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GENERALIZED COMPTON EFFECT

The Compton effect equations were derived and verified experimentally in 1922 when analyzing the collision of x-ray photons, with energies around several kilo electron volts (keV), and conduction electrons with energies of a few electron volts (eV). For many years this was considered to be the only case of interest; that is, where the incident energy of the photons exceeded that of the electrons. It was during the second half of the last century that the so called "inverse Compton effect", involving the collision of relativistic electrons with laser light photons, was developed. It is interesting to regard both situations above as limiting cases of a unique equation which is derived from the relativistic equations for energy and momentum conservation in their general form. The generalized Compton effect is thus appropriate for describing the collision of a photon and an electron (or, for that matter with any charged particle) regardless of their energies.

The occurrence of the Compton effect in astrophysical scenarios or in the laboratory is presented here for ranges of photon and electron energies spanning twenty-two orders of magnitude, in order to illustrate the importance of this generalized effect. Examples include the generation of high-energy gamma photons (of order TeV) and electrons as observed in cosmic radiation, the experiments of photonuclear reactions with gamma ray photons with hundreds of MeV, and the conversion of laser photons to x-ray photons. The beams thus produced inherit certain properties of laser beams, such as high intensity and collimation and high degrees of monochromaticity and polarization.

Key Words: electrons, photons, Compton effect.

I. INTRODUCTION

Synchrotron radiation, or electron bremsstrahlung radiation, is produced abundantly in circular accelerators (synchrotrons) where electrons are kept accelerated to ultra-relativistic energies. These accelerators have been named "synchrotron light sources" and there are a number of them around the world in countries such as the USA, Japan, Germany, France, Brazil, Australia, etc., where they are in high demand for basic and applied research. Synchrotron radiation has outstanding properties for research in chemistry (ultrafast reactions), biology (cell structure), material science (structure of compounds), medicine (diagnostic and therapy treatments), etc.

Almost simultaneously and independently in 1922⁽¹⁾, Arthur H. Compton in the USA and Peter Debye in Germany studied the collision of electrons with photons, assuming that both behaved as particles. Due to the significant experimental effort of Compton,

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this effect bears only his name and he was awarded the Nobel Prize in Physics for its discovery in 1927.

For many years the Compton effect (CE) was considered to be the transfer of energy from a photon to an electron of lower energy. In this context, in 1929, Dumond(2) developed a theory to interpret the relation between the experimental broadening of Compton lines and the distribution of the electron's momenta in atoms, thereafter named "Compton profiles". M. Cooper(3), in England, revived interest in these studies in the sixties and numerous applications to condensed matter problems were made and are being carried out to this day by Pratt(14) and others.

In the early fifties, with the development of energy dispersive gamma ray photon detectors, the CE made it possible to explain some details of the gamma ray spectra emitted by radionuclides. In 1948, twenty six years after discovery of the CE, physicists proposed the inverse process, that is, energy transfer from an electron to a photon (of lower energy), in order to explain the existence of photons of extremely high energies in the primary cosmic radiation flux.(5), but it was in 1965 that the term "inverse Compton effect" (ICE) was coined(6). Another fifteen years lapsed before this effect was used to produce high energy gamma rays in high energy electron accelerators(7). It should be stressed that the gamma ray beam produced by ICE inherits the properties of high intensity, monochromaticity, collimation and polarization of the laser beam, properties which have enabled studies of nuclear structure with high detail, including studies that harness the nuclear CE(4) to probe for stable structures, such as alpha particles, within heavy nuclei(8).

During the last several years many laboratories around the world have started to build compact sources based on ICE, so called "table-top synchrotron radiation sources"(9). Improvements in linear accelerator technology, spurred by potential applications in cancer therapy and elsewhere, have resulted in higher currents, shorter pulses, greater stabilities and repetition rates of electron beams, which in conjunction with commercially available table-top terawatt (T^3) lasers, has made it possible to obtain x-ray beams with similar, or better, properties than is generally available from synchrotron radiation produced by electron synchrotrons operating at much higher energies.

Such sources could be adapted for addressing several critical applications. For example, intense x-ray beams with energies within 10 to 100 keV are important for diagnostic image quality and dose reduction, and would be applicable to a broad range of basic studies in applied physics. Gamma ray beams with energies around several MeV might be used for improved cancer therapy treatments. Finally, the use of these gamma beams in the treatment of radioactive wastes from nuclear power plants is being explored, since it has been observed that they accelerate the radioactive decay(15).

Nowadays, there are three alternatives to the conventional x-ray tube to generate x-rays with special characteristics: synchrotron radiation, free electron lasers (wigglers and undulators) and inverse Compton effect. All of them have significantly increased the use of x rays in all sorts of applications.

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II. COMPTON EFFECT GENERALIZATION.

Derivation of the general equation.

It is a rather simple exercise of relativistic kinematics to write down the equations of energy and momentum conservation for the collision of a photon and electron in the general case. The result of solving these equations is(7):

$$\frac{h\nu'}{h\nu} = \frac{1+\beta\cos\alpha}{1-\beta\cos\theta+\dfrac{h\nu\cdot[1+\cos(\alpha-\theta)]}{m_i c^2}} \qquad (1)$$

where hν is the energy of the incident photon , hν' is the energy of the scattered photon, β is the velocity of the electron in terms of the velocity of light, p_i and p_f are the initial and final linear momenta of the electron, θ is the scattering angle of the electron and α is the incident angle of the laser beam (see Fig. 1).

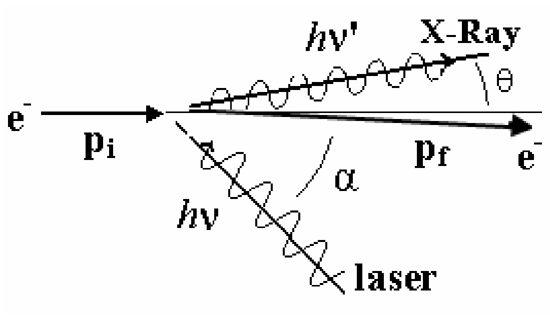

Figure 1. Schematic representation of the Generalized Compton effect.

When the incident electron is not relativistic (β→0), and instead of a laser photon it is an x-ray that collides with an electron, Eq.1 reduces to the familiar equation for the CE:

$$\frac{h\nu'}{h\nu} = \frac{1}{1+\dfrac{h\nu}{m_i c^2}(1-\cos\theta)}$$

Example 1. The photon beam, in Fig. 1, is incident from the right and the electron beam from the left collinearly. Then α = 0 and θ ≈ 0 (as we shall see) in Eq. 1. After the collision, we are only concerned with the photon beam which bounces to the right. If incident electrons have large kinetic energies such that $m_i c^2 >> h\nu$, the scattered photon energy will be:

$$h\nu' \approx (1+\beta)\, h\nu/(1-\beta) \approx 4\gamma^2 h\nu \qquad (2)$$

where $\gamma = (1-\beta^2)^{-1/2}$ and $\beta = v_e/c$.

Let us consider γ = 41 (E_e = 20MeV) and hν = 1.9 eV (red laser light), then hν' ≈ 13 keV. Currently there are available ultraviolet light lasers (hν > 5eV) which will allow one to obtain x-rays with energies over 30 keV. Note that in synchrotrons radiation sources, it is necessary to keep electrons continuously accelerated to 25 GeV to produce synchrotron radiation of energies of up to a few tens of keV.

By making use of the Klein-Nishina formula (see below), we can calculate that for $I_\nu = 10^7$ photons/s and $I_e = 10^{10}$ e^-/s, then $I_x = 10^7$ photons/s(11).

Example 2. The momenta of the incident electron and photon are orthogonal, that is, α = 90° and θ = 0°. Again, if the electron energy is large, equation 1 can be written as:

$$h\nu' \approx 2\gamma^2 h\nu,$$

which implies that the energy of the recoiling photon is a factor of two lower than in the first example (see equation 2). While this arrangement would be less efficient with respect to x-ray yield and collimation of the x-ray beam, the outgoing x-ray beam would not strike the laser apparatus.

The Klein-Nishina differential cross section.

We will mention briefly the collimation properties of the x-ray beam produced as a

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consequence of the collision of well collimated electron and laser photon beams. To consider the other important property of the x-ray beam, its degree of polarization, exceeds the scope of this note. The Klein-Nishina equation holds for both relativistic and non-relativistic electrons and allows one to calculate the scattering cross sections in terms of the angular arrangement of the beams in a Compton collision, the respective energies and the angular aperture of the beam (13).

If $E_e \gg m_0c^2$ the Klein-Nishina equation has the form:

$$\frac{d\sigma_{KN}}{d\Omega} = \frac{1}{2} r_0^2 \frac{(\nu')^2}{\nu^2}\left(\frac{\nu'}{\nu}+\frac{\nu}{\nu'}-\sin^2\theta\right)$$

where r_0 is the "classical radius" of the electron. Figure 2 shows calculated results of the angular distribution of the x-ray photon beam produced in a head-on collision of laser light photons with high energy electrons. The aperture is defined as the angle where the intensity of the x-ray beam drops to 50% of the intensity at 0°. The beam aperture is approximately given by $\theta \approx \gamma^{-2}$, as calculated by Chouffani(10), and in this range of energies it is a few milliradians, i.e., the beam has a diameter of one centimeter ten meters away from the collision region.

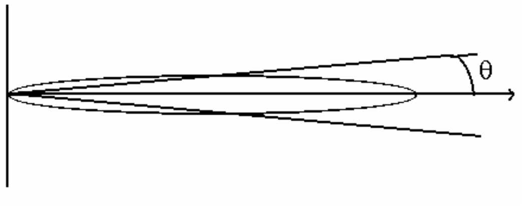

Figure 2. Calculated polar distribution of x-ray beam intensity emitted in the "forward direction", that is, along the incident electron beam propagation direction.

In Figure 3 there are depicted regions, spanning a range of 24 orders of magnitude in energy, where the Compton effect has its most striking consequences.

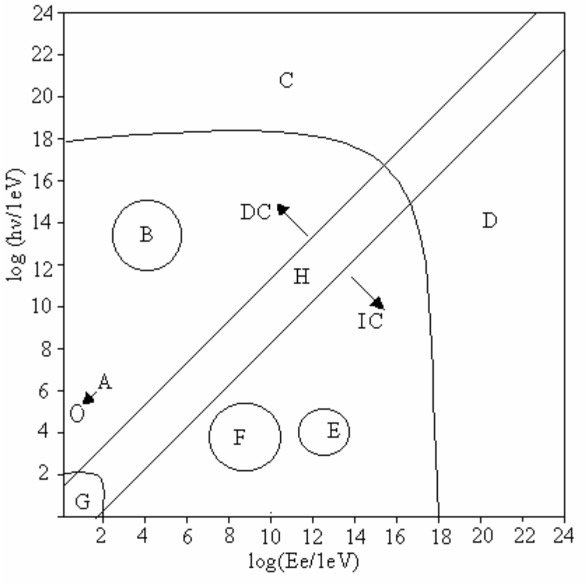

Figure 3. Main regions of interest of the generalized Compton effect. Vertical and horizontal axes indicate the scale of the incident energies of photon and electron, respectively.

In order to be complete, it would be necessary to draw this figure in 4-D, with axes representing initial and final energies of the electron and the photons. However, for purposes of illustration, we will consider only the initial energies of the electron and the photon, and in each different region we will refer to the characteristics of the outgoing particles.

Region "A" contains the range of electron and photon energies where A.H. Compton carried out his groundbreaking experiments.

Region "B" is where "nuclear Compton" effects have been and are currently performed, including Compton scattering with quarks inside nucleons.

Regions "C" and "D" are where astrophysical processes everywhere in the universe produce electrons and photons of the highest energies, and the generalized Compton effect is responsible for producing the highest observed energies for electrons and photons arriving to the earth.

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In regions "E" and "F" inverse Compton experiments of high and low energies take place. In zone "G" Compton collision effects are unobservable because $\lambda << \lambda_C$, where λ is the photon wavelength and λ_C is the Compton wavelength of the electron, given by $h/m_o c$.

Finally, along the diagonal "H", photons and electrons have roughly the same energy, so that energy transfer between them is negligible. The top arrow indicates the region of the direct Compton (DC) and the lower arrow the region where inverse Compton effect (ICE) are dominant.

III. CONCLUSIONS

Several textbooks and countless research articles treat the "direct" and "inverse Compton effects" as two separate effects, as if the physical phenomena involved were different. As equation 1 shows, it is one and only one effect with different outcomes depending on the relative energies of the particles. It must be noted that in Eq. 1 the photon is considered a particle and that electrons need to be treated with the relativistic kinematical expressions for conservation of energy and momentum. It is seen that a joint treatment has a greater teaching value and exposes the consequences and applications beyond the usual ones so frequent in radiation detection and measurement.

Finally it is worth mentioning that there are research efforts being made to produce the inverse Compton effect with an arrangement such that the outgoing x-ray beam does not impinge on the laser.

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