge of teaching when he planned the lesson ‘Skin-side out’. He demonstrated that he did not have a sound knowledge of how to sequence activities so that they were appropriate for his class. He moved from simple
to complex activities far too quickly, resulting in confusion and minimal student understanding from the process. Colin planned this lesson as a problem-solving lesson, but its implementation consisted of a procedural set of instructions that the children followed with little thought as to why each step was important.

Linda also had weak knowledge of teaching. She planned activities in which she dominated and the children’s role was passive, such as the observation of her making milkshakes. When the children had an active role, it was centred on filling containers with water and counting rather than learning about capacity and the appropriate units for measuring capacity. As a result, the focus of Linda’s lessons often lacked clarity and any significant engagement with mathematics. So great was her need to control the behaviour of the children in her class that this issue dominated her planning. Consequently, the children in her class developed only minimal understanding of the measurement process. Many were confused as a result of her activities, poor modelling and inability to use appropriate mathematical language in her explanations. When Linda planned, issues such as assessment and evaluation were not given consideration.

Lack of a strong knowledge of teaching had severe effects on planning. This was evident more by the obvious omissions than by what was included. Effective sequencing, appropriate strategies, meaningful assessment, introducing mathematical language were all glaring omissions, particularly with Linda, who demonstrated the weakest knowledge of teaching.

**Impact on teachers’ use of representations.**

Knowledge of specific representations directly relates to knowledge of mathematics and knowledge of teaching. A teacher cannot effectively represent
mathematics if they do not have an understanding of the mathematics. However, it is possible to understand mathematics and not have a representational repertoire to assist others in developing an understanding of the same concepts. Simply knowing the importance of representations is part of general teaching knowledge. Colin, Lois and Carla all expressed beliefs in the importance of representations. Colin often articulated the importance of building mental models and that this was linked to external representations of the abstract mathematical concepts. Carla was even more emphatic that she needed to model mathematics appropriately so that her students could develop meaningful understanding. This modelling process required appropriate representations.

Linda was the only teacher of the four who never discussed or even suggested that representation was important. When one considers representations from any of the five modes discussed by Lesh et al. (1987), it was clear that Linda did not consider these in her planning. It did not appear to be part of her knowledge of teaching. She did not demonstrate through her own modelling that she saw representation as a critical part of teaching. When she measured for the milkshakes, her modelling of inconsistent amounts she called ‘a cup’ was quite a concern. The way in which Linda represented the measurement process was poor. Linda appeared to be unaware that as a teacher, she was involved in representation, no matter how she taught. Her representations shown to the class were mainly representations of inaccuracy, rather than useful representations that would assist her students to develop relational understanding.

Teachers, in their knowledge of teaching, need to have knowledge about the role and purpose of representation in their planned instruction. Selecting specific
representations constitutes more than knowledge of teaching, as it relates to both knowledge of mathematics and knowledge of students—it is a fundamental component of pedagogical content knowledge.

**Impact on teachers’ ability to make accurate judgments.**

Each teacher was asked to reflect on his or her lessons during each post-lesson interview. While Carla and Lois recognised aspects they could teach differently and aspects that worked well, Colin and Linda both experienced difficulty in reflecting upon their teaching and making judgments. While lack of mathematical knowledge and knowledge of their students contributed to this, as has already been discussed, this impact on their pedagogical content knowledge was made even more severe due to weaknesses in their general knowledge of teaching.

Linda did not plan for assessment and evaluation. Questions dealing with this aspect of her teaching left her awkward and often without an answer. When she did answer, it was generally based on her feelings, rather than from evidence based upon the incidents throughout the lesson. Linda would often respond with general comments like, ‘I think it went alright’, but not offer any qualifying support.

Colin also did not respond well to questions asking him to make judgments about his teaching. He was limited in his knowledge of activities and although he tried to be creative in his lesson titles, the content and activity did not always prove to be appropriate as far as sequence and helping him fulfil his intended outcomes. When asked how he might have done things differently a common response would be, ‘Ah, I see what you mean … I haven’t really thought about that’. Colin lacked the experience to reflect on his teaching and to make accurate judgments about the effectiveness and value of his planned lessons.
The results of this study provide evidence that when a teacher’s general knowledge of teaching is weak, then the impact on pedagogical content knowledge is diminished. These first three parts of this chapter have shown that when each of the three knowledge types are weak, there is a diminished pedagogical content knowledge. As a result, the effectiveness and quality of that teacher’s classroom performance is restricted.

**Impact of Variation in Teachers’ Knowledge of Multiple Areas**

The first three parts of this chapter presented an analysis and discussion of the impact on pedagogical content knowledge of weak knowledge in each of the three areas first introduced in Figure 2.2 in Chapter Two. It has been demonstrated that whether the weakness is in knowledge of mathematics, knowledge of students or knowledge of teaching, the result is for teachers to have diminished pedagogical content knowledge. Within this section of this chapter, a brief extrapolation will be made to examine the impact on pedagogical content knowledge given a teacher who might be weak in more than one of the three types of knowledge.

Table 4.9 revealed that Carla was strong in all three types of knowledge. A reasonable representation of Carla’s pedagogical content knowledge is thereby illustrated in Figure 5.1 at the commencement of this chapter. Being strong in all three types of knowledge is best represented with congruent circles. This is not to be interpreted that there are equal amounts of knowledge. It needs to be stressed this model and its variations are representations and are not based upon actual measurements of teacher knowledge. The model is based upon the interpretive data presented in Chapter Four and further analysed in this chapter.
Lois was reported in Table 4.9 as only being weak in her knowledge of mathematics. Hence, Figure 5.3 represents the impact on her pedagogical content knowledge. While only one of the three knowledge types is classified as weak, it was argued that this was sufficient to cause diminished pedagogical content knowledge and represented by the variation to the model in the first part of this chapter.

Linda and Colin were reported as having weaknesses in all three areas of their required knowledge, although Linda’s weaknesses were more severe that Colin’s. The impact on the model given these multiple weaknesses is represented in Figure 5.10. Within the model, it can be observed that with each type of knowledge that is weak, the circles in the model shrink, causing a severe diminution in the area of the central intersection. This represents severely diminished pedagogical content knowledge for these two teachers. In the case of Colin, the severity was not as drastic as in the case of Linda, who provided evidence that her pedagogical content knowledge was very weak throughout the teaching of her measurement lessons.

A limitation of this study (comprising only four teachers) was that no teacher was shown to have weaknesses in exactly two types of knowledge. However, it would be reasonable to extrapolate from the evidence presented in this chapter that weakness in two of the three types of knowledge would impact on pedagogical content knowledge more than in the case of Lois, but arguably less than in the case of the two novice teachers, Colin and Linda.
Figure 5.10. The impact on PCK when all three areas of knowledge are weak.

Figure 5.11 represents variations of the model showing the impact on pedagogical content knowledge given the possibilities if two types of knowledge were weak. As has been represented by these variations to the model, the impact on pedagogical content knowledge is severe. When teachers have limited knowledge in two of the essential areas it must impact on their ability in many, if not all of the following:

- to plan effectively
- to determine appropriate outcomes
- to select appropriate activities
- to know what representations will help their students gain an understanding of the concepts being studied
• to use appropriate behaviour-management strategies

• to engage their students

• to assess and evaluate in appropriate ways.

Figure 5.11. Variations to the model if two knowledge types are weak.

As has been demonstrated throughout this chapter, a weakness in any one, or in multiple types of knowledge, will impact on pedagogical content knowledge. The model used to represent this is dynamic in nature as it allows all of these variations to
be represented for any teacher. The use of this model has also demonstrated and reinforced the notion of Shulman (1987) and Ball et al. (2008) that pedagogical content knowledge is a complex phenomenon and care needs to be taken when discussing weak pedagogical content knowledge that accurate causes are attributed to the weaknesses disclosed.

The final section of this chapter will briefly examine other factors besides the knowledge of the teachers that were found to impact on how teachers used their knowledge and, consequently, impacted upon their pedagogical content knowledge.

**Other Factors Impacting on Teachers’ Pedagogical Content Knowledge**

Pedagogical content knowledge is a complex phenomenon. The model, while a dynamic and useful way to represent pedagogical content knowledge, is not without its limitations. One cannot reduce such a complex phenomenon to a few simple circles without acknowledging that there are other factors that influence what teachers plan, how they plan, and the way they actually teach. Three such factors were considered:

- teacher beliefs
- teacher self-efficacy
- the cultural context of the school.

The final section of this chapter discusses each of these three factors, examining data collected while observing and interviewing each teacher. Table 5.1 provides a condensed summary of this data. This was then used as an interpretive
device to examine whether these factors had any impact upon the teacher’s pedagogical content knowledge.

There were occasions when teachers provided evidence of having certain knowledge, but one or more of these three factors prevented them from using this knowledge. As is evidenced in Table 5.1, considerably more data related to the teachers’ beliefs than for self-efficacy and the cultural context of the school. This was due to the opportunities provided for each teacher to share their beliefs throughout the interviews.

Table 5.1
Other Factors that Impacted on Pedagogical Content Knowledge

<table>
<thead>
<tr>
<th>Factor</th>
<th>Colin</th>
<th>Linda</th>
<th>Carla</th>
<th>Lois</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher beliefs</td>
<td>Importance of prior knowledge, need for maths to be fun and relevant, importance of estimation, need to plan extension work, importance of representations and models, importance of problem solving, cooperative group work, need for accuracy.</td>
<td>Importance of prior knowledge, need for maths to be fun and relevant, importance of estimation, need to plan extension work, importance of representations and models, importance of problem solving, cooperative group work, need for accuracy.</td>
<td>Importance of prior knowledge, need for maths to be fun and relevant, importance of estimation, need to plan extension work, importance of representations and models, importance of problem solving, cooperative group work, need for accuracy.</td>
<td>Importance of prior knowledge, need for maths to be fun and relevant, importance of estimation, need to plan extension work, importance of representations and models, importance of problem solving, cooperative group work, need for accuracy.</td>
</tr>
<tr>
<td>Cultural context of the school</td>
<td>Isolated from other staff, minimal support from others.</td>
<td>Isolated from other staff, considerable parent</td>
<td>Usually plans with other teachers, good support within</td>
<td>Isolated from other staff by choice.</td>
</tr>
</tbody>
</table>
Teacher beliefs.

Colin, Carla and Lois were all quite emphatic about their beliefs. However, Carla was the only teacher whose beliefs matched her performance and her teaching provided evidence that she transferred her beliefs into practice. Each of the other three teachers demonstrated a degree of disequilibrium between their stated beliefs and their actual practice.

For one to believe, one must first have knowledge about what it is that one believes. Pedagogical content knowledge is an amalgam of knowledge of mathematics, knowledge of students and knowledge of teaching. This can result in teachers holding strong beliefs in one area but not necessarily being able to translate those beliefs into their teaching, due to a weakness in another area of knowledge.

This study suggested that teachers’ beliefs are not always evident within their teaching. Both Lois and Colin held beliefs about the importance of teaching for relational understanding. The use of representations was important to them, as they believed that external representations were necessary for the development of internal representations or mental models. This belief was held by both teachers as a result of professional development programmes they had attended and general teaching knowledge to which they had been exposed. Colin’s belief in representation was what made him take his class out onto the oval to experience a kilometre. However, although his belief in representation was strong, his own instrumental knowledge let him down when he needed to apply this to specific situations.
When a teacher represents a concept, they first must have a clear understanding of the salient features of that concept, as representations fundamentally must work in the same manner as the real concept. Colin’s attempt to represent a large linear distance with a coiled representation and then with an estimation of that distance based upon time, failed to recognise the salient features of a kilometre. Hence, there was an incongruity between his belief related to his knowledge of teaching and his ability to put this into practice, due to his weak knowledge of mathematics.

Colin and Linda provided another example of where beliefs influenced pedagogical content area when they included estimation within their teaching. Both of these teachers expressed strong beliefs about the importance of estimation, apparently based upon ideas that they had been given during teacher education programmes. From their knowledge of general mathematics teaching strategies, estimation was considered important. Whether either teacher really understood the purpose of estimation or indeed, whether they understood what estimation involved is questionable, since both included estimation in their teaching in a mechanistic manner. It was always a step prior to actually measuring.

However, neither teacher actually used estimates correctly within their teaching or made an effort to help their children determine how they could improve their estimates. Linda saw estimation as just making ‘wild guesses’ and Colin responded to all estimates with ‘Right’, which was interpreted by the children as an affirmation of their estimate. Their belief impacted upon their pedagogical content knowledge as the knowledge they had from one area resulting in their belief was not matched by knowledge from another key area. Hence, they included estimation in
their lesson but their weaker mathematical knowledge prevented them from doing so with any effect.

Whether beliefs are attached to the teacher’s knowledge of mathematics, knowledge of students or knowledge of teaching, these beliefs have a meaningful impact on their teaching. If there is a weaker knowledge in one of the other two areas, then there will be a degree of disequilibrium in the teacher’s pedagogical content knowledge, as illustrated with the examples above. Figure 5.12 illustrates teacher beliefs impacting directly on pedagogical content knowledge.

Beliefs will always have an impact on teachers’ pedagogical content knowledge. When all knowledge types are strong, as in the case of Carla, then the impact of these beliefs on pedagogical content knowledge is positive. However, a belief from one knowledge area that is not supported by strong knowledge in the other two areas is likely to result in a negative impact.

*Figure 5.12. The impact of teacher beliefs on pedagogical content knowledge.*
Self-efficacy.

Carla, Lois and Colin all demonstrated high self-efficacy. Each was confident, believed that they were effective teachers and were helping students in their classes to learn. Their responses to the items on the reflective questionnaire indicated this level of confidence. Each of these three teachers had well-organised and well-managed classrooms. The efficiency of classroom operation for each of these teachers also led them to believe they were effective teachers. This high self-efficacy resulted in all three teachers planning activities that engaged children and allowed them to participate in groups and use a variety of materials. Figure 5.13 illustrates self-efficacy being added to the model as another factor that can impact on pedagogical content knowledge.

Figure 5.13. The additional impact of self-efficacy on pedagogical content knowledge.
There was clearly an impact upon their pedagogical content knowledge and how this transferred to their day-to-day teaching. Each was prepared to put into practice much of what they knew because they felt confident that they could do so successfully and that their children would receive more effective learning experiences as a result.

It is of particular interest that Colin believed he was an effective teacher and that his students gained positive learning experiences as a result of his teaching. He did not believe his own knowledge was weak, although he kept a mathematics dictionary on his desk to help him with mathematical language as the need arose. Yet, the results of this study have indicated his knowledge to be weaker than he realised and, as a result, his pedagogical content knowledge is weak. With weak pedagogical content knowledge, his teaching was not as effective as he believed. High self-efficacy did not transfer in the way Colin believed to highly successful teaching.

Linda was the only teacher in this study whose self-efficacy was quite low and her classroom did not display the same well-organised and well-managed environment. Her class was noisy and disruptive much of the time. As a result, Linda did not feel confident about her teaching and often her knowledge was put to one side, as the major focus of much of her teaching was to maintain order. Her selection of activities and content was not determined by her knowledge as much as it was by her need to keep her class under control.

Linda expressed beliefs in the importance of problem solving and the need for mathematics to be enjoyable and relevant, yet the knowledge to put her beliefs into practice was not demonstrated, as she constantly chose approaches that were intended to maintain control, due to her fear of her class becoming disruptive. Even her lessons
in which the children were allowed to be actively involved in measuring were under
the supervision of parent helpers. Linda ensured she had enough parents to supervise
each small group. Linda’s low self-efficacy directly affected her teaching choices and
it clearly impacted on her pedagogical content knowledge.

**Cultural context of the school.**

Each teacher worked in a school whose culture influenced them and the
choices they made. Linda, Lois and Carla all taught in the same school and yet the
culture and the environment of the school affected each one differently. The school
was designed to be open architecturally, encouraging a team-teaching approach.
However, the school had grown in population more quickly than could be
accommodated with permanent buildings and a number of temporary, demountable
buildings had been situated on the outer perimeter of the complex. The Year Three
and Year Four classes were allocated to these temporary classrooms.

Carla was teaching in a quite large, double classroom with one other teacher.
She stated clearly that she and the other teacher planned together and taught in a
team-teaching approach. Teaching separately was something she engaged in
specifically for this study, as the other teacher preferred not to be involved in the
study. Carla and her fellow teacher worked well together. They not only had a good
working relationship with each other but they also had developed effective classroom
routines and their children were always engaged and on-task. Their room was quiet,
other than acceptable ‘work noise’. Carla was comfortable within the context in
which she was required to teach. The school culture of team teaching worked for her
and suited her own teaching style. The temporary classroom appeared to have no
adverse impact on her teaching, as she was confident in performing effectively and
was secure in her working relationship with the other teacher sharing the
demountable classroom.

Lois was an experienced teacher who had confidence in her own teaching.
She preferred to plan alone and although others in the school worked in teams, she
did not. She was the senior teacher responsible for the middle primary years. Part of
her responsibility was to supervise the other Year Three and Year Four teachers and
encourage them to plan cooperatively. This included supervising Carla and Linda.
While she encouraged others to teach and plan with other teachers, she did not insist
on this, as she knew her preference was to plan and teach alone. The culture of the
school did not impact significantly on Lois, as she was strong in her own beliefs and
practices.

Linda was the only teacher on whom the culture of the school clearly made an
impact. Linda realised that the other teachers insisted on a quiet working atmosphere
in their classrooms. She knew this was her weakness, and as discussed in previous
sections, her need to control her class dominated her planning and decision making
about what and how to teach. Unlike the other teachers, Linda was allocated to a
smaller, transportable classroom and was the only teacher in the building. This left
Linda feeling isolated but nonetheless conscious that there was always considerable
noise coming from her room.

Being physically isolated should not have prevented Linda from planning
with other teachers and sharing ideas. However, this was not the case. Linda planned
by herself and with her inexperience and her lack of effective classroom behaviour-
management strategies found herself in a situation where she was not receiving the
mentoring and support she needed. This was made clear when Linda wrote on her
reflective questionnaire that she had volunteered to be a participant in this study because she hoped she could also learn from her involvement. Linda was looking for help with her teaching, but the way the school was structured, she received none. The pressure to perform in a quiet classroom like the others around her forced her to make decisions about her teaching that potentially impacted on her knowledge of what could have been taught if classroom management had not been such a high priority. Figure 5.14 illustrates the addition of the culture of the school as a final factor that was shown to impact on pedagogical content knowledge within this study.

Figure 5.14. The impact of all three factors on pedagogical content knowledge.
Colin, although at a different school, found himself teaching in a similar situation to Linda. He too was a new teacher who was teaching in a separate, traditional type classroom and left to plan by himself. All the teachers at Colin’s school taught in traditional classrooms and each teacher took responsibility for their own planning. The cultural context in Colin’s school followed a philosophy of ‘each teacher for themselves’. Colin expressed frustration about this on several occasions, but was learning to survive by himself and believed he was doing so quite well.

It may be that if Colin had been in a situation where he could plan and share with others, that some of his weakness could have been rectified. In shared planning sessions, teachers could have discussed mathematical language and how they would introduce it. Appropriate representations could have been discussed and perhaps errors like representing a kilometre with estimates of time could have been anticipated. The culture in Colin’s school left each teacher to learn to survive by their own means. Colin reported to the researcher that no other professional person had entered his classroom that year.

When teachers find themselves operating in a school where the culture has allowed teachers to be isolated and not to feel part of a wider educational community, there must be a significant impact on those teachers in terms of how they teach, what they teach, as well as having some overall accountability for their teaching. Linda operated in isolation and taught in a manner that she believed was appropriate to demonstrate that she was in control of her class. However, it has been reported in this study that her lessons were often lacking any significant mathematical focus, yet there was no real accountability for this placed upon Linda. In both Linda and Colin’s teaching, others on staff, including their principals, did not have any awareness of
what was happening in their classrooms on a day-by-day basis. It is clear that the culture of a school can influence teachers in terms of what they teach and the knowledge they either draw on or ignore to achieve their outcomes.

**Conclusion**

This chapter has reviewed the application of the model to the four case studies of Chapter Four and analysed its components in some detail. The chapter has shown that the model can effectively represent, explain and account for variations in teachers’ pedagogical content knowledge and classroom behaviours in teaching mathematics. The adequacy of the model to represent the relationship between knowledge of mathematics, knowledge of students and general knowledge of teaching was presented. In particular, the conceptual power of the model has been demonstrated and the evidence of its interpretive adequacy has been shown as it has been applied to each of the case studies.

The model used in this thesis clearly shows while two teachers may both have weak pedagogical content knowledge, the reasons for this can be quite different. This is an important step for understanding how and why pedagogical content knowledge changes from one teacher to another.

Finally, this chapter has suggested that teachers can be influenced and affected by other factors that impact on their decisions and on how they use their pedagogical content knowledge. Factors such as teacher beliefs, self-efficacy and the culture of the school were discussed, as the results of this study have suggested there are clear influences on teacher’s decisions resulting from each of these factors.
Chapter Six: Conclusion

Mathematics education has never been more important. It has been said that ‘one of the biggest problems of mathematics is to explain to everyone else what it is all about’ (Stewart, 1996, p. 1). Never before has this claim held more significance, not only to those involved in education but to society in general. The responsibility for mathematics teachers to be sufficiently skilled in order for them to ‘explain’ mathematics to their students is critical, given the rapidly emerging knowledge economy and a rise in knowledge intensity, ‘driven by the combined forces of the information technology revolution and the increasing pace of technological change’ (Houghton & Sheehan, 2000, p. 2)

Mathematics is also an essential component of the new services economy that is being driven by the information and communications technologies (ICT) revolution. As Lundvall and Foray (1996) state: ‘The ICT system gives the knowledge-based economy a new and different technological base which radically changes the conditions for the production and distribution of knowledge’ (p. 14).

Information technology is providing new ways of representing mathematical information, resulting in new challenges for both students and teachers by adding to the knowledge required by teachers and their skills in the representation of content. Students are finding more of their explanations are visual but with less emphasis upon physical manipulation of three-dimensional models. Globalisation is also having a considerable effect on education as ‘the pace and extent of the current phase of globalisation is without precedent’ (Houghton & Sheehan, 2000, p. 4).

All of these influences mean that there has never been a greater need to make mathematics accessible and understandable to all students. It follows that teachers
accepting the responsibility for preparing students for a world experiencing such rapid changes need to have strong pedagogical content knowledge.

Further, as Chapter One argued, the contemporary policy context has increased pressure for higher standards and for teachers to deliver quality instruction. National policy and implementation of initiatives like NAPLAN, the National Curriculum and the establishment of the Australian Institute for Teaching and School Leadership (AITSL), all demonstrate the need for teachers to have high quality pedagogical content knowledge as they implement and adjust to changes in curricula. This places demands on teachers to have the required knowledge to adapt to curricular change.

**The Problem and its Significance**

In framing the research inquiry in Chapter Two, it was asserted that, generally, research in education has not done justice to the complexity of teacher knowledge and, in particular, to pedagogical content knowledge. Moreover, a considerable amount of the research that has been conducted dealing with pedagogical content knowledge has fallen short of the mark, in that nearly one-third of the studies have been conducted without a close examination of a specific discipline, such as mathematics (Ball et al., 2008). It was argued the quality of teacher knowledge will only improve as a clearer understanding of this topic is achieved.

The goal of the study was to investigate pedagogical content knowledge by examining each of the three bodies of knowledge and their interaction in its complexity. A model was proposed showing how the dynamic synthesis of all three types of knowledge resulted in variations in pedagogical content knowledge
extending, developing Shulman’s (1986) work. The study demonstrated that teachers who are weak in pedagogical content knowledge may be weak for different reasons. The findings of this study suggest that each type of knowledge (knowledge of mathematics, knowledge of teaching and knowledge of students) are all critically important and a deficit in any one of these types of knowledge will lead to diminished pedagogical content knowledge.

The study has sought to provide a theoretical and practical contribution to better understand teacher knowledge by examining how Shulman’s theory concerning pedagogical content knowledge could be investigated as a multidimensional phenomenon, providing, elaborating and clarifying how required teacher knowledge is a condition for successful classroom instruction.

Chapter One identified four important issues dealing with this investigation of teacher knowledge:

1. How evident is the teacher’s depth of mathematical knowledge of measurement within their teaching?

2. How do teachers show that they address the needs of students when teaching?

3. How to teachers demonstrate their general pedagogical knowledge when teaching?

4. How is a teacher’s knowledge and practice impacted by other factors when teaching and what are these major factors?

The detailed findings of this study support the claim that the nature of the model presented in this study is dynamic and that pedagogical content knowledge
needs to be viewed as a synthesis of three knowledge types. This study has provided evidence that any model for pedagogical content knowledge is most useful when all three knowledge types are taken into account. Studies of teachers’ subject pedagogical content knowledge cannot be one-dimensional and must show how different components and their interaction affect teaching practice.

The study warrants the conclusion that each knowledge type is critical and teachers need to be assessed in terms of all three knowledge types if an accurate assessment of their pedagogical content knowledge is to be made. It has been demonstrated that there is a variety of reasons for why a teacher exhibits strong or weak pedagogical content knowledge. Any lack in a specific type of knowledge will result in a weakness in their pedagogical content knowledge. As a specific area of weakness is addressed, the overall effect will be to strengthen the teacher’s pedagogical content knowledge. Figure 6.1 illustrates this dynamic conception of the model designed to portray Shulman’s theory.
Figure 6.1. The model, showing its dynamic nature.

The green, purple and blue arrows within the three circles represent the variations that can exist for individual teachers within each type of knowledge. In the analysis presented in Chapter Five in Figures 5.3, 5.6 and 5.8, this variation was shown as limited knowledge in each area by using a dashed circle. The model represents teacher knowledge and is able to accommodate teachers with differing strengths in each knowledge type, making the model both adaptable and dynamic.

This study contributes to knowledge through its thorough and close-grained analysis of four teachers, applying the model operationalising Shulman’s theory from extensive data gathered from classroom observations, interviews and questionnaires. The research vigorously applied an interpretive framework built on the foundations of
Shulman’s concept. The research has developed and shown the application of the model in four distinct areas.

**Theoretical value.**

The research contributes to a theoretical clarification of a complex phenomenon. Shulman’s notion of pedagogical content knowledge has been demonstrated to be more than a unitary type of knowledge. Rather, it is both dynamic and synthetic in character. The model enables each of the components of pedagogical content knowledge to be examined separately along with the impact of variations in any of the three types of knowledge on pedagogical content knowledge. Strong or weak pedagogical content knowledge has been demonstrated to be a possible result from strengths or weaknesses in any one or more of the three knowledge types that synthesise to produce pedagogical content knowledge.

**Methodological value.**

The research has demonstrated that the model provides a robust platform for interpretive research. Among the teachers studied, Carla was the only one who demonstrated strong knowledge across each of the three knowledge types investigated. As a result, she was classified as demonstrating strong pedagogical content knowledge.

There were several significant factors in accomplishing the interpretive analysis of the extensive data and enabling such judgments of teacher knowledge. Firstly, the development of a methodological approach that enabled a deep and thorough analysis of the four participants discussed in Chapter Three. Secondly, the development of a comprehensive analysis of the multiple data sources generated, classifying observed meanings in a vigorous way. In this, the conceptual strength of
the model was both applied and tested. Thirdly, and equally importantly, the analysis could not have been accomplished without reliance on the researcher’s expert knowledge and experience as a mathematics educator.

**Diagnostic value.**

A further strength of the model is that it has demonstrated it is possible to use the model to identify the weakness in a particular teacher’s knowledge base and thereby determine a teacher’s individual needs for professional development. Evidence collected through interviews and classroom interactions suggested that teachers whose understanding of mathematics was largely instrumental impacted on their ability to teach measurement lessons in a relational manner. It was sufficient for weakness in only one of the three knowledge types to diminish pedagogical content knowledge.

The two beginning teachers demonstrated weak knowledge in all three areas and, consequently, diminished pedagogical content knowledge. The model proposed throughout this study used diagnostically would allow for tailor-made professional development for each of the teachers demonstrating weak knowledge in one or more areas. The effectiveness of the vignettes in capturing key incidents that point to teacher difficulties suggests a possible approach using self-reporting or peer learning to focus on ‘subject’ difficulties in the context of teacher professional development.

**Generalisability.**

A fourth area of significance of the study is its application beyond mathematics. Since this study has shown that a weakness in mathematical knowledge can impact negatively on a teacher’s pedagogical content knowledge, it is a reasonable generalisation to suggest that a weak knowledge of other subjects’
discipline knowledge, such as science or music, would also lead to weakened pedagogical content knowledge. The model used throughout this study to examine pedagogical content knowledge for primary teachers teaching measurement could be used when researching teacher knowledge in other discipline areas. The synthetic and dynamic nature of the model offers flexibility that makes it well suited to the investigation of other content areas, particularly those with a strong hierarchical organisation of knowledge. Science is an obvious area for further investigation.

**Pedagogical value for teacher education.**

A fifth value of the study is that the model offers insights into teacher knowledge that are valuable for those involved in mathematics education in teacher education programs. As programs are designed for pre-service teachers, a better understanding of pedagogical content knowledge can influence and guide the design of mathematics education units. The balance of the three knowledge types that constitute pedagogical content knowledge needs to be carefully considered by mathematics educators and decisions as to how these are taught need to be carefully made.

A key issue arising is whether to teach mathematical content and pedagogy separately, or whether to design using a more integrated approach. It may be, for example, that for primary pre-service teachers, an integrated approach may be more meaningful to create stronger links between the mathematical subject matter and the pedagogical approaches used to teach content in a relational manner. Again, the demonstrated value of a ‘critical incident’ approach to understanding teacher difficulties could find application in pre-service teacher education preparation,
sensitising novices to the need to take into account learners, teaching and subject together.

**Contribution to the Field**

This study contributes to the field of mathematics education in terms of advancing the understanding of teachers’ pedagogical content knowledge. The investigation has helped to reveal some of the complex nature of pedagogical content knowledge in mathematics education and, by implication, to other areas of the curriculum by examining its parts: knowledge of mathematics, in this case measurement, knowledge of students and knowledge of teaching. The relationship between these three knowledge types was put forward as being represented within the model first introduced in Chapter Two. The findings of this study suggest that a weakness in any one part of the model directly affects the central intersection, the area of the model representing pedagogical content knowledge.

This highlights again the diagnostic value of the model in helping to understand that teachers who demonstrate weak pedagogical content knowledge may do so for a number of reasons. This study has shown that a weakness in any one knowledge type will result in diminished pedagogical content knowledge. Weaknesses in more than one knowledge type are likely to amplify the degree of diminished pedagogical content knowledge. It was shown that whether the weakness is in the teacher’s knowledge of mathematics, knowledge of students or knowledge of teaching, this weakness impacted on four aspects of their pedagogical content knowledge, that is, on children’s learning, on the teacher’s planning, on teacher’s representations and how they incorporated these representations into their teaching. These weaknesses affect the teacher’s ability to make accurate judgments. The
evidence will reveal itself in different ways depending on which knowledge type is weak, but all affected the above four areas. Hence, this study has provided behavioural indicators that are likely to provide evidence of weak pedagogical content knowledge, as Figures 5.2, 5.5 and 5.7 demonstrate.

How far the findings of this study can be generalised is an open question. Each teacher has a unique combination of knowledge that is personally constructed and attached to many personal experiences. Yet, the study has demonstrated the robustness of model is in its application and suggests the approach used for this study is generalisable and applicable across settings into other curriculum areas and teachers of other year levels. Much of a teacher’s knowledge would be shared with other teachers and it would be reasonable to speculate that the findings of this study would apply to other teachers. Further research needs to be done replicating and extending the framework developed for this study, testing in other contexts how a weakness in any of the knowledge types will impact directly on a teacher’s pedagogical content knowledge and what may be done, in school and professional contexts, to address such weaknesses.

Implications for Further Research

Unravelling the complexities of pedagogical content knowledge is worthy of continuing research. This study was limited to looking at primary school teachers of Years Three and Four teaching measurement lessons and there is a need to examine other topics from the mathematics curriculum. How does any one individual teacher’s pedagogical content knowledge vary as they change from one topic to another? Can a teacher who is strong in their knowledge of measurement be weak when teaching fractions? Would a teacher like Carla be shown to have strong pedagogical content
knowledge if the topic had not been measurement? Is pedagogical content knowledge highly variable within one teacher’s practice or is it consistently weak or strong?

Many questions remain unanswered from such an evidence base. This study has demonstrated that pedagogical content knowledge is dependent upon three types of knowledge teachers are expected to have as part of their professional knowledge. The study has demonstrated how the model representing Shulman’s theory is rich in research potential and illustrated the value of in-depth analysis of teachers’ knowledge. However, there is a need to investigate this model further. How do changes in any one part of the model affect pedagogical content knowledge? For example, if an experienced teacher judged to be strong in pedagogical content knowledge changed class to another year level, would this teacher’s knowledge of the new class be as thorough as the knowledge demonstrated for the previous year level? What effect would changing to a new group have on pedagogical content knowledge? Further studies need to engage with the complexity of the phenomenon in the manner this research has indicated.

**Implications for Policy and Practice**

The implications of this study result in key recommendations for practice. A major reason for this study was to investigate Shulman’s notion of pedagogical content knowledge in an attempt to gain a better understanding of the knowledge teachers possess and need. As this knowledge becomes more clearly understood, the benefits of this can flow on to improving the quality of teaching practice. There are three areas where the findings of this study can lead to recommendations for practice.
**Recommendations for school-based programs.**

The weaker teachers in the study were teaching in schools where they felt isolated and lacked support from other staff. They were beginning teachers and needed to develop stronger knowledge in each of the three types of knowledge discussed throughout this study through support and mentoring from more experienced teachers on staff to overcome low self-efficacy and isolation. Less experienced teachers will benefit from teaching in the same school where shared planning and support are part of regular teaching.

Findings from this study could have implications for policy development for the close mentoring of beginning teachers. It is to be expected that schools provide adequate support for teachers of probationary status. There are important implications for policy given the current emphasis on teacher registration and the professional standards teachers are required to demonstrate. School principals and senior teachers need to become more aware of the needs of beginning teachers and to provide adequate mentoring programs to ensure beginning teachers continue to develop, particularly in all areas of required knowledge.

It is not always beginning teachers who need support and professional development. Again, the vignettes relied on in this research suggest how mentoring could be sensitive to cultural incidents, pointing to difficulties with pedagogical content knowledge. Senior teachers with responsibility for others may have knowledge of mathematics that is predominantly instrumental. Schools need to ensure that professional development is available for all staff to work on areas of need.
Within schools, professional conversations need to take place between teachers, with particular emphasis on senior school staff leading and directing these conversations. Above all, these conversations need to focus on teacher knowledge in its three aspects, but more particularly, subject knowledge. Identification processes need to be initiated within the school and ways of identifying areas where teachers require further development need to be established. How this could be more extensively achieved is itself a further area in need of research, in addition to the areas already identified in the previous section.

**Recommendations for system-based support for professional development in mathematics teaching.**

Most teachers find themselves teaching in schools that are part of a larger system. Whether, as in the case of this study, the system is government controlled, or whether it is a system controlled by another organisation such as the Catholic Education Office, teachers are influenced by policies and curricula from a systemic level. Therefore, the system needs to take considerable responsibility for the ongoing professional development of its teachers. In doing so, it needs to move from the traditional ‘one size fits all’ professional development course that assumes all teacher needs are the same, to a more diagnostic approach to professional development.

Professional development needs to be based upon a design model that recognises that pedagogical content knowledge consists of different knowledge types. This recognition will lead to courses that through their design will accommodate multiple needs and cater for individual teachers according to their needs, to strengthen their pedagogical content knowledge. Again, there needs to be a conscious focus on teacher professional knowledge, rather than pedagogical ‘practice’.
This study demonstrates conclusively that teachers may have weak pedagogical content knowledge for a number of reasons, so system-wide professional development courses need to be designed and offered to teachers addressing different weaknesses. Professional development focusing on the introduction of new curriculum may not be appropriate if the teacher’s knowledge of mathematics is fundamentally the problem, or if their knowledge of students is weak. Only when this complexity of pedagogical content knowledge is acknowledged and incorporated into professional development design will teacher standards for teacher registration be realised more successfully. A planned approach to professional development should consciously address the different components of teacher professional knowledge.

Findings from this study can provide evidence to system personnel for the need to offer tailor-made, professional development that includes all three types of knowledge. Teachers need to be selected for courses that closely match their area of need. Ideally, the three areas of teacher knowledge should become part of the professional development design language. A key challenge in realising this goal is developing instruments that enable individual teachers to map their own pedagogical content knowledge in terms of these three knowledge types.

Systems need to accept their role and responsibility for ensuring appropriate and continued professional development for their teachers. The findings of this study have many implications in identifying the needs of teachers concerning their further professional development. This study has highlighted the importance of each of the three types of knowledge in terms of their impact on pedagogical content knowledge. More explicit inclusion of content for all three knowledge types needs to be included in future curricula documents.
Recommendations for teacher education.

Universities have the responsibility for the initial education and preparation of teachers during their pre-service years. This study has implications for practice by influencing the decisions made by teacher education faculties. It has been widely researched that there is an ‘inadequacy of mathematical skills of many students entering primary and early childhood education courses’ (Taplin, 1995, p. 498). This consequently affects their ability to feel confident when confronted with the reality of having to teach mathematics. Those responsible for designing units for mathematics education need to ensure that the three knowledge types are taught in an integrated manner.

At the very least, mathematics educators need to be aware of the importance of the knowledge of mathematics, the knowledge of students and the knowledge of teaching in their combined contribution to helping teachers form their own strong, rich, pedagogical content knowledge. Graduates of teacher education courses who intend to teach mathematics to children need to enter the profession with a strong mathematical understanding and an awareness of appropriate pedagogical practices for the students they will teach.

Conclusion

The understanding of pedagogical content knowledge can only become clearer and richer as teachers allow researchers into their classrooms to examine and document their teaching and work with them in collegial ways to improve teacher quality. Within this study, four such teachers generously allowed their practice to be examined while teaching a series of measurement lessons, showing in acute detail how teachers like Carla are helping children discover the ‘real nature’ of
mathematics. They are enabling children to decode the symbolism to reveal and understand ‘its substance’. The other teachers who participated are representative of the professionals who are trying to achieve those same results but are restricted by weaknesses in their own knowledge base. Due to the openness and willingness of these four teachers, this study has been able to contribute to an increased understanding of pedagogical content knowledge and has helped to clarify future directions for professional development in mathematics teaching.

While there have been calls for exemplary teachers to be studied to investigate what constitutes effective pedagogy, this study has demonstrated the value of examining the practices of a variety of teachers and their practice. By examining teachers with weaknesses in one or more knowledge types, this study has been able to demonstrate the effect and implications of these deficiencies upon pedagogical content knowledge.

Pedagogical content knowledge, like mathematics, may be complex and difficult to explain. However, like mathematics, it has significance and an improved understanding of this complex phenomenon will lead to improved teaching and a more mathematically informed population.
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Appendix A: Initial Interview Questions

Initial Interview Schedule

The following questions formed the basis of the initial interviews. However, they were not the only questions asked. Each teacher’s responses were followed through with more probing questions if further elaboration was required.

1. How long have you been teaching?

2. How well do you feel you were prepared for teaching mathematics in your initial teacher education programme?

3. In the time since you graduated, what have you done to develop professionally in the area of mathematics teaching?

4. What is mathematics?

5. What would you say about your own background in mathematics? How do you perceive yourself?

6. Why is mathematics taught in schools? What are your basic beliefs?

7. Tell me about how you plan your mathematics programme and explain to me the school’s expectations of you when you develop your mathematics programme.

8. What do you think are the important priorities that you have in mind when you're planning?

9. Do you use manipulative materials in your approach to teaching mathematics? If so, what determines when and how you use them?
10. How do you incorporate problem solving into your overall mathematics programme?

11. How do you incorporate the use of technology into your overall mathematics programme?

12. How would you describe the role the students in your class play in a mathematics lesson?
Appendix B: Questions for Interviews Prior to Lessons

The following questions formed the basis of the interviews prior to each lesson. The questions were deliberately few in number. Their purpose was to provide the researcher with a clear statement of intent for each lesson. However, they were not the only questions asked. Each teacher’s responses were followed through with more probing questions if further elaboration was required.

1. What are the expected outcomes of today’s lesson? What are the key concepts for your students to learn?

2. What resources did you use to help in the preparation of the lesson?

3. How do you plan to engage the children within this lesson?

4. How will you determine the success of the lesson? What will you be looking for when assessing throughout the lesson and making final judgments about the success of the lesson?
Appendix C: Questions for Interviews After Lessons

The following questions formed the basis of the interviews after each lesson. Their purpose was to provide the researcher with a clear statement of how the teacher perceived the success of each lesson in terms of their original, intended outcomes. However, these questions were not the only questions asked. Each teacher’s responses were followed through with more probing questions if further elaboration was required.

1. How successful was today’s lesson? What did you set out to achieve and to what extent did you achieve it?

2. If you were now going to teach another class the same lesson, is there anything you would change based on what’s happened with this group?

3. What sort of things were you looking for as you moved around the room throughout the lesson?

4. When you follow on from this lesson, what do you think would be logical flow-on activities? Where would you go from here?

5. When you planned this lesson, had you anticipated any problems? Were there any unanticipated problems?

6. How do you think the activities you planned today helped the students in your class develop understanding?
Appendix D: Samples of Vignettes Written as Part of the Data Analysis Process

The following samples have been selected to provide an illustration of how specific incidents were initially interpreted and written as brief vignettes. Each vignette elaborated on an incident portraying an aspect of one the three types of knowledge: knowledge of mathematics, knowledge of teaching and knowledge of students.

On the following pages there are four vignettes, a sample from each of the four teachers:

Vignette #14—Colin, ‘Estimating mass’—provides insights into Colin’s knowledge of teaching.

Vignette #1—Linda, ‘Instant child sweetener’—provides insights into Linda’s knowledge of students.

Vignette #3—Lois, ‘Selecting an appropriate unit’—provides insights into Lois’ knowledge of mathematics.

Vignette #11—Carla, ‘Determining errors’—provides insights into Carla’s knowledge of students.
Vignette #14—Colin

Estimating mass.

After stating that the activity the class would be doing was called ‘A tonne of children’, Colin introduced the lesson on mass with the question, ‘What do you think an elephant would weigh?’ The purpose of this question was to focus the children’s attention on things that were obviously heavy. He wanted to lead the children to think about the unit of mass the ‘tonne’.

The responses to his question would indicate that the children did not necessarily provide much thought to their responses, but took the title of the activity to provide them with contextual clues to the answer to his question. Two different responses were forthcoming but both directly related to his initial comment, ‘This activity is called “A tonne of children”’. The first response came from Jessica, ‘About five maybe, ten, twenty, children’.

It is evident that because Colin has called the activity ‘A tonne of children’, Jessica has assumed that an elephant’s mass is a tonne and the question needs to be interpreted as how many children in a tonne, hence her reply. It is clear from the answer, ‘five maybe, ten, twenty’, that she is guessing. Colin then realised that Jessica had connected the title of the activity and his question, ‘What do you think an elephant would weigh?’ He was not expecting this, but was hoping for an estimate of an elephant’s mass in formal units (i.e. kilograms). He continued with the following discussion:

Colin:  Alright. If you're going to, let's not think about how much they'd weigh in children. How many they'd weigh in, what would we measure an elephant in do you think?
Child: (Indiscernible.)

Colin: Okay a, you say kilograms, a tonne. A lot of us said a tonne. What is a tonne? Anna, what's a tonne? Other people called out a tonne. What's a tonne, Rene?

Rene: A thousand kilograms.

Colin: A thousand kilograms. How do you know that?

As is evident from this segment of transcript, many of the children called out ‘a tonne’. It appeared that if Colin did not want to know how many children would be the same as an elephant, then the other significant component of the activity title was the word ‘tonne’. Hence, it was almost in unison that ‘tonne’ was offered as an answer.

At this point, Colin proceeded to talk about a kilogram. He asked the children what they had in front of them that might weigh a kilogram and then to hold something up. The following interactions took place:

Colin: Hold up something that might weigh approximately about a kilogram. Hold up a couple of, what have you got there Angus? One book. Let me feel your book. Alright, maybe, maybe that'd be about half a kilo. That's definitely not going to be a kilo. Think about, what are some of the packets that we have? Ah, listening, looking this way, Andrew. Now think about some of the packaging that we have that you buy at the shops. What does margarine come in, containers, what does?

Child: Two hundred and fifty grams.
Colin: Two hundred and fifty grams. You can also have it in five hundred grams and the big one is often a kilogram. Alright. What else do we have? Do you think that would weigh about a kilogram? That's getting close that one there.

Child: What about this one?

Colin: That's not too bad. Not too bad. That's getting close too, that's a heavy book. Alright. You reckon that nearly, yeah. Alright. Okay, you've all, people, all of us have all felt a kilogram before, we've touched a kilogram, we've been shopping, we've picked up a kilogram, things like that. You all know how much you weigh, in kilograms. You should!

Colin is placing considerable emphasis on life experiences that he assumes children have encountered. Comments like, ‘all of us have all felt a kilogram before, we've touched a kilogram, we've been shopping, we've picked up a kilogram’ infers that he believed he did not need to include this in his teaching. It was assumed that children knew what a kilogram was. It is also evident that while he did not explore the relationship between grams and kilograms, this was assumed to be knowledge that the children would have.

Colin had no way of testing to see whether the objects the children were holding up did have a mass of approximately a kilogram. Therefore, the belief that Colin frequently put forward that children need to develop mental models of mathematical ideas is in conflict with his practice at this point of time. Mass is not a visual aspect of an object, but rather needs to be experienced by hefting; it would not be unreasonable to have expected Colin to have objects available for the children to heft which had a mass of one kilogram. This also would have reinforced the fact that
mass is not determined by size (i.e. he could have had a kilogram of nails, a kilogram of wool etc.). It is not clear that the children actually had a developed concept of mass. The attribute of mass was not explicitly dealt with for the children to abstract the idea.

Instead, Colin asked the children to hold up objects based on if they thought it might be a kilogram. Based on his reaction, ‘That's definitely not going to be a kilo’ to one child, it is clear that not all children had realistic ideas about what measured a kilogram. Yet, he either had no way of dealing with these misconceptions or chose not to. Therefore, when the class refocused on, ‘What do you think an elephant would weigh?’ each child was estimating with a different referent unit of mass. They may have all been using the same name—kilogram—but based on their personal experiences (or lack of), each of their ‘kilograms’ was different. Added to this is the fact that it is unlikely that any of the children really knew how heavy an elephant might be. It is unlikely that this was part of the children’s life experiences. Hence, the following estimates and discussion were forthcoming:

*Angus:* Um, three tonnes.

*Colin:* Three tonnes. So how many kilograms would that be?

*Angus:* Three thousand.

*Colin:* Three thousand kilograms.

*Child:* Seven hundred and seventy-one kilograms.

*Colin:* You think about seven hundred and seventy one. How did you come to that figure?

*Child:* I don't know. I just.
Colin: That's just what you think in your head?

Child: Yeah.

Colin: Nathan, what do you think?

Nathan: About three or four thousand kilograms.

Colin: Three or four thousand kilograms. How many tonnes would that be?

Nathan: Three or four.

Colin: Three or four tonnes. Alright.

Colin immediately moved on from the elephant situation to:

Colin: What about a car? What do you think a car would weigh? Just say um, say my car. Say a Holden Commodore or a Camira or something like that. What do you think?

Child: A tonne and a quarter.

Colin: A tonne and a quarter. What do you think? If you've got something to say, put your hand up. What do you think it'd be, Brent?

Brent: About five or twenty tonnes.

Colin: Five or twenty tonnes. Um, five or twenty, what do you think it would be? Do you think it's something in between there?

Brent: About seven.

Colin: About seven. Evelyn, what do you think?

Evelyn: About one and a half tonnes.

Colin: One and a half.
Child: Oh, it depends what's in the car as well.

Colin: Alright, that's good. It depends on what's in the car.

Brent’s answer of ‘about five or twenty tonnes’ provides an indication of what kind of mental model of ‘kilogram’ he was operating with. When asked if he thought it might be something in between, ‘about seven’ was offered in what appeared to be a random guess. After some further discussion about trucks that are called ‘one tonners’, Colin moved on to asking what the collective mass of the class would be. It is worth noting that although the questions about the elephant and car were asked, at no stage was the mass of either given. After all the discussion about each, the children were no wiser on whether their estimates were close or not.

Nothing was done throughout this lesson by Colin to help the children gain an understanding or to experience how heavy a kilogram is. For a teacher who professes the belief in hands-on teaching, enabling children to construct mental models and helping children make sense of mathematics (i.e. make meaning)—very little of this lesson appeared to reflect this. The major experience of this lesson about measuring mass was through talk. Without physical experiences in hefting, any refinement of the children’s estimation of mass seemed unlikely to occur. The small amount of estimating that children were engaged in seemed to be inappropriate and heavily influenced by Colin’s own introductory comments. If you tell a class the lesson is about a ‘tonne’, there is a fair chance that when asked to estimate the mass of an object the desired response will be a ‘tonne’.
Vignette #11—Linda

‘Instant child sweetener’.

This lesson was one from a series of lessons taught by Linda on measuring capacity. All the lessons so far had been aimed at Year Three and in each of the previous lessons, Linda had provided different work for her Year Four children. During this lesson, both Year Three children and Year Four children were involved in the same activity.

The lesson consisted of one activity followed by the children writing a report on what they had done in the activity. The lesson was intended to provide the children with practice at measuring using millilitres. To motivate the children and provide an across-curriculum link, Linda related the activity to what they had been doing in language. She introduced the activity as follows:

Right. And we've been reading the witches and the witches are out to turn children into mice. Well we're going to try and stop the witches from coming to Australia and turning you into mice and we're going to make an instant child sweetener. I'll just get the recipe ready. I want you to watch.

She then demonstrated the procedure to the children so that they would know what to do. All that was required was for the children to measure 20 ml of one ingredient, 20 ml of another and add a teaspoon of sugar.

Linda continued with the following instructions:

Linda:  Okay, it says measure into a cup 20 mils of wombleberry juice, and the wombleberry juice, each group will have two containers. You have to be very careful because it's filled very high. Sit down please Matthew. And you'll find you've got red wombleberry juice and you have to pour it into
the jug. I've just gone a bit over twenty mils and you pour that into your cup. Okay. In the other container, you've got some blue liquid and that is royal beetle's milk.

Children: Oh (and other noises).

Child: It wouldn't be that bad.

Linda: It's not. It's yum. Then you pour that into the cup. Who knows what colour it might turn? Put your hand up if you know. Damon.

Damon: Purple.

Linda: Purple. What do you think?

Child: Blue.

Linda: Who thinks it will turn blue? What do you think?

Child: Purple.

Linda: Purple. No, we're not working with sticky tape just now. Now, then we put, we put in a teaspoon of sugar and it just has to be a very tiny teaspoon. A teaspoon of sugar. You put all of that into your cup and you mix it up and you have to drink it immediately and it'll turn you into sweet children.

The children were then divided into four groups and commenced making their ‘brew’. Linda had arranged for two parents to be present to assist in supervising the activity because of the nature of the activity and the possible spillage that could occur. Each child took their turn at making the recipe and drinking it, once all three ingredients were combined.
When all the children had made their drink, Linda asked them to sit at their desks and to write up a report on what they had just done. In writing the report, the guidelines Linda provided was to write the recipe, explain any problems they had measuring, why they had them, how they manoeuvred around them and to include anything else they thought was important. This constituted an entire, 45-minute mathematics lesson. There was no time at the end of the lesson for discussion or sharing of reports.

From this lesson emerged several aspects worthy of discussion.

**Purpose of the lesson.**

As explained by Linda in the interview after the lesson, the purpose of this lesson was to:

*Okay, once again it was following on with the mils and trying to get them to use a smaller unit; we were using the twenty mils. Um, and I think it was important for the kids to realise that they had to have their measurements exact today and um, so they had to have exactly twenty mils and um, using the colours to get the colour right. I think that was a good evaluation. Um, there were problems with the size of the measuring glasses but that was just all the school had to use.*

Linda demonstrated an interesting perception in this explanation that had been inferred in previous lessons but not as explicitly as here. She refers to the 20 millilitres as ‘a smaller unit’—‘we were using the twenty mils’. In previous vignettes dealing with Linda’s lessons, the lack of modelling the unit ‘millilitre’ has been discussed. In each previous lesson where Linda has claimed to have been teaching millilitres, she has, in fact, focused on a specific quantity such as a small, 50 ml
bottle, two of these bottles at a time being 100 ml; a 100 ml measuring cylinder; a
600 ml milk bottle; a litre milk container. At no stage did she make a conscious effort
to develop an understanding of the unit ‘millilitre’.

Her explanation above, along with the emphasis she placed on activities in her
lessons would suggest that Linda does not see the importance of the single millilitre
as a unit. Rather, she sees ‘the unit’ being whatever number of millilitres one
designates. This description of 20 ml as ‘a smaller unit’ and the expression ‘we were
using the twenty mils’ are not just accidental expressions or slips of the tongue. These
expressions are consistent with the way in which Linda treated the unit ‘millilitre’
throughout the entire series of lessons.

Linda obviously acknowledges that single millilitres do exist but does not
appear to consider them important enough to include in her teaching. She seems
happy in the belief that you can nominate any given number of millilitres to be the
basic unit of measurement (although evidence would suggest that she believes
multiples of ten make more appropriate units, e.g. 100 ml, 50 ml, 20 ml). Once you
nominate 20 ml to be the amount to be measured, it appears as if Linda believes that
is the unit. Hence, in the recipe for this lesson (see p. 127) each child had to measure
two units of 20 ml.

When an attempt was made during the interview to have Linda elaborate
further on this issue, she appeared unsure of what the researcher was leading to with
his questions. Her responses were as follows:

*Researcher:* Okay, so the mathematics of the lesson was specifically further
consolidation of using millilitres?

*Linda:* Hmm, hmm.
Researcher: Um, and to preciseness?

Linda: Hmm.

Researcher: So, using the unit accurately?

Linda: Yep.

Other than these expressions of agreement, Linda could not be drawn to provide any further explanation. She had already stated that the purpose was to have the children measure exactly 20 ml and there simply was no further elaboration necessary. She appeared both uncomfortable and unsure about the continued questioning. Sensing this discomfort, the researcher did not pursue this issue further.

**Measuring accurately.**

Many of the children experienced some problem measuring 20 ml exactly. Although the measuring cylinders were clearly marked, most initially poured more than 20 ml into the cylinder. This was not a major problem and not one that most children could not overcome. In most cases, it meant pouring back and forth until they were happy that they had an accurate measurement.

This problem was caused predominantly by two factors. First, the fact that a millilitre is a small amount meant that if the children poured too quickly then, before they realised, they had poured more than the required 20 ml. The second factor concerned the equipment they were using. The coloured water was in margarine-like containers that meant the children had to pour from the corner of the container.

The measuring cylinder that they were pouring into was quite high and in many cases, even when the children had thought they had stopped pouring at the appropriate time, by the time all of the water had settled they often found they had
too much. Once all the water had reached the bottom, the amount actually read closer to 30 ml.

Rather than pouring back and forth until 20 ml had been measured, some children used strategies to overcome this problem. Some simply made sure they poured very slowly. They had realised that if they poured too quickly they would inevitably end up with more than the 20 ml. A couple of children solved the problem of measuring accurately by pouring slowly and stopping before they had 20 ml. They then used a teaspoon, as illustrated below, to add small amounts of coloured water until they had exactly 20 ml. These children avoided the need to pour water back and forth until their measurement was accurate.

One final factor related to accuracy was the mixing of the blue coloured water with the red coloured water. Linda discussed with the class what the resulting colour would be and indicated that checking the colour was an effective evaluation of their accuracy:

Um, and I think it was important for the kids to realise that they had to have their measurements exact today and um, so they had to have exactly twenty mils and um, using the colours to get the colour right. I think that was a good evaluation.

Interestingly, in most instances, as soon as the child had put in the teaspoon of sugar and stirred their mixture, they drank it. Hence, in reality, Linda did not have the opportunity to check the colour of their mixture. One can only suspect that Linda was calling their bluff by saying she knew what colour the mixture should be, to encourage them to be as accurate as possible in their measurements.
There were children who did not measure accurately, were aware of this, and did not appear to be concerned. One example is a boy who was working with one of the parent helpers. The parent suggested he had too much and tried to persuade him to measure more accurately. The following discussion took place between them:

**Parent:** Are you sure you’ve got twenty mils? Put it down on the flat desk. And read the lines. No, you’ve got twenty there already.

**Child:** I know.

**Parent:** Now it’s too much. You’ll have to pour some out.

**Child:** I don’t care.

**Parent:** Are you sure you’ve got the right amount?

**Child:** Yeah.

**Parent:** I hope it’s the right colour.

**Child:** Yep.

**Parent:** Put your teaspoon of sugar in and pass these along to the next person. You’re going to be extra sweet this afternoon, aren’t you?

**Child:** Yep.

**Parent:** Good.

While most children attempted to measure accurately, this was not an isolated account of carelessness. If Linda had been serious about using colour as an evaluation of their accuracy, perhaps she could have had a sample of the mixture already made up to be used as a comparison before the children drank their mixture. If the children
thought that they had to compare their mixture with Linda’s accurately measured sample, it may have encouraged more to try to be accurate with their measuring.

**Engagement.**

Another factor that was evident throughout this lesson was the actual amount of time children spent on-task—the time they were engaged in mathematical activity. Earlier in this vignette, it was stated that this lesson’s duration was 45 minutes. Much of the time in this lesson was spent waiting for other children to take their turn measuring. If one looks at the level of engagement in this lesson for each child, it consisted of listening to the instructions and watching Linda demonstrate (introduction), making the recipe (the activity) and writing a report (consolidation).

The introduction took 6·5 minutes. This included a brief review of the previous Year Three lesson, showing the children the recipe for this lesson and the children watching while Linda followed the instructions to model for the class what they were to do. Based on video evidence, the time taken for children to make their drink varied from one minute and 24 seconds to three minutes. The mode was approximately 1·5 minutes. Linda took 4·5 minutes to explain how she wanted the reports written and what should be included in them. Most children took less than five minutes to write their brief report. In total, including listening to Linda, this meant that each child was actually engaged for approximately 17·5 minutes out of the 45 minute lesson. The worrying aspect of this time analysis is that each child was not on-task for a greater amount of lesson time than the time they were on-task. On average, each child was not engaged on anything meaningful for 27·5 minutes of the 45 minute lesson.
This was clearly a lesson that worried Linda from an organisational viewpoint. She had expressed in an earlier interview that when doing hands-on work, she often provided Year Three and Four children the same activity to reduce the ‘chaos’. In her planning, she had arranged for two parent helpers to be present for the lesson to reduce organisational problems. Yet, taking both of these factors into account, the lesson still resulted with many distractions occurring. This was due in large part to children having completed their task and becoming bored.

While it would not be beneficial to include all segments of transcript that demonstrate this problem, the following segments provide an example:

**Linda:** Shh. Sit down please. Sit down. Sit down. Hands down. Stop calling out.

We're not talking about that now, thanks. Shh. You ready? This is how I want it to work ...

**Linda:** Alright. Shh. We can't start until the noise stops. Sit on the floor. I can hear people calling out. Justin, that'll be a cross because you weren't listening. You need to show me you're listening face as well. Okay. A few questions. Samantha? ...

**Linda:** Listen, listen, listen. Come and talk to me, please.

**Child:** No. I want to work.

**Linda:** No. No. Out into the bag room.

**Child:** No-o.

**Linda:** You chose not to be in here. You need to wait in the bag room now because you chose not to be in here. Out you go. Go on …
Linda: Alright. (Claps.) Eyes this way please. Shh. Matthew W—eyes this way. Shh. Shane, ready. Eyes this way. Most people have had a go at drinking theirs now. Just sit there but watch me. Shh. Right. Eyes this way. Most people have made their drink. I am waiting for listening bodies. Shh. Stop. Shh. I'm still waiting. And it looks like. Right, people that I have to wait for are (writes on board). Most people have finished their drink now.

While this kind of behaviour was common throughout the lesson, much of the children’s behaviour went either unnoticed or ignored. They found a variety of ways of filling in time once they had completed what was required of them. It should be stated that this behaviour featured prominently once children had completed their task or while they were waiting for their turn. The problem highlighted by this behaviour is that Linda had under planned. This is an unfortunate situation as it was evident that Linda had planned and had done so carefully.

Linda’s careful planning was, in fact, the problem. As she knew of the potential for this class to display disruptive behaviour, she planned the lesson in a very structured manner. In her planning, she ensured that all children would do the same activity, parents would be available to assist in supervision, there was enough equipment for four groups, the recipe was written out on card for each group, she would demonstrate the procedure first, and she would evaluate their performance by whether their mixture was the right colour. This was a highly organised lesson. In focusing on potential classroom management problems, Linda had not realised that she had under planned. As a result, the children did not have sufficient work to keep them engaged for the full 45 minutes. The main aspect Linda appears to have planned to avoid—‘chaos’ (to use her own word)—resulted because she minimised her
activities to concentrate on control. Based on the video evidence that while the children were taking their turn at measuring the mixture they performed the task well, one must assume that if there had been additional activities in the lesson to keep them engaged, Linda’s management problems would have been minimal.

**Mathematical content.**

A final factor that was evident during this lesson was the limited mathematical content covered. While this factor is related to the previous one, it is in itself a significant aspect of Linda’s lesson. Certainly, the mathematical content was limited due to only providing one activity during the lesson. However, even if considering this as a 20-minute lesson rather than a 45-minute lesson, the activity selected was still mathematically limited. Essentially, the only measuring required was for each child to measure 20 ml of coloured water using a calibrated measuring cylinder and then to repeat this a second time. Adding a teaspoon of sugar would be the only other measurement that could be considered. The addition of one teaspoon of sugar is hardly a sophisticated measurement.

The report writing did not incorporate any new mathematics. In most instances, the children wrote out the recipe and stated that one of the problems they encountered was that at first they poured too much. Hence, they had to pour back and forth until their measurement was accurate. All reports were quite brief.

There is no question that this activity has a place in a mathematics lesson. However, this was one of a series of lessons on capacity (at least for the Year Three children), and to focus only on measuring 20 ml in a 45-minute lesson appears to be mathematically deficient. The Year Three children had already used the same measuring cylinders in a previous lesson. Therefore, it cannot be considered that part
of the new mathematics for this lesson was the introduction of new measuring instruments.

This factor returns to Linda’s planning. Her emphasis on organisational aspects of the lesson in an attempt to avoid chaos had not only resulted in the class not being sufficiently engaged, but also demonstrated that Linda had not given sufficient attention to the mathematics she was teaching. Mathematical content was hard to find.
Vignette #3—Lois

Selecting an appropriate unit.

During a lesson on length, the issue of determining appropriate units for measuring length was discussed. Lois had spent time looking at the relationships between millimetres, centimetres and a metre. She had used a metre ruler and helped the class work out that there were 1000 millimetres in a metre. The following discussion then took place:

Lois: *When would I use millimetres?*

Child: *When you’re measuring something short.*

Lois: *When I’m measuring something short or something ...?*

Child: *Small.*


Child: *Tiny.*

Lois: *Tiny. Good. When might I measure using kilometres?*

Child: *When you’re measuring the road.*

Lois: *When I’m measuring the road.*

Child: *Something real big.*

Lois: *Something really, really big. When? What do you travel in?*

Child: *Cars.*

Lois: *Right, in your cars.*
This discussion proves interesting as it places emphasis on only one aspect of selecting appropriate units. While it may be an important criterion in certain situations, for example, large distances would normally be measured using kilometres and small objects may often be measured using millimetres, another important criterion for determining the appropriateness of a unit is the precision or accuracy required. This aspect is often an essential consideration when determining the purpose of making the measurement in the first place. Lois overlooked this second and perhaps more important criterion for selecting appropriate units.

Lois recognised that mathematics needs to be made relevant to children. There is a clear understanding on Lois’ part that children need to ‘see’ mathematics, yet she had not considered the reasonableness the resulting measurements would attain if the only criterion used is ‘the size of the unit must be consistent with the length being measured’. It is almost certain that Lois realised the need for extreme accuracy in many real life situations. One then is left to wonder why a teacher who, in real life no doubt realises the need for accuracy in certain measurements, does not emphasise this important criterion when teaching mathematics in school.

It could be interpreted that Lois endeavoured to simplify measurement at this stage by teaching that the determination of an appropriate unit is based on a single criterion. In doing so, her teaching is counterproductive in that it overlooks the fact that measurements are associated with a purpose. In real life, many measurements need to be precise. Accuracy is more important than whether the object being measured is small or large and yet, at no stage was there any acknowledgement of this as a criterion.
Vignette #11—Carla

**Determining errors.**

One of Carla’s lessons required children to compare their height with their arm spans. The investigation was entitled, ‘Are you a square or a rectangle?’ Carla provided a variety of measuring materials from standard tape measures and height grids to rolls of streamer paper tape.

With this variety of measuring approaches available to the class, the children became actively engaged and set about measuring with a partner. Carla made it clear that she was conscious of all students in the class and their abilities. Several times, she expressed her concern for students and her willingness to assist with comments throughout the lesson, such as:

*Carla:*  
*If you get stuck, ask for help.*

*Carla:*  
*I’ll come and help these people.*

*Carla:*  
*If you need help to measure Matthew, come back and get me.*

As well as offering support to students, Carla also monitored how they measured. She wanted every child to succeed at measuring accurately. Seeing a child struggle with measuring his partner’s height with string, Carla moved over and offered: ‘I’ll hold the bottom for you because it’s a little bit hard’. To another child, when checking the measurement he had recorded, Carla responded: ‘I’d like to see your string for that because I don’t think you’d be that far out’. To another, ‘You have to lift your arms up a bit so they’re straight’.

The anticipation of student errors was clearly an important aspect of Carla’s teaching. This aspect of teaching requires sound knowledge in all three areas. Not
only did Carla need to have a good understanding of the measurement process and which aspects were likely to be difficult for her students, but she also needed to know appropriate ways of intervening and teaching the measurement process so that students did understand. This was a dominant characteristic of all Carla’s lessons.